

**PERFORMANCE COMPARISON OF THE SPECIALIZED ALPHA MALE GENETIC ALGORITHM WITH SOME EVOLUTIONARY ALGORITHMS**

**ÖZELLEŞTİRİLMİŞ ALFA ERKEK (ALPHA MALE) GENETİK ALGORİTMANIN EVRİMSEL ALGORİTMALARLA PERFORMANS KARŞILAŞTIRMASI**

**Mehmet Hakan SATMAN\* Emre AKADAL\*\***

*Geliş Tarihi: 08.08.2018*  
*(Received)*

*Kabul Tarihi: 27.03.2019*  
*(Accepted)*

**ABSTRACT:** Alpha Male Genetic Algorithms are sexist and population based optimization tools that mimic the swarm behavior of animals. The algorithm consists on a socially partitioned population of individuals where the partitions are formed by sexual selection of females. In this paper, we suggest to use Linear Crossover and Hooke-Jeeves method for crossover and hybridization operators of Alpha Male Genetic Algorithms, respectively. We perform a simulation study using a set of well-known test functions to reveal performance differences between the specialized algorithm and some other well-known optimization techniques including Genetic Algorithms, Differential Evolution, Particle Swarm Optimization, and Artificial Bee Colony Optimization. Simulation results show that the specialized algorithm outperforms its counterparts in most of the cases.

**Key Words:** Optimization, Evolutionary algorithms, Simulations.

**ÖZ:** Alfa erkek genetik algoritmalar cinsiyet farkı gözetken ve hayvan gruplarının hareketlerini taklit eden topluluk tabanlı bir optimizasyon aracıdır. Algoritma, dişilerin eş seçimi ile oluşturduğu sosyal olarak bölünmüş birey topluluklarına dayanmaktadır. Çalışmada, Alfa Erkek Genetik Algoritma'nın çaprazlama ve hibritleşme operatörü olarak sırasıyla Doğrusal Çaprazlama ve Hooke-Jeeves yöntemi kullanılması önerilmiştir. Çalışma kapsamında özelleştirilmiş algoritma ile Genetik Algoritmalar, Diferansiyel Evrim, Parçacık Sürü Optimizasyonu ve Yapay Arı Kolonisi Optimizasyonu gibi iyi bilinen algoritmalar arasındaki performans farklılıklarını ortaya çıkarabilmek için bilinen test fonksiyonları ile bir simülasyon çalışması gerçekleştirilmiştir. Simülasyon sonuçları, özelleştirilmiş algoritmanın çoğu durumda daha iyi performans sergilediğini göstermiştir.

**Anahtar Kelimeler:** Optimizasyon, Evrimsel algoritmalar, Simülasyon.

### 1. INTRODUCTION

Genetic Algorithms (GAs) are search and optimization tools that mimic the natural selection and principals of genetics. Since GAs are problem independent, the goal function under optimization may not be either continuous or differentiable. As a result of this, the function parameters can be in type of binary, integer, ordinal, categorical, or real. The simple GA, first developed by Holland (1975) and

\*Prof. Dr., İstanbul Üniversitesi İktisat Fakültesi, mhsatman@istanbul.edu.tr, ORCID: 0000-0002-9402-1982

\*\*Dr., İstanbul Üniversitesi Enformatik Bölümü, emre.akadal@istanbul.edu.tr, ORCID: 0000-0001-6817-0127

extended by Goldberg (1989), is defined for the optimization problems with binary variables and the algorithm is based on the main genetic operators including selection, crossover, mutation, and elitism. Since any member of the integer set  $\mathbb{Z}$  can be decoded using bits, the simple GA is also applied and well-performed on the integer variable optimization problems. By mapping the integer values onto the set of  $\mathbb{R}$ , real-parameter optimization is also possible with the simple GA by means of discretization.

