Some Curvature Properties of Generalized Complex Space Forms

Pegah Mutlu

Faculty of Engineering, Istanbul Gedik University
Kartal, 34876, Istanbul, Turkey
pegah.mutlu@gedik.edu.tr

Abstract: The object of the present paper is to study generalized complex space forms satisfying curvature identities named Walker type identities. Also It is proved that the difference tensor $R.\widetilde{C} - \widetilde{C}.R$ and the Tachibana tensor $Q(S, \widetilde{C})$ of any generalized complex space form $M(f_1, f_2)$ of dimensional $m \geq 4$ are linearly dependent at every point of $M(f_1, f_2)$. Finally generalized complex space forms are studied under the condition $R.R - Q(S,R) = L\, Q(g, \widetilde{C})$.

Keywords: Generalized complex space forms, Conharmonic curvature tensor, Walker type identity, Pseudosymmetric manifold, Tachibana Tensor.

1. INTRODUCTION

In 1989, Z. Olszak has worked on the existence of a generalized complex space form [1]. In [2], U.C. De and A. Sarkar studied the nature of a generalized Sasakian space form under some conditions regarding projective curvature tensor. They also studied Sasakian space forms with vanishing quasi-conformal curvature tensor and investigated quasi-conformal flat generalized Sasakian space forms, Ricci-symmetric and Ricci semisymmetric generalized Sasakian space forms [3]. Venkatesha and B.Sumangala [4], M. Atceken [5], S. Yadav and A. K. Srivastava [6] studied on generalized space form satisfying certain conditions on an M-projective curvature tensor, concircular curvature and psedo projective curvature tensor $\widetilde{P}$ satisfying $R.\widetilde{P} = 0$ and many authors studied on generalized Sasakian space form [7]. M.C. Bharathi and C. S. Bagewadi [8] extended the study to W_2 curvature, conharmonic and concircular curvature tensors on generalized complex space forms.

Motivated by these ideas, in the present paper, we study generalized complex space forms satisfying curvature identities named Walker type identities. The difference tensor $R.\widetilde{C} - \widetilde{C}.R$ and the Tachibana tensor $Q(S, \widetilde{C})$ of any generalized complex space form $M(f_1, f_2)$ of dimensional $m \geq 4$ are linearly dependent at every point of $M(f_1, f_2)$. Generalized complex space forms are studied under the condition $R.R - Q(S,R) = L\, Q(g, \widetilde{C})$.

A Kaehler manifold is an even dimensional manifold M^m, where $m=2n$ with a complex structure J and a positive definite metric g which satisfies the following conditions [9]

$$J^2 = -I, \quad g(JX, JY) = g(X, Y) \quad \text{and} \quad \nabla J = 0,$$

where ∇ denotes the covariant derivative with respect to Levi-Civita connection.

Let (M, J, g) be a Kaehler manifold with constant holomorphic sectional curvature $K(X \wedge JX) = c$, then is said to be a complex space form and it is well known that its curvature tensor satisfies the equation

$$R(X,Y)Z = \frac{c}{4}\{g(Y,Z)X - g(X,Z)Y + g(X,JZ)JY - g(Y,JZ)JX + 2g(X,JY)JZ\}, \tag{1}$$

for any vector fields X ,Y, Z on M.

An almost Hermitian manifold M is called a generalized complex space form $M(f_1, f_2)$ if its Riemannian curvature tensor R satisfies

$$R(X,Y)Z = f_1\{g(Y,Z)X - g(X,Z)Y\} + f_2\{g(X,JZ)JY - g(Y,JZ)JX + 2g(X,JY)JZ\}, \tag{2}$$

for any vector fields $X,Y,Z \in TM$, where f_1 and f_2 are smooth functions on M [10,11].

For a generalized complex space form $M(f_1, f_2)$ we have

$$S(X,Y) = \{(m-1)f_1 + 3f_2\}\, g(X,Y), \tag{3}$$

$$QX = \{(m-1)f_1 + 3f_2\}X, \tag{4}$$

$$r = m\{(m-1)f_1 + 3f_2\}, \tag{5}$$

where S is the Ricci tensor, Q is the Ricci operator and r is the scalar curvature of $M(f_1, f_2)$.

2. PRELIMINARIES

In this section, we recall some definitions and basic formulas which will be used in the following sections.

