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Research Article

Exact Solutions of Rosenzweig-Macarthur (RM) Model Equations by Using Exp Function Method

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Abstract

He's exp function method aims to finding exact solutions of nonlinear evolution equations in mathematical physics. The exact solutions of the Rosenzweig-MacArthur (RM) model equations is obtain by using the exp-function method. The method is by transformation used to construct solitary and soliton solutions of nonlinear evolution equations. The free parameters in the obtained generalized solutions might imply some meaningful results in physical process.

Keywords: Rosenzweig-MacArthur (RM) Model, Exact solution, Exp-Function Method, Populations

1. Introduction

The Rosenzweig-MacArthur foodchain model (RM model) is widely used in population dynamics to modelize the predator-prey relationship. The Rosenzweig-MacArthur model can be written in following dimensionlesssystem of two differential equations,

$$\frac{dx_1}{dt} = x_1(1 - x_1 - \frac{z_1 x_2}{1 + b_1 x_1})$$

$$\frac{dx_2}{dt} = x_2\left(\frac{z_2 x_1}{1 + b_1 x_1} - z_3\right)$$

where z_1, z_2, z_3 and b_1 are constants. The exact solutions of the nonlinear evolution equations plays an important role in the study of nonlinear physical phenomena. In addition, the exact solutions of those has been used extensively as a benchmark model for testing various numerical solution methods. The nonlinear systems of equation emerge in various fields of scientific and engineering, such as solid physics, optical fibers, biology, chemical physics and fluid mechanics. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations [1]. In the last two decades, a vast variety of simple and direct methods to find analytic solutions of nonlinear differential equations (and systems) and evolution equations have been developed such as the tanh–sech method [2], extended tanh method [3], sine–cosine method [4], homogeneous balance method [5], F-expansion method [6], homotopy perturbation method [7], Exp-function method [8].

The main aim of this paper is to apply the Exp-function function method with the help of symbolic computation to obtain exact soliton solutions of RM model. By using Exp-function method, many kinds of nonlinear partial differential equations have been solved successfully.

2. The Exp-Function Method

The Exp-function method was first proposed by He and Wu [8] and was successfully used in the solution of many types of nonlinear partial differential equations.

We consider a general nonlinear PDE in the form

$$P\,(x, y, u\,, u_x, u_y,\, u_{xx},\, u_{xy}, u_{yy}, \dots) = 0 \tag{2.1}$$

Let us introduce a complex variable ξ, as follows

$$\xi = kx + wt$$

where k and w are constants, we can rewrite Eq. (2.1) in the following nonlinear ODE:

$$Q\,(\,u, u', u'', u''', \dots\,) = 0$$

where the prime denotes the derivation with respect to ξ . According to Exp-function method, we assume that the solution can be expressed in the form [8]

$$u\,(\xi) = \frac{\sum_{n=-c}^{d} a_n e^{(n\xi)}}{\sum_{m=-p}^{q} b_m e^{(m\xi)}} \tag{2.2}$$

where c, d, p and q are positive integer which could be freely chosen, a_n and b_m are unknown constants to be determined. For simplicity, we set $d = q = c = p = 1$ then Eq. (2.2) can be writen as,

$$u\,(\xi) = \frac{a_{-1}e^{-\xi} + a_0 + a_1 e^{\xi}}{b_{-1}e^{-\xi} + b_0 + e^{\xi}} \tag{2.3}$$

3. Exact Solution of Rosenzweig-Macarthur (RM) Model

The dimensionless RM equations are given as following,

$$\frac{dx_1}{dt} = x_1\left(1 - x_1 - \frac{z_1 x_2}{1+b_1 x_1}\right) \tag{3.1}$$

$$\frac{dx_2}{dt} = x_2\left(\frac{z_2 x_1}{1 + b_1 x_1} - z_3\right) \tag{3.2}$$

