Yayın Geliş Tarihi (Submitted): 29/04/2019
Yayın Kabul Tarihi (Accepted): 20/06/2019
***Makele Türü (Paper Type):* Araştırma Makalesi – Research Paper**

ÜÇGEN SEZGİSEL BULANIK SAYILAR İÇİN GERGONNE NOKTASINA DAYALI YENİ BİR SIRALAMA YÖNTEMİ

Gültekin ATALIK[1] ve Sevil SENTURK[2]

ÖZET

Bulanık küme teorisi, araştırmacıların ölçüm hatası, belirsizlik ve insan düşüncelerinden kaynaklanan belirsizlikleri tespit etmelerini sağlar. Bulanık küme teorisi birçok araştırmacı tarafından pek çok farklı türe genişletilmiştir. Sezgisel bulanık kümeler, bu türlerden biridir. Sezgisel bulanık kümelerde iki fonksiyon vardır. Bunlar üyelik fonksiyonu ve üye olmama fonksiyonlarıdır. Sezgisel bulanık sayıların sıralanması birçok gerçek yaşam probleminin modellenmesinde temel bir rol oynamaktadır. Literatürde, sezgisel bulanık sayıları sıralamak için çeşitli yöntemler pek çok araştırmacı tarafından önerilmiştir. Üçgenin iç teğet çemberinin kenarlara değme noktalarını karşı köşe noktalarıyla birleştiren doğru parçalarının kesişim noktası, Gergonne noktasıdır. Bu çalışmada, üçgen sezgisel bulanık sayıyı sıralamak için Gergonne noktasına dayanan yeni bir yöntem önerilmiştir. Önerilen yöntemi diğer yöntemlerle karşılaştırmak için farklı üçgen sezgisel bulanık sayılar kullanılarak bir çalışma yapılmıştır. Elde edilen sonuçlar yorumlanmıştır.

Anahtar Kelimeler: Sezgisel bulanık kümeler, Gergonne noktası, bulanık sayıyı sıralamak, üçgen sezgisel bulanık sayılar.

[1] Sorumlu Yazar, Araş. Gör., İstatistik Bölümü, Fen Fakültesi, Eskişehir Teknik Üniversitesi, Eskişehir, Türkiye, ORCID: https://orcid.org/0000-0002-8795-539X
[2] Doç. Dr., İstatistik Bölümü, Fen Fakültesi, Eskişehir Teknik Üniversitesi, Eskişehir, Türkiye, ORCID:https://orcid.org/0000-0002-9503-7388

A NEW RANKING METHOD FOR TRIANGULAR INTUITIONISTIC FUZZY NUMBER BASED ON GERGONNE POINT

ABSTRACT

Fuzzy sets theory allows researchers to identify the uncertainties that arise from measurement error, vagueness and human thoughts. Fuzzy sets theory has been extended into various different types by many researchers. Intuitionistic fuzzy sets are one of these types. There are two functions in intuitionistic fuzzy sets. These are membership function and non - membership function. The ranking of intuitionistic fuzzy numbers plays the main role in modeling many real life problems. Several methods for ranking intuitionistic fuzzy numbers have been well discussed in the literature. In a triangle, the lines from the vertices to the points of contact of the opposite sides of the inscribed circle meet at a point. That point is the Gergonne point. In this paper, a new method based on the Gergonne point is proposed to rank triangular intuitionistic fuzzy numbers. An illustrative example and comparison study is performed with the existing methods by using different triangular intuitionistic fuzzy numbers. The results are interpreted as a conclusion.

Keywords: Intuitionistic fuzzy sets, Gergonne point, ranking fuzzy number, triangular intuitionistic fuzzy numbers

1. INTRODUCTION

Fuzzy sets, an extension of classical sets were proposed by Zadeh in 1965. Uncertainty can occur from lack of knowledge, from chance, from ignorance, from measurement errors, etc. Fuzzy sets are powerful tools for modelling the uncertainties that are encountered in daily life. Fuzzy sets theory includes many different types. These are interval – valued fuzzy sets, type 2 fuzzy sets, hesitant fuzzy sets, neutrosophic fuzzy sets, pythagorean fuzzy sets and intuitionistic fuzzy sets.