Let (M, g) be an n-dimensional, $n \geq 3$, semi-Riemannian connected manifold of class C^∞ with Levi-Civita connection ∇ and Ξ (M) being the Lie algebra of vector fields on M.

We define on M the endomorphisms $X \wedge_A Y$, $\mathcal{R}(X,Y)$ and $\widetilde{\mathcal{C}}(X,Y)$ of Ξ (M) by

$$(X \wedge_A Y) Z = A(Y,Z)X - A(X,Z)Y, \tag{6}$$

$$\mathcal{R}(X,Y) Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z, \tag{7}$$

$$\widetilde{\mathcal{C}}(X,Y) Z = \mathcal{R}(X,Y) Z - \frac{1}{n-2} [X \wedge_g QY - Y \wedge_g QX] Z, \tag{8}$$

respectively, where A is a symmetric (0,2)-tensor on M and $X,Y,Z \in \Xi$ (M). The Ricci tensor S, the Ricci operator Q and the scalar curvature r of (M , g) are defined by $S(X,Y) = \mathrm{tr}\{Z \to \mathcal{R}(Z,X)Y\}$, $g(QX,Y) = S(X,Y)$ and $r = \mathrm{tr}\, Q$, respectively. [X,Y] is the Lie bracket of vector fields X and Y. In particular we have $(X \wedge_g Y) = X \wedge Y$.

The Riemannian-Christoffel curvature tensor R, the conharmonic curvature tensor $\widetilde{C}$ and the (0,4)-tensor G of (M, g) are defined by

$$R(X_1, X_2, X_3, X_4) = g(\mathcal{R}(X_1, X_2) X_3, X_4),$$
$$\widetilde{\mathcal{C}}(X_1, X_2, X_3, X_4) = g(\widetilde{\mathcal{C}}(X_1, X_2) X_3, X_4),$$
$$G(X_1, X_2, X_3, X_4) = g((X_1 \wedge_g X_2) X_3, X_4),$$

respectively, where $X_1, X_2, X_3, X_4 \in \Xi$ (M).

From (8) it follows that

$$\widetilde{C}(X,Y,Z,W) = -\widetilde{C}(Y,X,Z,W),$$
$$\widetilde{C}(X,Y,Z,W) = -\widetilde{C}(X,Y,W,Z),$$
$$\widetilde{C}(X,Y,Z,W) = \widetilde{C}(Z,W,X,Y),$$
$$\widetilde{C}(X,Y,Z,W) + \widetilde{C}(X,Z,W,Y) + \widetilde{C}(X,W,Y,Z) = 0.$$

Let $\mathcal{B}(X,Y)$ be a skew-symmetric endomorphism of Ξ (M). We define the (0,4)-tensor B by $B(X_1,X_2,X_3,X_4) = g(\mathcal{B}(X_1,X_2) X_3, X_4)$. The tensor B is said to be a generalized curvature tensor if

$$B(X_1, X_2, X_3, X_4) = B(X_3, X_4, X_1, X_2),$$
$$B(X_1, X_2, X_3, X_4) + B(X_2, X_3, X_1, X_4) + B(X_3, X_1, X_2, X_4) = 0.$$

For a (0,k)-tensor field T, $k \geq 1$, a symmetric (0,2)- tensor field A and a generalized curvature tensor B on (M , g), we define the (0, k+2)-tensor fields B .T and Q(A, T) by

$$(B.T)(X_1, ..., X_k; X, Y) = -T(\mathcal{B}(X,Y)X_1, X_2, ..., X_k) - \cdots - T(X_1, X_2, \ldots, X_{K-1}, \mathcal{B}(X,Y) X_k),$$

$$Q(A,T)(X_1, ..., X_k; X, Y) = -T((X \wedge_A Y)X_1, X_2, ..., X_k) - \cdots - T(X_1, X_2, \ldots, X_{K-1}, (X \wedge_A Y) X_k),$$

respectively, where $X, Y, Z, X_1, X_2, ..., X_k \in \Xi$ (M).

Let (M, g) be covered by a system of charts $\{W; x^k\}$. We define by g_{ij}, R_{hijk}, S_{ij}, and

$$\widetilde{C}_{hijk} = R_{hijk} - \frac{1}{n-2}(g_{hk} S_{ij} - g_{hj}S_{ik} + g_{ij}S_{hk} - g_{ik} S_{hj}), \tag{9}$$

the local components of the metric tensor g, the Riemannian-Christoffel curvature tensor R, the Ricci tensor S, and the conharmonic curvature tensor $\widetilde{C}$, respectively. Further, we denote by $S_{ij} = S_{ir}\, g_j^r$ and $S_i^j = g^{jr} S_{ir}$.