Floating-point GAs (FPGAs), are another family of GAs in which the function parameters can take floating-point numbers as values and the algorithm does not require an encoding/decoding strategy for discretization mapping. Since the phenotype/genotype distinction does no longer exist, chromosomes in FPGAs are real values. However the crossover and mutation operations in simple GA are mainly based on bits, new algorithms and operators are devised for combining parent solutions to produce offspring. These operators are mainly based on linear or non-linear combination of numbers.

Some other members of the evolutionary optimization methods do not follow the same evolution process as in the GAs. For example, Particle Swarm Optimization (*PSO*) mimics the swarm intelligence of fish and birds (Eberhart & Kennedy, 1995). In *PSOs*, each candidate solution is a vector of real numbers. This property replaces the encoding/decoding strategy needed by classical GAs. The candidate solutions, or individuals, are modified directly by knowledge sharing during the searching process of right migration path or food sources.

Differential Evolution (*DE*), another member of the evolutionary algorithms, is similar with the *FPGAs* in terms of crossover, mutation, and selection operators (Storn & Price, 1997). However the method applies the operators in a different order. Although *FPGAs* apply crossover and mutation operators on the selected individuals, *DE* first applies crossover and mutation on the randomly selected solutions and the winner is selected at the last stage of the procedure. As in *PSO*, the individuals of population are real vectors and an encoding/decoding strategy is not needed.

Artificial Bee Colony (*ABC*) Algorithm (Karaboga & Basturk, 2007) is another population based optimization method that mimics the behavior of honey bee swarms. The initial population consists on the candidate solutions that represent locations of good food sources analogously. Socially partitioned honey bees visit and modify the locations. Quality of food sources is proportional to nectar amount which corresponds to fitness or cost in the other population based algorithms. The algorithm carries out the tasks of exploring new areas of the solution space, local search, and random search using the socially partitioned bees.

The Alpha Male Genetic Algorithm ( $\alpha$ -MGA) (Drezner & Drezner, 2018) is a sexist, hybrid, and population based optimization method. As it is described in Section 3, the algorithm is defined with general principals. In this paper, we suggest to use Linear Crossover as the crossover operator and Hooke-Jeeves algorithm as the hybridization operator in the  $\alpha$ -MGA. We propose a simulation study to reveal performance differences of  $\alpha$ -MGA with some other evolutionary optimization methods.

In Section 2, we give a brief review of gender-specific genetic algorithms. In Section 3, the  $\alpha$ -MGA and our suggestions are mentioned. In Section 4, we perform a simulation study and the results are presented. Finally in Section 5, we conclude.

## 2. GENDER-SPECIFIC GENETIC ALGORITHMS

The classical GA and its extensions do not take the sex or the gender into consideration. In other words, individuals in GAs are selected only by considering their fitness or cost values depending on the selection strategy. However, some GAs are developed with sexist properties in the literature.

Wagner & Affenzeller (2005) stated that the process of choosing mating partners in natural populations is different for male and female individuals as the male individuals try to spread their genes in a rather broad whereas the female individuals normally choose their partners depending on much harder criterion. Allenson (1992) developed, Lis & Eiben (1997) developed and Esquivel, Leiva, & Gallard (1999) extended a multisexual genetic algorithm in which the objective functions are mapped to genders of individuals. Ansotegui, Sellmann, & Tierney (2009) developed a GA in which the individuals assigned to a gender and the crossover operation is only permitted between the individuals with opposed genders. Rejeb & AbuElhajj (2000) applied a similar gender based GA on the graph partitioning problems. Sanchez-Velazco & Bullinaria (2003) introduced the sexual selection mechanism in GA. In their work, individuals not only have a gender property but selection, crossover, and mutation operators behave different depending on the gender of the individuals. Drezner & Drezner (2006) stated that the gender-specific algorithm more closely and more accurately simulates natural evolutionary processes. They also calculated Hamming distances of solutions at runtime and they concluded that the gender-specific approach promotes diversity of populations. Vrajitoru (2002) examined the effects of gender separation on the genetic diversity by simulations.