In view of the exp function method, we assume that the solution of Eq. (3.1)

$$x_1 = \frac{a_{-1}e^{-x} + a_0 + a_1e^{x}}{b_{-1}e^{-x} + b_0 + e^{x}} \tag{3.3}$$

$$x_2 = \frac{h_{-1}e^{-x} + h_0 + h_1e^{x}}{g_{-1}e^{-x} + g_0 + e^{x}} \tag{3.4}$$

a_n , b_m , h_n and g_m are unknown constants. By simple calculation, we have

$$(1 + b_1x_1)x_1' - x_1\big((1 - x_1)(1 + b_1x_1) - z_1x_2\big) = 0 \tag{3.5}$$

$$(1 + b_1x_1)x_2' - x_2(z_2x_1 - z_3(1 + b_1x_1)) = 0 \tag{3.6}$$

Substituting Eq. (3.3) and (3.4) into Eq. (3.1) and (3.2), and by the help of Maple14, we have

$$\frac{-1}{A}[C_3e^{3x} + C_2e^{2x} + C_1e^{x} + C_0 + C_{-1}e^{-x} + C_{-2}e^{-2x} + C_{-3}e^{-3x}] = 0 \tag{3.7}$$

$$\frac{-1}{B}[D_3e^{3x} + D_2e^{2x} + D_1e^{x} + D_0 + D_{-1}e^{-x} + D_{-2}e^{-2x} + D_{-3}e^{-3x}] = 0 \tag{3.8}$$

and for the solutions of Eq. (3.7) and (3.8) all coefficients must be zero.

$C_3 = c_1a_1h_1 - a_1 - a_1c_2h_1 + a_1^2 + a_1^2c_2h_1$

$C_2 = c_1a_0h_1 - a_0c_2h_1 + 2a_0a_1 - a_1c_2h_0 - a_1b_0 + wa_1b_0 + c_1a_1h_1b_0$

$-a_1b_0c_2h_1 + a_1^2c_2h_0 + c_1a_1h_0 - a_0 + wa_1b_0c_2h_1 - wa_0 + a_1^2g_0 - a_1g_0$

$+2a_0a_1c_2h_1 - wa_0c_2h_1$

$C_1 = 2wa_1b_{-1} + 2a_0a_1c_2h_0 - wa_0g_0 + a_1b_{-1}c_2h_1 + a_0^2c_2h_1 + a_1^2g_{-1}$

$+a_1b_0g_0 + wa_1b_0c_2h_0 + a_0^2 + c_1a_1h_{-1} + c_1a_1h_0b_0 - a_0c_2h_0 +$

$2a_1a_{-1} - a_0b_0c_2h_1 + c_1a_1h_1b_{-1} + wa_1b_0g_0 + c_1a_0h_0 - a_1b_{-1} - a_0g_0$

$+2a_1a_{-1}c_2h_1 + 2a_0a_1g_0 - 2wa_{-1} - a_1h_0b_0c_2 - a_{-1} + a_1^2c_2h_{-1} - 2wa_{-1}c_2h_1$

$-a_0b_0 - a_{-1}c_2h_1 - a_1c_2h_{-1} + 2wa_1b_{-1}c_2h_1 + c_1a_{-1}h_1 - a_{-1}g_{-1} + c_1a_0h_1b_0 - wa_0c_2h_0$

$C_0 = -a_{-1}b_0c_2h_1 + 2wa_1b_{-1}g_0 - a_{-1}b_0 + wa_1b_0g_{-1} + wa_0b_{-1}$

$-a_1b_0c_2h_{-1} + c_1a_0h_1b_{-1} - a_0b_0g_0 + wa_0b_{-1}c_2h_1 + 2a_0a_1g_{-1} - a_0b_{-1}$

$-2wa_{-1}g_0 - wa_{-1}b_0c_2h_1 - a_1b_{-1}g_0 + c_1a_0h_0b_0 - a_{-1}g_0 + 2a_{-1}a_0c_2h_1$

$+a_0^2c_2h_0 + 2a_{-1}a_1c_2h_0 - a_0c_2h_{-1} - 2wa_{-1}c_2h_0 + c_1a_0h_{-1} - a_0h_0b_0c_2$