The local components of the (0, 6)-tensor fields R.T and Q (g ,T) on M are given by

$$(R.T)_{hijklm} = -g^{rs}\big(T_{rijk}R_{shlm} + T_{hrjk}R_{silm} + T_{hirk}R_{sjlm} + T_{hijr}R_{sklm}\big), \tag{10}$$

$$Q(g,T)_{hijklm} = -g_{mh}T_{lijk} - g_{mi}T_{hljk} - g_{mj}T_{hilk} - g_{mk}T_{hijl} + g_{lh}T_{mijk} + g_{li}T_{hmjk} + g_{lj}T_{himk} + g_{lk}T_{hijm}, \tag{11}$$

where T_{hijk} are the local components of the tensor T.

In this part we present some considerations leading to the definition of Deszcz Symmetric (Pseudosymmetric in the sense of Deszcz) and Ricci-pseudosymmetric manifolds.

A semi-Riemannian manifold (M,g) satisfying the condition $\nabla R = 0$ is said to be locally symmetric. Locally symmetric manifolds form a subclass of the class of manifolds characterized by the condition

$$R.R = 0 \tag{12}$$

where R. R is a (0,6)-tensor field with the local components

$$(R.R)_{hijklm} = \nabla_m \nabla_l R_{hijk} - \nabla_l \nabla_m R_{hijk} = g^{rs}(R_{rijk}R_{shlm} + R_{hrjk}R_{silm} + R_{hirk}R_{sjlm} + R_{hijr}R_{sklm}).$$

Semi-Riemannian manifolds fulfilling (12) are called semisymmetric [12]. They are not locally symmetric, in general.

A more general class of manifolds than the class of semisymmetric manifolds is the class of pseudosymmetric manifolds.

A semi-Riemannian manifold (M,g) is said to be pseudosymmetric in the sense of Deszcz [13,14] if at every point of M the condition

$$R.R = L_R Q(g,R) \tag{13}$$

holds on the set $\mathcal{U}_R = \left\{x \in M \,\middle|\, R - \frac{r}{n(n-1)} G \neq 0 \text{ at } x\right\}$, where L_R is some function on $\mathcal{U}_R$.

A semi-Riemannian manifold (M, g) is said to be Ricci-pseudosymmetric [15] if at every point of M the condition

$$R.S = L_s Q(g,S) \tag{14}$$

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holds on the set $\mathcal{U}_s = \left\{x \in M \,\middle|\, S - \frac{r}{n} g \neq 0 \text{ at } x\right\}$, where L_s is some function on $\mathcal{U}_s$. Every pseudosymmetric manifold is Ricci-pseudosymmetric. The converse statement is not true. The class of Ricci-pseudosymmetric manifolds is an extension of the class of Ricci-semisymmetric (R.S= 0) manifolds as well as of the class of pseudosymmetric manifolds. Evidently, every Ricci-semisymmetric is Ricci-pseudosymmetric. There exist various examples of Ricci-pseudosymmetric manifolds which are not pseudosymmetric.
(13), (14) or other conditions of this kind are called curvature conditions of pseudosymmetry type [16].

3. WALKER TYPE IDENTITIES ON GENERALIZED COMPLEX SPACE FORMS

In this section, we present results on generalized complex space forms satisfying curvature identities named Walker type identities.

LEMMA 3.1 [17]. *For a symmetric (0,2)-tensor A and a generalized curvature tensor $\mathcal{B}$ on a semi-Riemannian manifold (M,g), $n \geq 3$, we have*

$$\sum_{(X_1,X_2)(X_3,X_4)(X,Y)} Q(A,\mathcal{B})(X_1,X_2,X_3,X_4;X,Y) = 0. \qquad (15)$$

It is well-known that the following identity

$$\sum_{(X_1,X_2)(X_3,X_4)(X,Y)} (R\,.\,R)(X_1,X_2,X_3,X_4;X,Y) = 0 \qquad (16)$$

holds on any semi-Riemannian manifold.