Ranking of a fuzzy number is an important and difficult task in fuzzy set theory. Ranking of two fuzzy numbers proves that one is larger or smaller than the other. Many fuzzy applications such as decision – making, hypothesis testing, forecasting and risk analysis need to rank fuzzy numbers. Therefore, it is crucial to rank the fuzzy numbers correctly.

Intuitionistic fuzzy sets (IFS), consider both membership and non - membership function, are an extension of ordinary fuzzy sets. As an extension of an ordinary fuzz number, intuitionistic fuzzy number (IFN) appears to suit more for modelling uncertainty. Many ranking methods for intuitionistic fuzzy numbers were proposed by many researchers in the literature.

Grzegorzewski (2003) proposed a ranking method based on a metric for intuitionistic fuzzy numbers. Mithcell (2004) developed a ranking method for triangular intuitionistic fuzzy numbers with a statistical point of view. Nayagam et. al. (2008) introduced a ranking method for triangular intuitionistic fuzzy number (TIFN) based on Chen and Hwang's (1992) method. Wang and Zhang (2009) gave definitions of the expected value, score function and accuracy function of trapezoidal intuitionistic fuzzy numbers and proposed a ranking method based on score and accuracy function. Li (2010a) gave definition of the values and ambiguities of the membership and non – membership functions and proposed a ranking method based on these values. Li et al. (2010b) proposed a ranking method for triangular intuitionistic fuzzy numbers based on the value and ambiguity indexes. Nehi (2010) introduced a ranking method for trapezoidal intuitionistic fuzzy numbers based on the characteristic value of an intuitionistic fuzzy number. Wei and Tang (2010) introduced a ranking method by using a possibility degree method for intuitionistic fuzzy numbers and used this method to rank alternatives in multi-criteria decision making problems (MCDM). Dubey and Mehra introduced an approach for triangular intuitionistic fuzzy number based on Li's(2010) method and used it to solve linear programming problems. Nayagam et. al (2011) introduced a ranking method for intuitionistic fuzzy numbers based on the score function and applied the proposed method to clustering problems. Salahshour et al.(2012) converted each triangular intuitionistic fuzzy number to related two triangular fuzzy numbers and introduced a ranking method based on this transformation. Seikh et. al. (2012) introduced a ranking method based on a ranking index to find out the relation between two generalized triangular intuitionistic fuzzy number. Nagoorgani and Ponnalagu (2012) defined a division operator for triangular intuitionistic fuzzy numbers based on α,β - cut and introduced a ranking method by using the score and accuracy functions. They used the proposed method to solve intuitionistic fuzzy linear programming problem. Das and Guha(2013) introduced a new ranking method by using the centroid point of an intuitionistic fuzzy number and compared the proposed method by giving some numerical examples. Kumar and Kaur (2013) introduced a new ranking method by modifying Nehi's (2010) method and showed the limitations of the existing ranking methods. Rezvani (2013) introduced a ranking method for trapezoidal intuitionistic fuzzy numbers based on value and ambiguity indexes. Roseline and Amirtharaj (2013) defined the magnitude of the membership function and non – membership function for trapezoidal intuitionistic fuzzy numbers and proposed a ranking method based on the magnitude value. Peng and Chen (2013) gave the definition of a center index and radius index for canonical intuitionistic fuzzy numbers and developed a new ranking method based on a ranking index. Also, they gave some examples to show the validity of the proposed method. Zhang and Nan (2013) proposed a ranking method for triangular intuitionistic fuzzy number and applied it to multi-attribute decision making (MADM) problems. Seikh et. al. (2013) gave detailed information about triangular intuitionistic fuzzy numbers and suggested a ranking method based on an average ranking index. Prakash et. al. (2016) introduced a ranking method for both triangular intuitionistic fuzzy numbers and trapezoidal intuitionistic fuzzy numbers based on a centroid concept and gave some examples to demonstrate the effectiveness of the proposed method. Bharati (2017) proposed a new ranking method based on fuzzy origin and signed distance for triangular intuitionistic fuzzy numbers and showed the validity of its axioms. Garg (2017) introduced a new improved score function for intuitionistic multiplicative set and used it to rank the alternatives in MDM problems. Nayagam et. al (2017) defined eight different scores for the class of trapezoidal intuitionistic fuzzy numbers and used them to rank trapezoidal fuzzy numbers. They discussed the significance of the proposed method. Tao et. al (2017) presented a new ranking method based on the intuitionistic fuzzy possibility degree for interval – valued fuzzy numbers. Uthra et. al. (2017) gave the definition of generalized intuitionistic pentagonal fuzzy number and proposed a new ranking method. They gave some illustrative examples. Garg and Kumar (2018) proposed a ranking method based on an improved possibility degree method for intuitionistic fuzzy numbers and used this method to solve MADM problems. Hao and Chen (2018) defined the maximum, minimum and ranking function for interval –valued intuitionistic fuzzy numbers and introduced a new ranking method. The proposed method was used to solve MADM problems with interval – valued intuitionistic fuzzy numbers. Uthra et. al. (2018) defined generalized intuitionistic hexagonal, octagonal and pentagonal fuzzy numbers and proposed a new ranking method. Xing et. al. (2018) introduced a ranking method based on Euclidean distance for intuitionistic fuzzy numbers and generalized the proposed method by using Minkowski distance.