Ansotegui, Sellmann, & Tierney (2009) devised a different sexual strategy. They proposed to partition the population into two groups. The first group defines a single gender. Individuals in the first group compete for the right of mating whereas individuals in the second group are randomly selected. The fittest individual in the first group plays the role of forming the new generation.

Drezner (2008) performed a simulation study and showed that assigning gender to individuals and permitting recombination only between opposite genders provide better results. They suggest to introduce the gender strategy as follows:

1. Randomly assign genders to individuals of initial population with equal probability.
2. In mating process, select the first individual using the selecting scheme by not considering the gender. The second individual must be selected from the pool of opposite gender.
3. Assign a gender to generated offspring with equal probability.

Drezner & Drezner (2018) proposed the Alpha Male Genetic Algorithm ( $\alpha$ -MGA) which is based on a similar sexist approach. Differently, assignment of sex is not basically labelling individuals but the method also mimics the mating process of animals which includes social dominance. The next section discuss the method in more detail.

### 3. ALPHA-MALE GENETIC ALGORITHMS

The Alpha Male Genetic Algorithm ( $\alpha$ -MGA) (Drezner & Drezner, 2018) is a genetic algorithm in which the individuals in a population are partitioned into social groups. A social group is formed by productive females which select the same alpha male. In each social group, an individual is labeled as alpha male and the remainder are productive females. Offspring are generated by mating the alpha male and each productive females in encapsulated groups. Each productive female is permitted to generate only one offspring in a generation. The generated offspring form new groups in which a member is alpha male and the remainder are productive females again. After sorting the population, the best *popSize* individuals form the new population in the next generation where *popSize* is the population size parameter of algorithm. Generated offspring are not permitted to mate in the generation they are created.

Algorithm randomly selects  $P$  and  $K$  parameters before a generation is started where  $K$  is the number of alpha males,  $P - K$  is the number of reproductive females,  $P$  is the population size and  $K < P$ . Process of initial population creation is similar with the standard GA except the algorithm labels the individuals with a gender property.

In each generation, the productive females select an alpha male to produce an offspring. The generated offspring is added to another population and does not play role on generating new offspring in the current generation. After generating  $2P - K$  individuals in total, the best  $P$  individuals form the new population for the next generation. The hybrid version of the algorithm has four general steps: *selection*, *crossover*, *improvement*, and *replacement*.

By the *replacement* strategy, if the generated offspring is better than the worst population member and is not identical to any existing member then the

offspring is added to the population and the worst member is removed. Otherwise, the population is not changed.

Since it is not defined in the original study, the authors suggest to *merge* individuals as the crossover operator does. Addition to this, *improving* an individual is not defined explicitly.

Referring the previous studies, we suggest to use the *Linear Crossover* as the crossover operator (Satman & Akadal, 2017). The Linear Crossover operator produces three offspring

$$J_1 = \frac{1}{2}A + \frac{1}{2}B$$

$$J_2 = \frac{3}{2}A - \frac{1}{2}B$$

$$J_3 = -\frac{1}{2}A + \frac{3}{2}B$$

where  $A$  and  $B$  are parents, and  $J_1, J_2, J_3$  are produced offspring (Herrera, Lozano & Sanchez, 2003). An additional selection mechanism can be performed to obtain single offspring using this operator.

The *improvement* step basically stands for the mutation operator but it is known that the mutation operator in *GAs* is totally blind. Instead, a local search algorithm can improve the success of a solution by means of a higher fitness value. Satman (2015) suggested to use *Hooke-Jeeves* algorithm to obtain better solutions after recombination as its success is proven by the simulations in many cases. *Hooke-Jeeves* (Moser, 2009) algorithm is a direct search method in which the goal function can be either continuous or discrete. It is shown that a good starting point results good solutions and it can be used as a local *fine-tuning* algorithm to improve solutions found by a *GA*.