THEOREM 3.2. *Let (M,g), $n \geq 4$, be a semi-Riemannian manifold. Then the following three equalities are equivalent :*

$$\sum_{(X_1,X_2)(X_3,X_4)(X,Y)} (R\,.\,\widetilde{C})(X_1,X_2,X_3,X_4;X,Y) = 0, \qquad (17)$$

$$\sum_{(X_1,X_2)(X_3,X_4)(X,Y)} (\widetilde{C}\,.\,R)(X_1,X_2,X_3,X_4;X,Y) = 0 \qquad (18)$$

and

$$\sum_{(X_1,X_2)(X_3,X_4)(X,Y)} \left(R.\widetilde{C} - \widetilde{C}\,.\,R\right)(X_1,X_2,X_3,X_4;X,Y) = 0 \qquad (19)$$

on M.

Proof. In view of (10) , we have

$$(R\,.\,\widetilde{C})_{hijklm} = g^{rs}(\widetilde{C}_{rijk}R_{shlm} + \widetilde{C}_{hrjk}R_{silm} + \widetilde{C}_{hirk}R_{sjlm} + \widetilde{C}_{hijr}R_{sklm}) \qquad (20)$$

$$(\widetilde{C}\,.\,R)_{hijklm} = g^{rs}(R_{rijk}\widetilde{C}_{shlm} + R_{hrjk}\widetilde{C}_{silm} + R_{hirk}\widetilde{C}_{sjlm} + R_{hijr}\widetilde{C}_{sklm}) \qquad (21)$$

Using (9) in (20) we obtain

$$(R\,.\,\widetilde{C})_{hijklm} = (R\,.\,R)_{hijklm} - \frac{1}{n-2}[g_{ij}(A_{hklm} + A_{khlm}) + g_{hk}(A_{ijlm} + A_{jilm}) - g_{ik}(A_{hjlm} + A_{jhlm}) - g_{hj}(A_{iklm} + A_{kilm})], \qquad (22)$$

where $A_{mijk} = S^r_m\, R_{rijk}$.
Applying, in the same way, (9) in (21) we get

$$\begin{aligned}(\widetilde{C}\,.\,R)_{hijklm} = (R\,.\,R)_{hijklm} &- \frac{1}{n-2}[Q(S,R)_{hijklm} + g_{hl}A_{mijk} - g_{hm}A_{lijk} \\ &- g_{il}A_{mhjk} + g_{im}A_{lhjk} + g_{jl}A_{mkhi} \\ &- g_{jm}A_{lkhi} - g_{kl}A_{mjhi} + g_{km}A_{ljhi}]. \qquad (23)\end{aligned}$$

We set

$$\begin{aligned}\mathcal{F}_{hijklm} = -\frac{1}{n-2}[\, &g_{ij}(A_{hklm} + A_{khlm}) + g_{hk}(A_{ijlm} + A_{jilm}) \\ &- g_{ik}(A_{hjlm} + A_{jhlm}) - g_{hj}(A_{iklm} + A_{kilm}) \\ &+ g_{kl}(A_{mjhi} + A_{jmhi}) + g_{jm}(A_{klhi} + A_{lkhi}) \\ &- g_{km}(A_{jlhi} + A_{ljhi}) - g_{jl}(A_{kmhi} + A_{mkhi}) \\ &+ g_{mh}(A_{lijk} + A_{iljk}) + g_{li}(A_{mhjk} + A_{hmjk}) \\ &- g_{mi}(A_{lhjk} + A_{hljk}) - g_{lh}(A_{mijk} + A_{imjk})].\end{aligned}$$

Symmetrizing (22) with respect to the pairs (h,i), (j,k) and (l,m) and applying (15) and (16) we obtain

$$\sum_{(X_1,X_2)(X_3,X_4)(X,Y)} (\widetilde{C}\,.\,R)(X_1,X_2,X_3,X_4;X,Y) = -\mathcal{F}_{hijklm}$$

In the same way , using (20), we have

$$\sum_{(X_1,X_2)(X_3,X_4)(X,Y)} (R\,.\,\widetilde{C})(X_1,X_2,X_3,X_4;X,Y) = \mathcal{F}_{hijklm}$$

From the last two equations we get

$$\sum_{(X_1,X_2)(X_3,X_4)(X,Y)} (R\,.\,\widetilde{C} - \widetilde{C}\,.\,R)(X_1,X_2,X_3,X_4;X,Y) = 2\,\mathcal{F}_{hijklm}$$

If $\mathcal{F}_{hijklm} = 0$, then (17) (equivalently (18), (19)) holds on M. This completes the proof.