In this paper, a new ranking method based on the Gergonne point for triangular intuitionistic fuzzy numbers is proposed for the first time. As it can be seen from the existing papers in the literature, the existing methods assign a real number to a fuzzy number to rank them. The proposed method associates an intuitionistic fuzzy number with triplets and ranks them in lexicographical order.

The remainder of the paper is organized as follows. Section I gives a brief definition of the intuitionistic fuzzy sets and intuitionistic fuzzy numbers. The theory of the proposed method is explained in section 2. Section 3 gives an illustrative example and a comparative study. Some conclusions are given in the last section.

2. INTUITONISTIC FUZZY SETS

In this section, the basic definition of intuitionistic fuzzy sets and triangular intuitionistic fuzzy numbers are given.

Intuitionistic fuzzy sets were introduced by Atanassov (1986). Intuitionistic fuzzy sets are a generalization of Zadeh's (1965) ordinary fuzzy set. Membership function and non – membership function characterize the intuitionistic fuzzy set. The definition of the intuitionistic fuzzy set is given as follows.

Definition 1: Let $X \neq \emptyset$ be a given set. An intuitionistic fuzzy set in $\tilde{A}$ over X is an object having the form

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle | x \in X\} \tag{1}$$

Where, $\mu_{\tilde{A}}(x): X \to [0,1]$ is the degree of the membership and $v_{\tilde{A}}(x): X \to [0,1]$ is the degree of non – membership with the condition $0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1,\ \forall\ x \in X$. For each element of x, "the hesitancy degree" of an intuitionistic fuzzy set of $x \in X$ in $\tilde{A}$ computed as follows:

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - v_{\tilde{A}}(x) \tag{2}$$

Definition 2: An intuitionistic fuzzy set $\tilde{A}$ of the real line is called intuitionistic fuzzy number if;

i. $\tilde{A}$ is intuitionistic fuzzy – normal i.e. there exists at least two points $x_0, x_1 \in X$ such that $\mu_{\tilde{A}}(x_0) = 1$ and $v_{\tilde{A}}(x_1) = 1$,

ii. $\tilde{A}$ is intuitionistic fuzzy – convex i.e. its membership function μ is a fuzzy convex and its non – membership function v is a fuzzy concave,

iii. $\mu_{\tilde{A}}$ is upper semicontinuous and $v_{\tilde{A}}$ is lower semicontinuous,

iv. $upp\ \tilde{A} = \{x \epsilon X | v_{\tilde{A}}(x) < 1\}$ is bounded. (Kahraman et. al.,2017)

Definition 3: For any intuitionistic fuzzy number $\tilde{A}$ there exist four functions $f_A, g_A, h_A, k_A: R \rightarrow [0,1]$ called, the sides of a fuzzy number, where f_A and k_A are nondecreasing and g_A and h_A are nonincreasing. The membership and non – membership function of $\tilde{A}$ are given as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & if \quad x < a_1 \\ f_A(x) & if \quad a_1 \leq x < a_2 \\ 1 & if \quad a_2 \leq x < a_3 \\ g_A(x) & if \quad a_3 \leq x < a_4 \\ 0 & if \quad a_4 \leq x \end{cases} \tag{3}$$

$$v_{\tilde{A}}(x) = \begin{cases} 0 & if \quad x < a'_1 \\ h_A(x) & if \quad a'_1 \leq x < a'_2 \\ 1 & if \quad a'_2 \leq x < a'_3 \\ k_A(x) & if \quad a'_3 \leq x < a'_4 \\ 0 & if \quad a'_4 \leq x \end{cases} \tag{4}$$