Finally, we suggest to specialize the optional parts of the Alpha Male *GA* as follows:

1. Select the parents as defined in the original algorithm.
2. Use the *Linear Crossover* operator to generate three offspring, select the best one depending on its fitness value.
3. Improve the selected offspring using the *Hooke-Jeeves* algorithm.
4. Apply *replacement* as defined in the original algorithm.

#### 4. SIMULATION STUDY

We performed a simulation study to reveal performance differences with some well-known evolutionary algorithms using a set of test functions. In the

simulation study, performance of the  $\alpha$ -MGA is compared with the GA with floating-point encoding scheme, *Differential Evolution*, *Particle Swarm Optimization*, and *Artificial Bee Colony* algorithms.

Since the curvature of functions, number of parameters or the dimension of the space that the function is defined have significant effects on the optimization, the simulation study consists on many different scenarios. Algorithms are performed on the test functions for  $p = 2$ ,  $p = 10$ , and  $p = 25$  where  $p$  is the number of function parameters. For each dimension selection, algorithms are performed for  $N = 20$ ,  $N = 50$ , and  $N = 100$ , where  $N$  is the population size. Since the  $\alpha$ -MGA is sexist, population is partitioned into two groups. The sexist population division is formed for

- 2 males and 10 females
- 5 males and 25 females
- 10 males and 50 females.

The parameter number of generations is limited as  $maxiter = 10$  and  $maxiter = 50$ . This parameter is also used as the stopping rule. In the cases  $maxiter = 10$  is used, the earlier convergence speed is compared. For a larger selection, for example  $maxiter = 50$ , it is tested that the algorithm either converges or not. Each single configuration is iterated 1000 times.

Besides the configuration, algorithms are performed on a rich set of test functions including Test Tube Holder, Holder Table, Carrom Table, Cross in Tray, Crowned Cross, Cross, Cross Leg Table, Pen Holder, Bird, Modified Schaffer 1-4, Egg Holder, Chichinadze, Mc Cormick, Three Humps Camel Back, Zettle, Styblinski-Tang, Bukin, Leon, Giunta, Schaffer, Schwefel, Ackley, Bohachevsky, Griwank, Holzman, Hyperellipsoid, Levy, Maxmod, Multimod, Rastrigin, Rosenbrock, Sine Envelope, Sphere, and Sumsquares (Mishra, 2006). Some of the functions are defined with 2 parameters and a list and definitions of these functions are given in Appendix 1. The rest of the functions are defined for  $m$  parameters and a list and definitions of these functions are given in Appendix 2.

Table 1, Table 2, and Table 3 summarize the reports for 2-variables cases. In Table 1, the first line indicates that the GA yields  $-10.343$  in average for maximum number of iterations is 10 and the population size is 20. The values in parenthesis indicate the standard deviations of obtained optimum. Since the global optimum of the Test Tube Holder function is 10.8723, it can be said that the algorithms except PSO obtain results precisely. ABC and  $\alpha$ -MGA obtains closer solutions with lower standard deviations. These interpretations are current for the whole cases in Table 1, Table 2, and, Table 3.