The equations (17) – (19) are named the Walker type identities. We also can consider the following Walker type identity

$$\sum_{(X_1,X_2)(X_3,X_4)(X,Y)} (\widetilde{C}\,.\,\widetilde{C}\,)(X_1,X_2,X_3,X_4;X,Y) = 0\,. \qquad (24)$$

THEOREM 3.3. *Let $M(f_1, f_2)$ be an m-dimensional ($m \geq 4$) generalized complex space form. Then we have*

$$R\,.\,\widetilde{C} - \widetilde{C}\,.\,R = \frac{2}{m-2}[(m-1)f_1 + 3\,f_2]\,Q(g\,,R)$$

$$= \frac{2}{m-2}[(m-1)f_1 + 3\,f_2]\,Q\big(g\,,\widetilde{C}\big)$$

$$= \frac{2}{m-2}Q(S\,,R)$$

$$= \frac{2}{m-2}Q\big(S\,,\widetilde{C}\big). \qquad (25)$$

Proof. By using (3) the equations (22), (23) and (9) reduce to

$$R\,.\,\widetilde{C} = R\,.\,R \qquad (26)$$

$$\widetilde{C}\,.\,R = R\,.\,R - \frac{2}{m-2}[(m-1)f_1 + 3\,f_2]\,Q(g\,,R) \qquad (27)$$

and

$$\widetilde{C} = R - \frac{2}{m-2}[(m-1)f_1 + 3\,f_2]\,G \qquad (28)$$

respectively. Hence we have

$$R\,.\,\widetilde{C} - \widetilde{C}\,.\,R = \frac{2}{m-2}[(m-1)f_1 + 3\,f_2]\,Q(g\,,R).$$

and so $Q(g\,,R) = Q(g\,,\widetilde{C})$. This completes the proof.

In view of the above theorem an m-dimensional ($m \geq 4$) generalized complex space form satisfying the following conditions:

- the tensors $R\,.\,\widetilde{C} - \widetilde{C}\,.\,R$ and $Q(g\,,R)$ are linearly dependent at every point of $M(f_1, f_2)$,
- the tensors $R\,.\,\widetilde{C} - \widetilde{C}\,.\,R$ and $Q\big(g\,,\widetilde{C}\big)$ are linearly dependent at every point of $M(f_1, f_2)$,
- the tensors $R\,.\,\widetilde{C} - \widetilde{C}\,.\,R$ and $Q(S\,,R)$ are linearly dependent at every point of $M(f_1, f_2)$,
- the tensors $R\,.\,\widetilde{C} - \widetilde{C}\,.\,R$ and $Q\big(S\,,\widetilde{C}\big)$ are linearly dependent at every point of $M(f_1, f_2)$.

COROLLARY 3.4. *Let $M(f_1, f_2)$, ($m \geq 4$), be an m-dimensional generalized complex space form satisfying $R.\,\widetilde{C} = 0$, then $M(f_1, f_2)$ is semisymmetric.*

THEOREM 3.4. *Let $M(f_1, f_2)$, be an m-dimensional ($m \geq 4$) generalized complex space form. Then the Walker type identities (17) – (19) and (24) hold on $M(f_1, f_2)$.*

Proof. In view of theorem 3.3., we have

$$R\,.\,\widetilde{C} - \widetilde{C}\,.\,R = \frac{2}{m-2}[(m-1)f_1 + 3\,f_2]\,Q(g\,,R)$$

and using (15) we get (19) (equivalently (17) and (18)).

Further, we note that $\widetilde{C} = R - \frac{2}{m-2}[(m-1)f_1 + 3\,f_2]\,G$. This gives

$$\widetilde{C}.\widetilde{C} = \widetilde{C}.\left(R - \frac{2}{m-2}[(m-1)f_1 + 3\,f_2]\,G\right) = \widetilde{C}.R$$

$$= \left(R - \frac{2}{m-2}[(m-1)f_1 + 3\,f_2]\,G\right).R$$

$$= R.R - \frac{2}{m-2}[(m-1)f_1 + 3\,f_2]\,Q(g\,,R).$$

Now using (15) and (16) complete the proof.