Definition 4: A triangular intuitionistic fuzzy number $\tilde{A}$ is a subset of IFS in R with following membership function and non – membership function as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & if \quad a_1 \leq x < a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & if \quad a_2 \leq x < a_3 \\ 0 & , \quad otherwise \end{cases} \tag{5}$$

$$v_{\tilde{A}}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a'_1} & if \quad a'_1 \leq x < a_2 \\ \dfrac{x - a_2}{a'_3 - a_2} & if \quad a_2 \leq x < a'_3 \\ 1 & , \quad otherwise \end{cases} \tag{6}$$

where, $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$, $0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1$ and TIFN is denoted by $\tilde{A}_{TIFN} = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ as in figure 1. (Nehi and Maleki,2005).

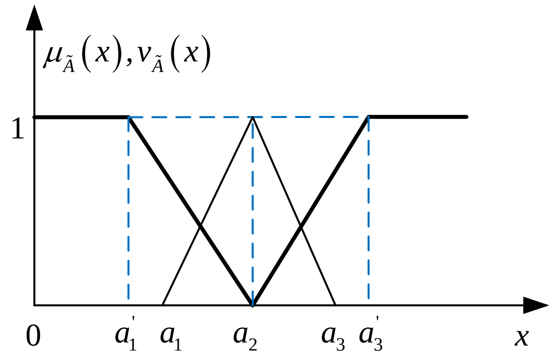

Figure 1. Membership and non – membership function of TIFN

Kahraman et. al. (2017) modified the demonstration of TIFN. The new demonstration of the membership and non – membership function of TIFN is shown in figure 2.

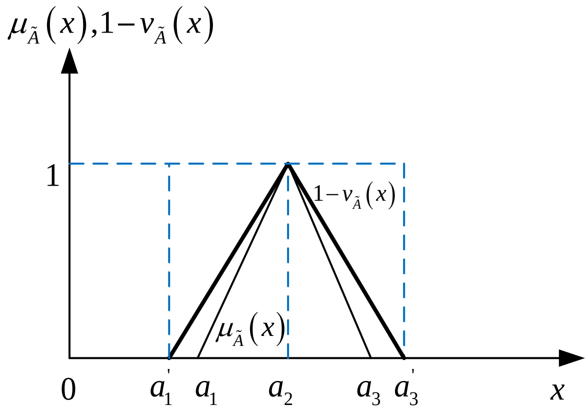

Figure 2. New demonstration of TIFN (Kahraman et. al, (2017)).

3. THE THEORY OF PROPOSED METHOD

In a triangle, the lines from the vertices to the points of contact of the opposite sides with the inscribed circle meet at a point called the Gergonne Point. The Gergonne point is well – known as the center of a triangle. Akyar and Akyar (2016) introduced a ranking method based on the Gergonne point for triangular fuzzy numbers. In this paper, a new ranking method for TIFN is proposed by using Kahraman et. al's (2017) demonstration and Akyar and Akyar's (2016) method .

There are two triangles shown in figure 3 for TIFN with the help of Kahraman et. al's demonstration. These are; one for the membership function $\mu_{\tilde{A}}(x)$ and one for non – membership function $v_{\tilde{A}}(x)$ shown as $\triangle DBE$ and $\triangle ABC$ respectively. Therefore, for a given

TIFN $\tilde{A}_{TIFN} = (a_1, a_2, a_3; a'_1, a_2, a'_3)$, a new lexicographic ranking based on the Gergonne point namely $Rank_{\mathcal{G}}(\tilde{A})$ is calculated as follows.