**Table1:** Simulation results for 2-vars

Function Name	Iter	Pop	GA	DE	PSO	ABC	M-F	$\alpha$ -MGA	
Test Tube Holder (-10.8723)	10	20	-10,343 (0,413)	-10,559 (0,241)	-7,521 (2,156)	-10,748 (0,108)	2-10	-10,87 (0,001)	
		50	-10,612 (0,192)	-10,704 (0,114)	-8,681 (1,513)	-10,814 (0,047)	5-25	-10,87 (0)	
		100	-10,718 (0,105)	-10,766 (0,068)	-9,229 (1,148)	-10,827 (0,035)	10-50	-10,87 (0)	
	50	20	-10,786 (0,098)	-10,81 (0,048)	-8,638 (2,2)	-10,86 (0,009)	2-10	-10,87 (0)	
		50	-10,816 (0,055)	-10,838 (0,024)	-10,269 (1,109)	-10,867 (0,004)	5-25	-10,87 (0)	
		100	-10,839 (0,029)	-10,851 (0,013)	-10,527 (0,711)	-10,868 (0,002)	10-50	-10,87 (0)	
	Holder Table (-26.92)	10	20	-25,624 (0,965)	-25,5 (2,116)	-7,011 (6,827)	-26,635 (0,411)	2-10	-26,924 (0)
			50	-26,182 (0,762)	-26,495 (0,519)	-15,739 (8,521)	-26,9 (0,02)	5-25	-26,924 (0)
			100	-26,65 (0,442)	-26,733 (0,208)	-20,13 (7,306)	-26,913 (0,012)	10-50	-26,924 (0)
50		20	-26,856 (0,32)	-26,922 (0,006)	-7,68 (7,064)	-26,924 (0)	2-10	-26,924 (0)	
		50	-26,921 (0,058)	-26,923 (0,002)	-17,166 (8,351)	-26,924 (0)	5-25	-26,924 (0)	
		100	-26,924 (0)	-26,923 (0,001)	-22,257 (6,288)	-26,924 (0)	10-50	-26,924 (0)	
Carrom Table (-24.157)		10	20	-21,862 (1,681)	-21,848 (2,884)	-3,015 (5,576)	-23,739 (0,644)	2-10	-24,157 (0)
			50	-22,979 (1,316)	-23,433 (0,885)	-10,464 (8,816)	-24,12 (0,032)	5-25	-24,157 (0)
			100	-23,739 (0,751)	-23,817 (0,377)	-15,126 (8,12)	-24,14 (0,018)	10-50	-24,157 (0)
	50	20	-24,084 (0,438)	-24,153 (0,015)	-3,37 (5,924)	-24,157 (0)	2-10	-24,157 (0)	
		50	-24,156 (0,015)	-24,156 (0,002)	-12,309 (9,066)	-24,157 (0)	5-25	-24,157 (0)	
		100	-24,157 (0)	-24,156 (0,001)	-17,764 (7,537)	-24,157 (0)	10-50	-24,157 (0)	
	Cross in Tray (-2.063)	10	20	-2,059 (0,004)	-2,06 (0,004)	-1,822 (0,168)	-2,062 (0,001)	2-10	-2,063 (0)
			50	-2,061 (0,001)	-2,062 (0,001)	-1,95 (0,12)	-2,062 (0)	5-25	-2,063 (0)
			100	-2,062 (0,001)	-2,062(0)	-2,009 (0,073)	-2,063 (0)	10-50	-2,063 (0)
50		20	-2,063 (0)	-2,063 (0)	-1,845 (0,168)	-2,063 (0)	2-10	-2,063 (0)	
		50	-2,063 (0)	-2,063 (0)	-2 (0,115)	-2,063 (0)	5-25	-2,063 (0)	
		100	-2,063 (0)	-2,063 (0)	-2,045 (0,059)	-2,063 (0)	10-50	-2,063 (0)	
Crowned Cross (0)		10	20	0,027 (0,152)	0,916 (0,117)	1,307 (0,197)	0 (0)	2-10	0,035 (0,06)