$-wa_{-1}b_0 + 2a_{-1}a_1g_0 - wa_0g_{-1} + wa_1b_0c_2h_{-1} - a_0g_{-1} + c_1a_{-1}h_0$

$+c_1a_1h_0b_{-1} + c_1a_1h_{-1}b_0 + 2wa_1b_{-1}c_2h_0 + c_1a_{-1}h_1b_0 - a_1b_{-1}c_2h_0$

$+2a_0a_1c_2h_{-1} - a_1b_0g_{-1} - a_0b_{-1}c_2h_1 + a_0^2g_0 - wa_0c_2h_{-1} - 2a_{-1}a_0 - a_{-1}c_2h_0$

$C_{-1} = -a_{-1}b_0g_0 + 2a_{-1}a_1g_{-1} + c_1a_{-1}h_{-1} + a_0^2c_2h_{-1} + 2a_{-1}a_0g_0$

$-a_1b_{-1}c_2h_{-1} + wa_0b_{-1}c_2h_0 - wa_{-1}b_0c_2h_0 - a_0b_{-1}c_2h_0 - a_{-1}g_{-1}$

$+wa_0b_{-1}g_0 - a_0b_0g_{-1} - a_0b_0c_2h_{-1} + 2a_{-1}a_0c_2h_0 - a_{-1}b_{-1}c_2h_1 - a_0b_{-1}g_0$

$-a_{-1}b_0c_2h_0 - a_{-1}c_2h_{-1} + c_1a_0h_0b_{-1} + c_1a_0h_{-1}b_0 - a_{-1}b_{-1} - wa_{-1}b_0g_0$

$+2wa_1b_{-1}g_{-1} + a_{-1}^2c_2h_1 + a_{-1}^2 + a_0^2g_{-1} + 2wa_1b_{-1}c_2h_{-1} + c_1a_1h_{-1}b_{-1}$

$-2wa_{-1}c_2h_{-1} - a_1b_{-1}g_{-1} + 2a_{-1}a_1c_2h_{-1} - 2wa_{-1}g_{-1} + c_1a_{-1}h_1b_{-1}$

$C_{-2} = c_1a_0h_{-1}b_{-1} + wa_0b_{-1}c_2h_{-1} - a_{-1}b_{-1}g_0 - wa_{-1}b_0c_2h_{-1} - wa_{-1}b_0g_{-1}$

$+2a_{-1}a_0c_2h_{-1} + a_{-1}^2g_0 - a_{-1}b_0g_{-1} + c_1a_{-1}h_{-1}b_0 - a_0b_{-1}c_2h_{-1} - a_0b_{-1}g_{-1} + a_{-1}^2c_2h_0$

$-a_{-1}b_0c_2h_{-1} - a_{-1}b_{-1}c_2h_0 + 2a_{-1}a_0g_{-1} + wa_0b_{-1}g_{-1} + c_1a_{-1}h_0b_{-1}$

$C_{-3} = a_{-1}^2c_2h_{-1} - a_{-1}b_{-1}c_2h_{-1} - a_{-1}b_{-1}g_{-1} + a_{-1}^2g_{-1} + c_1a_{-1}h_{-1}b_{-1}$

$D_3 = -c_2c_3a_1h_1 + d_1h_1 + d_1h_1c_4a_1$

$D_2 = d_1h_1g_0c_4a_1 + d_1h_1g_0 - c_2c_3a_1h_0 + wh_1g_0c_4a_1 + d_1h_1b_0 - c_2c_3a_1h_1g_0 + d_1h_0$

$-wh_0 + d_1h_0c_4a_1 - wh_0c_4a_1 + d_1h_1c_4a_0 - c_2c_3a_0h_1 + wh_1g_0$

$D_1 = -c_2c_3a_0h_0 + d_1h_{-1} + d_1h_0g_0c_4a_1 + d_1h_{-1}c_4a_1 + d_1h_1g_0b_0 - c_2c_3a_1h_0g_0$