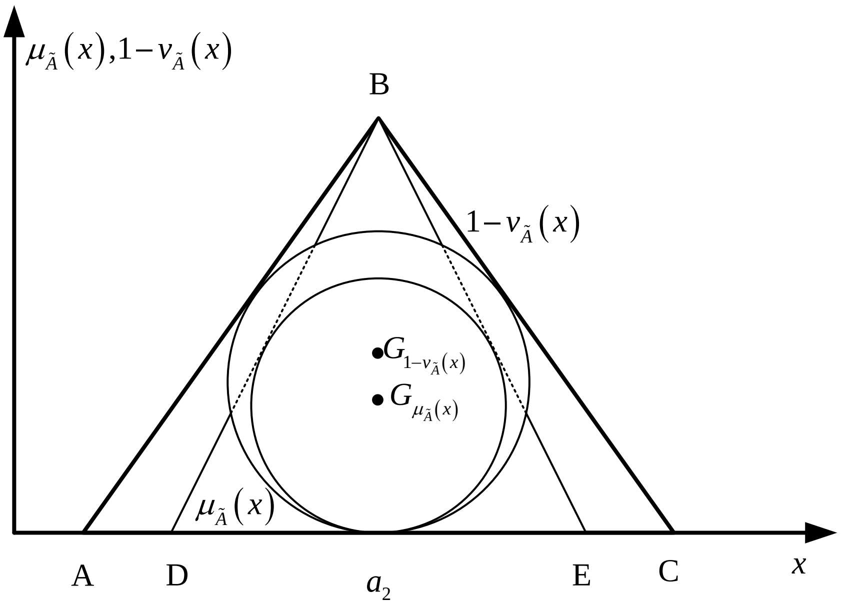

Figure 3. Gergonne points and triangles of TIFN

Let be $a = |BC|$, $b = |CA|$, $c = |BA|$, the trilinear coordinates of Gergonne point for triangle ABC is defined in Eq.(7).

$$\frac{bc}{b+c-a} : \frac{ca}{c+a-b} : \frac{ba}{a+b-c} \tag{7}$$

Let $\tilde{A}_{TIFN} = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ be a given TIFN and then we get $= |BC| = \sqrt{(a_2 - a'_3)^2 + 1}$, $b = |CA| = a'_3 - a'_1$, $c = |BA| = \sqrt{(a_2 - a'_1)^2 + 1}$, $d = |BE| = \sqrt{(a_2 - a_3)^2 + 1}$, $b = |DE| = a_3 - a_1$ and $e = |BD| = \sqrt{(a_2 - a_1)^2 + 1}$ as shown.

The trilinear coordinates $(\alpha:\beta:\gamma)_{1-v_{\tilde{A}}}$ of Gergonne point $G_{1-v_{\tilde{A}}}$ for triangle ABC is calculated in the following.

$$\alpha_{1-v_{\tilde{A}}} = \frac{(a'_3 - a'_1)\sqrt{(a_2 - a'_1)^2 + 1}}{(a'_3 - a'_1) + \sqrt{(a_2 - a'_1)^2 + 1} - \sqrt{(a_2 - a'_3)^2 + 1}} \tag{8}$$

$$\beta_{1-v_{\tilde{A}}} = \frac{\sqrt{(a_2 - a'_1)^2 + 1}\,\sqrt{(a_2 - a'_3)^2 + 1}}{(a'_1 - a'_3) + \sqrt{(a_2 - a'_1)^2 + 1} + \sqrt{(a_2 - a'_3)^2 + 1}} \tag{9}$$

$$\gamma_{1-v_{\tilde{A}}}=\frac{(a_3'-a_1')\sqrt{(a_2-a_3')^2+1}}{(a_3'-a_1')+\sqrt{(a_2-a_3')^2+1}-\sqrt{(a_2-a_1')^2+1}} \tag{10}$$

Thus, the Cartesian coordinates corresponding to $G_{1-v_{\tilde{A}}}$ is defined in Eq. (11).

$$G_{1-v_{\tilde{A}}}=\left(\frac{\alpha_{1-v_{\tilde{A}}}aa_1'+\beta_{1-v_{\tilde{A}}}ba_2+\gamma_{1-v_{\tilde{A}}}ca_3'}{\alpha_{1-v_{\tilde{A}}}a+\beta_{1-v_{\tilde{A}}}b+\gamma_{1-v_{\tilde{A}}}c},\frac{\beta_{1-v_{\tilde{A}}}b}{\alpha_{1-v_{\tilde{A}}}a+\beta_{1-v_{\tilde{A}}}b+\gamma_{1-v_{\tilde{A}}}c}\right) \tag{11}$$