	50	0 (0)	0,83 (0,108)	1,17 (0,171)	0 (0)	5-25	0,018 (0,033)
	100	0 (0)	0,768 (0,1)	1,114 (0,16)	0 (0)	10-50	0,021 (0,032)
	20	0,008 (0,071)	0,701 (0,109)	0,844 (0,269)	0 (0)	2-10	0,026 (0,049)
	50	0 (0)	0,626 (0,099)	0,783 (0,233)	0 (0)	5-25	0,013 (0,028)
	100	0 (0)	0,58 (0,083)	0,745 (0,211)	0 (0)	10-50	0,01 (0,024)
Cross (0)	20	0 (0)	0 (0)	0 (0)	0 (0)	2-10	0 (0)
	10	0 (0)	0 (0)	0 (0)	0 (0)	5-25	0 (0)
	100	0 (0)	0 (0)	0 (0)	0 (0)	10-50	0 (0)
	20	0 (0)	0 (0)	0 (0)	0 (0)	2-10	0 (0)
	50	0 (0)	0 (0)	0 (0)	0 (0)	5-25	0 (0)
	100	0 (0)	0 (0)	0 (0)	0 (0)	10-50	0 (0)
Cross Leg Table (-1)	20	-0,922 (0,268)	0 (0)	0 (0)	-1 (0)	2-10	-0,962 (0,191)
	10	-1 (0)	0 (0)	0 (0)	-1 (0)	5-25	-0,991 (0,094)
	100	-1 (0)	0 (0)	0 (0)	-1 (0)	10-50	-0,997 (0,055)
	20	-0,967 (0,178)	0 (0)	0 (0)	-1 (0)	2-10	-0,996 (0,063)
	50	-1 (0)	0 (0)	0 (0)	-1 (0)	5-25	-1 (0)
	100	-1 (0)	0 (0)	0 (0)	-1 (0)	10-50	-1 (0)
Pen Holder (-0.96354)	20	-0,958 (0,009)	-0,962 (0,002)	-0,766 (0,187)	-0,963 (0)	2-10	-0,964 (0)
	10	-0,962 (0,002)	-0,963 (0,001)	-0,891 (0,077)	-0,964 (0)	5-25	-0,964 (0)
	100	-0,963 (0,001)	-0,963 (0)	-0,921 (0,048)	-0,964 (0)	10-50	-0,964 (0)
	20	-0,963 (0)	-0,964 (0)	-0,779 (0,187)	-0,964 (0)	2-10	-0,964 (0)
	50	-0,964 (0)	-0,964 (0)	-0,92 (0,067)	-0,964 (0)	5-25	-0,964 (0)
	100	-0,964 (0)	-0,964 (0)	-0,946 (0,034)	-0,964 (0)	10-50	-0,964 (0)
Bird (- 106.7645)	20	-99,945 (9,347)	-101,298 (6,585)	-27,262 (40,196)	-105,885 (1,463)	2-10	-106,764 (0)
	10	-103,904 (4,302)	-104,734 (2,602)	-60,253 (36,042)	-106,572 (0,259)	5-25	-106,764 (0)
	100	-105,745 (1,406)	-105,813 (0,995)	-70,784 (31,887)	-106,701 (0,079)	10-50	-106,764 (0)
	20	-106,721 (0,683)	-106,761 (0,013)	-33,402 (44,721)	-106,764 (0,001)	2-10	-106,764 (0)
	50	-106,761 (0,022)	-106,764 (0,001)	-82,907 (37,297)	-106,764 (0)	5-25	-106,764 (0)
	100	-106,763 (0,007)	-106,764 (0,001)	-97,235 (22,866)	-106,764 (0)	10-50	-106,764 (0)
Modified Schaffer 1 (0)	10	0,11 (0,082)	0,095 (0,063)	0,419 (0,11)	0 (0)	2-10	0,001 (0,006)
	50	0,051 (0,046)	0,054	0,341	0 (0)	5-25	0 (0)