4. GENERALIZED COMPLEX SPACE FORM SATISFYING R. R – Q(S , R) = L Q(g ,$\widetilde{C}$)

In this section we consider m-dimensional, ($m \geq 4$), generalized complex space forms satisfying the condition

$$R.\,R - Q(S\,,R) = L\,Q(g\,,\widetilde{C}) \qquad (29)$$

on $\mathcal{U}_{\widetilde{C}} = \{x \in M(f_1\,,f_2)\,|\,\widetilde{C} \neq 0 \ at\ x\}$, *where L is some function on* $\mathcal{U}_{\widetilde{C}}$.

THEOREM 4.1. *Let $M(f_1, f_2)$ be an m-dimensional ($m \geq 4$) generalized complex space form. If the relation (29) fulfilled on $\mathcal{U}_{\widetilde{C}} \subset M(f_1\,,f_2)$, then $M(f_1\,,f_2)$ is pseudo-symmetric with the function $L_R = L + (m-1)f_1 + 3\,f_2$.*

Proof. Using (3) and (28) in (29), we have

$$R.\,R - [(m-1)f_1 + 3\,f_2]\,Q(g\,,R) = L\,Q(g\,,\widetilde{C})$$

and so

$$R.\,R = [\,L + (m-1)f_1 + 3\,f_2]\,Q(g\,,R).$$

This completes the proof.

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REFERENCES

[1] Z. Olszak, "The existance of generalized complex space form", Israel.J.Math., vol.65, pp.214- 218, 1989.
[2] U.C. De and A. Sarkar,"On the projective curvature tensor of generalized Sasakian space forms", Quaestines Mathematicae, vol.33, pp.245-252, 2010.
[3] Sarkar. A. and De. U.C, "Some curvature properties of generalized Sasakian space forms", Lobachevskii journal of mathematics, Vol.33, no.1, pp.22-27, 2012.
[4] Venkatesha and B. Sumangala, "On M-Projective curvature tensor of a generalized Sasakian space form", Acta Math. Univ. Comenian, Vol.82(2) pp.209-217, 2013.
[5] M. Atceken , "On generalized Saskian space forms satisfying certain conditions on the concircular curvature tensor", Bulletin of Mathemaical analysis and applications, vol.6(1), pp.1-8, 2014.
[6] S. Yadav, D.L. Suthar and A. K. Srivastava , "Some results on $M(f_1, f_2, f_3)$ 2n+1-manifolds", Int. J. Pure Appl. Math., vol.70(3), pp.415-423, 2011.
[7] H. G. Nagaraja and Savithri Shashidhar, "On generalized Sasakian space forms", International Scholarly Research Network Geometry, 2012.
[8] M. C. Bharathi and C. S. Bagewadi, "On generalized complex space forms", IOSR Journal of Mathematics, vol.10, pp. 44-46, 2014.
[9] K. Yano, *Differential geometry on complex and almost complex spaces*, Pergamon Press, 1965.
[10] F. Tricerri and L. Vanhecke "Curvature tensors on almost Hermitian manifolds", Trans. Amer. Math. Soc., vol. 267, pp.365-398, 1981.
[11] U.C. De and G.C. Ghosh "On generalized Quasi-Einstein manifolds" Kyungpoole Math.J., vol.44, pp. 607-615, 2004.
[12] Z.I. Szabó, "Structure theorems on Riemannian spaces satisfying R(X,Y).R=0", I. The local version, J. Differential Geom., vol.17, pp.531-582, 1982.
[13] R. Deszcz, "On pseudosymmetric spaces", Bull. Soc. Math. Belg. Ser., Vol. A44, pp.1-34, 1992.
[14] R. Deszcz and Ş. Yaprak, "Curvature properties of certain pseudosymmetric manifolds",Publ. Math. Debr., vol.45, pp.334-345, 1994.
[15] R. Deszcz and M. Hotlos , "Remarks on Riemannian manifolds satisfying a certain curvature condition imposed on the Ricci tensor", Prace. Nauk. Pol. Szczec., vol. 11, pp. 23-34, 1989.
[16] M. Belkhelfa, R. Deszcz, M, Głogowska, M. Hotlos, D. Kowalczyk and L. Verstraelen, " A Review on pseudosymmetry type manifolds", Banach Center Publ., vol.57, pp. 179-194, 2002.
[17] R. Deszcz, J. Deprez and L. Verstraelen, "Examples of pseudo-symmetric conformally flat warped products", Chinese J. Math., vol. 17, pp.51-65, 1989.