$+d_1h_1g_0c_4a_0 - wh_0b_0 + 2wh_1g_{-1} - c_2c_3a_0h_1g_0 + wh_1g_0b_0 + d_1h_1c_4a_{-1} + wh_1g_0c_4a_0$

$+d_1h_1g_{-1} - c_2c_3a_{-1}h_1 - 2wh_{-1} + d_1h_0c_4a_0 + 2wh_1g_{-1}c_4a_1 + d_1h_0b_0 + d_1h_1g_{-1}c_4a_1$

$-c_2c_3a_1h_{-1} - wh_0c_4a_0 - 2wh_{-1}c_4a_1 + d_1h_1b_{-1} - c_2c_3a_1h_1g_{-1} + d_1h_0g_0$

$D_0 = -2wh_{-1}c_4a_0 + d_1h_1g_{-1}c_4a_0 + d_1h_{-1}c_4a_0 - c_2c_3a_0h_1g_{-1} + d_1h_1g_0b_{-1} - c_2c_3a_{-1}h_0$

$+d_1h_{-1}g_0 + d_1h_1g_0c_4a_{-1} + d_1h_{-1}b_0 - wh_0c_4a_{-1} - wh_{-1}g_0 - c_2c_3a_0h_{-1} - c_2c_3a_0h_0g_0$

$-c_2c_3a_1h_0g_{-1} + d_1h_0c_4a_{-1} + d_1h_0g_{-1}c_4a_1 + wh_1g_0b_{-1} + d_1h_0g_0b_0 + d_1h_{-1}g_0c_4a_1$

$+d_1h_1g_{-1}b_0 - wh_{-1}g_0c_4a_1 + 2wh_1g_{-1}b_0 + d_1h_0g_{-1} + 2wh_1g_{-1}c_4a_0 + wh_0g_{-1}$

$+wh_1g_0c_4a_{-1} + wh_0g_{-1}c_4a_1 - c_2c_3a_1h_{-1}g_0 - c_2c_3a_{-1}h_1g_0 + d_1h_0g_0c_4a_0 + d_1h_0b_{-1}$

$-2wh_{-1}b_0 - wh_0b_{-1}$

$D_{-1} = d_1h_{-1}b_{-1} - 2wh_{-1}b_{-1} - c_2c_3a_{-1}h_0g_0 + d_1h_{-1}g_{-1}c_4a_1 + d_1h_{-1}c_4a_{-1}$

$-c_2c_3a_1h_{-1}g_{-1} + 2wh_1g_{-1}b_{-1} - c_2c_3a_{-1}h_1g_{-1} - wh_{-1}b_0g_0 - c_2c_3a_{-1}h_{-1}$

$+d_1h_0g_0c_4a_{-1} - wh_{-1}g_0c_4a_0 + d_1h_1g_{-1}b_{-1} + d_1h_{-1}g_{-1} - 2wh_{-1}c_4a_{-1} + wh_0g_{-1}c_4a_0$

$+d_1h_0g_{-1}c_4a_0 + wh_0g_{-1}b_0 + d_1h_0g_0b_{-1} + d_1h_1g_{-1}c_4a_{-1} + d_1h_{-1}g_0c_4a_0 - c_2c_3a_0h_{-1}g_0$

$+d_1h_{-1}g_0b_0 + 2wh_1g_{-1}c_4a_{-1} + d_1h_0g_{-1}b_0 - c_2c_3a_0h_0g_{-1}$

$D_{-2} = d_1h_{-1}g_{-1}b_0 + d_1h_{-1}g_{-1}c_4a_0 + d_1h_0g_{-1}b_{-1} + d_1h_{-1}g_0c_4a_{-1} + d_1h_{-1}g_0b_{-1}$

$+d_1h_0g_{-1}c_4a_{-1} + wh_0g_{-1}c_4a_{-1} - wh_{-1}g_0b_{-1} - c_2c_3a_{-1}h_0g_{-1} + wh_0g_{-1}b_{-1}$