The trilinear coordinates $(\alpha:\beta:\gamma)_{\mu_{\tilde{A}}}$ of Gergonne point $G_{\mu_{\tilde{A}}}$ for triangle DBE is calculated as follows.

$$\alpha_{\mu_{\tilde{A}}}=\frac{(a_3-a_1)\sqrt{(a_2-a_1)^2+1}}{(a_3-a_1)+\sqrt{(a_2-a_1)^2+1}-\sqrt{(a_2-a_3)^2+1}} \tag{12}$$

$$\beta_{\mu_{\tilde{A}}}=\frac{\sqrt{(a_2-a_3)^2+1}\sqrt{(a_2-a_1)^2+1}}{(a_1-a_3)+\sqrt{(a_2-a_1)^2+1}+\sqrt{(a_2-a_3)^2+1}} \tag{13}$$

$$\gamma_{\mu_{\tilde{A}}}=\frac{(a_3-a_1)\sqrt{(a_2-a_3)^2+1}}{(a_3-a_1)+\sqrt{(a_2-a_3)^2+1}-\sqrt{(a_2-a_1)^2+1}} \tag{14}$$

Thus, the Cartesian coordinates corresponding to $G_{\mu_{\tilde{A}}}$ is calculated as in Eq.(15).

$$G_{\mu_{\tilde{A}}}=\left(\frac{\alpha_{\mu_{\tilde{A}}}da_1+\beta_{\mu_{\tilde{A}}}ba_2+\gamma_{\mu_{\tilde{A}}}ea_3}{\alpha_{\mu_{\tilde{A}}}d+\beta_{\mu_{\tilde{A}}}b+\gamma_{\mu_{\tilde{A}}}e},\frac{b\beta_{\mu_{\tilde{A}}}}{\alpha_{\mu_{\tilde{A}}}d+\beta_{\mu_{\tilde{A}}}b+\gamma_{\mu_{\tilde{A}}}e}\right) \tag{15}$$

After the calculating the Gergonne points of TIFN, getting a common point to represent these points is useful in terms of ranking. Let, be this common Gergonne point $G_{\tilde{A}}$. $G_{\tilde{A}}$ is calculated as shown in Eq. (16)

$$G_{\tilde{A}}=pG_{\mu_{\tilde{A}}}+qG_{1-v_{\tilde{A}}} \tag{16}$$

Here, $p+q=1$. To make calculation easier, $G_{\tilde{A}}$ can be taken as follows

$$G_{\tilde{A}}=(x_{\tilde{a}},y_{\tilde{a}})=\frac{G_{\mu_{\tilde{A}}}+G_{1-v_{\tilde{A}}}}{2} \tag{17}$$

By using Eq.(11),(15) and (17), the new ranking $Rank_{\mathcal{G}}(\tilde{A})$ is obtained in the following.

$$Rank_{\mathcal{G}}(\tilde{A})=(x_{\tilde{a}},1-y_{\tilde{a}},a_2) \tag{18}$$

Let $\tilde{A}=(a_1,a_2,a_3;a_1',a_2,a_3')$ and $\tilde{B}=(b_1,b_2,b_3;b_1',b_2,b_3')$ be two TIFN, by using Eq.(18) we can rank these two number as shown below.

$$\tilde{A} < \tilde{B} \quad iff \quad Rank_{\mathcal{G}}(\tilde{A}) <_L Rank_{\mathcal{G}}(\tilde{B}) \tag{19}$$

Here, $<_L$ means lexicographical order. Lexicographical order is defines as in Eq. (20)

$$(x_1, x_2, x_3) <_L (y_1, y_2, y_3) \Leftrightarrow (\exists m = 1,2,3)(\forall i < m)(x_i = y_i) \wedge (x_m < y_m) \tag{20}$$

4. ILLUSTRATIVE APPLICATION AND COMPARATIVE STUDY

In this section, an example will be given to prove the applicability of the proposed method firstly. Then, a comparative study between the proposed method and existing methods in the literature will be performed to demonstrate the validity of the proposed ranking method.