$-wh_{-1}g_0c_4a_{-1} - c_2c_3a_{-1}h_{-1}g_0 - c_2c_3a_0h_{-1}g_{-1}$

$D_{-3} = -c_2c_3a_{-1}h_{-1}g_{-1} + d_1h_{-1}g_{-1}b_{-1} + d_1h_{-1}g_{-1}c_4a_{-1}$

Equating all the coefficients of e^{nx} to zero, we obtain a system of algebraic equations which can be solved by the Maple, to obtain the following cases of solutions

<u>Case 1:</u> $h_{-1} = 0, a_{-1} = 0,\ z_3 = \frac{b_1a_1 - a_1 - a_1{}^2b_1 + 1}{h_1}, g_0 = 0, b_{-1} = 0,$

$$z_2 = \frac{z_3\,(b_1a_1 + 1)}{a_1}, g_{-1} = 0, h_0 = 0, b_0 = 0, a_0 = 0$$

Substituting these results into (3.1) and (3.2), the following exact solution will be derived,

$$x_1 = \frac{e^t}{\left(\frac{g_0^2 b_1 - 2g_{-1} + g_0^2 - 2b_1 g_{-1}}{g_0}\right) + e^t}$$

$$x_2 = \frac{h_1 e^t}{g_{-1}e^{-t} + g_0 + e^t}$$

where b_1, g_{-1} and g_0 are free parameters.

<u>Case 2:</u> $h_{-1} = 0, a_{-1} = 0, z_3 = -1 , b_{-1} = 0, b_0 = g_0 , z_2 = \frac{-1}{a_1}$

$$z_2 = \frac{z_3(b_1 a_1 + 1)}{a_1}, g_{-1} = 0, h_0 = 0, \quad a_0 = 0 , z_1 = \frac{1-a_1}{h_1}, b_1 = 0$$

Substituting these results into (3.1) and (3.2), we obtain the following exact solution,

$$x_1 = \frac{a_1 e^t}{g_0 + e^t}$$

$$x_2 = \frac{h_1 e^t}{g_0 + e^t}$$

where a_1,h_1 and g_0 are free parameters.

<u>Case 3:</u> $g_{-1} = 0, a_{-1} = 0, h_1 = \frac{h_{-1} z_2}{b_0 g_0}, a_0 = 0 , z_3 = 1 , b_1 = z_2 - 1$

$$b_{-1} = 0, a_1 = 1, h_0 = \frac{h_{-1}(b_0 + z_2 g_0)}{b_0 g_0}, z_1 = 0$$

Substituting these results into (3.1) and (3.2), the following exact solution will be derived,

$$x_1 = \frac{e^t}{b_0 + e^t}$$

$$x_2 = \frac{h_{-1}(b_0 g_0 e^{-t} + b_0 + z_2 g_0 + z_2 e^t)}{b_0 g_0 (g_0 + e^t)}$$

where b_0, h_{-1}, z_2 and g_0 are free parameters.

<u>Case 4:</u> $g_{-1} = 0, a_{-1} = 0, h_0 = \frac{-h_{-1} z_2}{a_0}, b_{-1} = -a_0^2, b_0 = 0, z_3 = 2$

$$a_1 = 1, b_1 = \frac{1}{2} z_2 - 1, g_0 = 0, z_1 = 0, h_1 = \frac{1}{4}\frac{h_{-1} z_2^2}{a_0^2}$$

Substituting these results into (3.1) and (3.2), the following exact solution will be derived,

$$x_1 = \frac{a_0 + e^t}{-a_0^2 e^{-t} + e^t}$$

$$x_2 = \frac{h_{-1}(4a_0^2 e^{-t} - 4a_0 z_2 + z_2^2 e^t)}{a_0^2 e^t}$$

where a_0, h_{-1} and z_2 are free parameters.

4. Conclusions

In this article, we have been looking for the exact solution of the nonlinear Rosenzweig–MacArthursystem. The free parameters can be determined using any relevant to initial or boundary conditions. The obtained results show that the Exp-function method is an effective and powerful mathematical tool for solving nonlinear dynamical systems in mathematical physics.

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