It is used the TIFN taken from Bharati (2017). These numbers and calculation are shown as below.

Example: Let $\tilde{A} = (2,3,4; 1,3,5)$ be a TIFN. $a = |BC| = 2.236$, $b = |CA| = 3$, $c = |BA| = 2.236$, $d = |BE| = 1.414$, $b = |DE| = 2$ and $e = |BD| = 1.414$

$G_{1-v_{\tilde{A}}}$'s trilinear coordinates are calculated in the following.

$$\alpha_{1-v_{\tilde{A}}} = \frac{(5-2)\sqrt{(3-1)^2+1}}{(5-1)+\sqrt{(3-1)^2+1}-\sqrt{(3-5)^2+1}} = 2.236$$

$$\beta_{1-v_{\tilde{A}}} = \frac{\sqrt{(3-1)^2+1}\,\sqrt{(3-5)^2+1}}{(1-5)+\sqrt{(3-1)^2+1}+\sqrt{(3-5)^2+1}} = 3.396$$

$$\gamma_{1-v_{\tilde{A}}} = \frac{(5-1)\sqrt{(3-5)^2+1}}{(5-1)+\sqrt{(3-1)^2+1}-\sqrt{(3-1)^2+1}} = 2.236$$

$$G_{1-v_{\tilde{A}}} = (2.495, 0.505)$$

The trilinear coordinates of $G_{\mu_{\tilde{A}}}$ is calculated as below.

$$\alpha_{\mu_{\tilde{A}}} = \frac{(4-2)\sqrt{(3-2)^2+1}}{(4-2)+\sqrt{(3-2)^2+1}-\sqrt{(3-4)^2+1}} = 1.414$$

$$\beta_{\mu_{\tilde{A}}} = \frac{\sqrt{(3-4)^2+1}\sqrt{(3-2)^2+1}}{(2-4)+\sqrt{(3-2)^2+1}+\sqrt{(3-4)^2+1}} = 2.414$$

$$\gamma_{\mu_{\tilde{A}}} = \frac{(4-2)\sqrt{(3-4)^2+1}}{(4-2)+\sqrt{(3-4)^2+1}-\sqrt{(3-2)^2+1}} = 1.414$$

$$G_{\mu_{\tilde{A}}} = (3,0.547)$$

After calculation $G_{\mu_{\tilde{A}}}$ and $G_{1-v_{\tilde{A}}}$, the common point $G_{\tilde{A}}$ is calculated as $G_{\tilde{A}} = (2.748,0.526)$. Finally, $Rank_{\mathcal{G}}(\tilde{A})$ is founded as below.

$$Rank_{\mathcal{G}}(\tilde{A}) = (2.748,1 - 0.526,3) = (2.748,0.474,3)$$

Similarly, let and $\tilde{B} = (1,2,3; 0,2,6)$ be TIFN. $a = |BC| = 4.123$, $b = |CA| = 5$, $c = |BA| = 2.236$, $d = |BE| = 1.414$, $b = |DE| = 2$ and $e = |BD| = 1.414$. $\alpha_{1-v_{\tilde{B}}} = 3.592$, $\beta_{1-v_{\tilde{B}}} = 6.783$, $\gamma_{1-v_{\tilde{B}}} = 2.993$ and $G_{1-v_B} = (1.214,0.612)$. $\alpha_{\mu_{\tilde{B}}} = 1.414$, $\beta_{\mu_{\tilde{B}}} = 2.414$, $\gamma_{\mu_{\tilde{B}}} = 1.414$ and $G_{\mu_{\tilde{B}}} = (2,0.547)$. The common point $G_{\tilde{B}}$ is calculated as $G_{\tilde{B}} = (1.607,0.579)$. Finally, $Rank_{\mathcal{G}}(\tilde{B}) = (1.607,0.421,2)$

After calculation $Rank_{\mathcal{G}}(\tilde{A})$ and $Rank_{\mathcal{G}}(\tilde{B})$, we can rank these two number as below.

$$Rank_{\mathcal{G}}(\tilde{A}) >_L Rank_{\mathcal{G}}(\tilde{B})$$

So, we can say $\tilde{A}$ is greater than $\tilde{B}$.

A comparison study is performed by using five TIFN between the proposed method and other methods in the literature. The results are shown in table 1. The first three TIFN are taken from Bharati (2017).

Table 1. The comparison of the proposed method with existing method

Examples	Bharati's method	Nayagam et. al's method	Prakash et al's method	Li's method	Dubey and Mehra's method	Roseline and Amirtharaj's method	Rezvani's method	Proposed method
$\tilde{A} = (2,3,4; 1,3,5)$ $\tilde{B} = (1,2,3; 0,2,6)$	$A > B$	$A > B$	$A > B$	$A > B$	$A > B$	$A > B$	$A > B$	$A > B$
$\tilde{A} = \begin{pmatrix} -4,-3,-2; \\ -6,-3,-1 \end{pmatrix}$ $\tilde{B} = \begin{pmatrix} -4,-3,-2; \\ -8,-3,0 \end{pmatrix}$	$B > A$	$A > B$	$B > A$	$A \approx B$	$A \approx B$	$A > B$	$A > B$	$B > A$
$\tilde{A} = (1,2,3; 0,2,4)$ $\tilde{B} = \begin{pmatrix} -3,-2,-1; \\ -4,-2,0 \end{pmatrix}$	$A > B$	$A > B$	$B > A$	$A > B$	$A > B$	$A > B$	$A > B$	$A > B$
$\tilde{A} = \begin{pmatrix} 1.1,1.5,1.8; \\ 1.05,1.5,1.85 \end{pmatrix}$ $\tilde{B} = \begin{pmatrix} 1.1,1.48,1.78; \\ 1.07,1.48,1.8 \end{pmatrix}$	$B > A$	$B > A$	$A > B$	$A > B$	$A \approx B$	$A > B$	$A > B$	$A > B$
$\tilde{A} = \begin{pmatrix} 0.05,0.1,0.2; \\ 0.00,0.1,0.21 \end{pmatrix}$ $\tilde{B} = \begin{pmatrix} 0.25,0.3,0.4; \\ 0.2,0.3,0.45 \end{pmatrix}$	$B > A$	$A > B$	$A > B$	$B > A$	$B > A$	$B > A$	$B > A$	$B > A$
$\tilde{A} \begin{pmatrix} 4.02,4.72,4.83; \\ 4,4.72,4,92 \end{pmatrix}$ $\tilde{B} \begin{pmatrix} 4.021,4.721,4.831 \\ 4.01,4.721,4.921 \end{pmatrix}$	$B > A$	$B > A$	$B > A$	$B > A$	$B > A$	$B > A$	$B > A$	$B > A$

When the table is examined, all methods have the same result for the first TIFN $\tilde{A}$ and $\tilde{B}$. Although, the other methods ranked the second number differently, Bharati's method, Prakash's method and the proposed method gave the same result. All methods ranked the third number in the same way except Prakash's method. For the fourth number, Prakash's method, Li's method, Roseline's method, Rezvani's method and the proposed method showed the same result. Except Nayagam's method and Prakash's method, the other methods ranked the fifth number the same. For sixth numbers, all ranking methods gave the same results. This situation occurs if two numbers are too close to each other. According to the table, it can be said that the proposed method is consistent with other methods in the literature.

5. CONCLUSION

When one works with fuzzy numbers, it is crucial to rank fuzzy numbers correctly. Because it can affect the results of the analysis. To deal with this situation, many researchers gave their attention to this topic and a great deal of papers that include fuzzy ranking methods are proposed in the literature. In this study, a new ranking method for triangular intuitionistic fuzzy numbers was proposed. The proposed method is based on the Gergonne point of the membership and non – membership function of TIFN and uses the lexicographic order to rank the numbers. In this manner, it is the first paper in the literature concerning the Gergonne point of triangles for the triangular intuitionistic fuzzy numbers. An illustrative example and a comparison study were performed to examine the validity of the proposed method. The results showed that, the proposed method is consistent with other methods in the literature. The proposed method is not affected by the sign of the number or being close to each other. So, it can be considered as the advantage of this method. For further research, the proposed method can be applied to fuzzy hypothesis tests and fuzzy linear programming problems.

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