				(0,034)	(0,145)			
		100	0,024 (0,025)	0,034 (0,024)	0,274 (0,145)	0 (0)	10-50	0 (0)
		20	0,013 (0,015)	0,004 (0,003)	0,398 (0,131)	0 (0)	2-10	0 (0)
	50	50	0,006 (0,007)	0,001 (0,002)	0,241 (0,183)	0 (0)	5-25	0 (0)
		100	0,002 (0,003)	0,001 (0,001)	0,108 (0,145)	0 (0)	10-50	0 (0)
Modified Schaffer 2 (0.002)		20	0,105 (0,08)	0,1 (0,065)	0,422 (0,107)	0 (0)	2-10	0 (0,005)
	10	50	0,051 (0,045)	0,052 (0,036)	0,348 (0,141)	0 (0)	5-25	0 (0)
		100	0,022 (0,025)	0,032 (0,024)	0,284 (0,146)	0 (0)	10-50	0 (0)
		20	0,012 (0,019)	0,001 (0,002)	0,4 (0,133)	0 (0)	2-10	0 (0)
	50	50	0,003 (0,006)	0 (0)	0,24 (0,184)	0 (0)	5-25	0 (0)
		100	0 (0,002)	0 (0)	0,11 (0,146)	0 (0)	10-50	0 (0)
Modified Schaffer 3 (0.0015)		20	0,097 (0,07)	0,094 (0,059)	0,414 (0,113)	0,073 (0,051)	2-10	0,002 (0)
	10	50	0,052 (0,038)	0,05 (0,033)	0,328 (0,147)	0,04 (0,027)	5-25	0,002 (0)
		100	0,027 (0,019)	0,032 (0,021)	0,263 (0,14)	0,024 (0,016)	10-50	0,002 (0)
		20	0,014 (0,019)	0,005 (0,003)	0,382 (0,141)	0,003 (0,002)	2-10	0,002 (0)
	50	50	0,004 (0,003)	0,003 (0,001)	0,231 (0,181)	0,002 (0,001)	5-25	0,002 (0)
		100	0,002 (0,001)	0,002 (0,001)	0,089 (0,125)	0,002 (0)	10-50	0,002 (0)
Modified Schaffer 4 (0.2925)		20	0,351 (0,037)	0,339 (0,029)	0,469 (0,043)	0,334 (0,029)	2-10	0,293 (0)
	10	50	0,325 (0,023)	0,32 (0,018)	0,442 (0,057)	0,313 (0,015)	5-25	0,293 (0)
		100	0,313 (0,014)	0,309 (0,011)	0,415 (0,062)	0,305 (0,009)	10-50	0,293 (0)
		20	0,304 (0,01)	0,294 (0,002)	0,462 (0,051)	0,294 (0,001)	2-10	0,293 (0)
	50	50	0,3 (0,006)	0,293 (0,001)	0,404 (0,076)	0,293 (0)	5-25	0,293 (0)
		100	0,296 (0,003)	0,293 (0)	0,344 (0,064)	0,293 (0)	10-50	0,293 (0)

**Table2: Simulation Results for 2-vars (Continued)**

Function Name	Iter	Pop	GA	DE	PSO	ABC	M-F	$\alpha$ -MGA
Egg Holder (- 959.64)	10	20	-831,356 (87,405)	-811,582 (83,336)	-353,754 (193,479)	-933,859 (42,282)	2-10	-955,126 (12,629)
		50	-898,755 (50,966)	-872,128 (55,929)	-494,758 (189,417)	-942,641 (19,28)	5-25	-959,52 (1,389)

	100	-922,728 (39,165)	-901,721 (35,02)	-581,453 (174,733)	-948,06 (11,934)	10-50	-959,641 (0)
	20	-906,518 (56,547)	-915,565 (32,578)	-385,8 (197,582)	-953,188 (14,935)	2-10	-958,051 (7,69)
	50	-951,301 (23,125)	-935,645 (20,026)	-603,564 (187,643)	-956,638 (6,353)	5-25	-959,641 (0)
	100	-958,517 (8,587)	-947,413 (11,187)	-687,331 (164,77)	-959,215 (1,448)	10-50	-959,641 (0)
Chichinadze (- 43.3159)	20	-42,165 (1,047)	-42,132 (1,036)	36,088 (160,402)	-42,216 (1,263)	2-10	-43,314 (0,036)
	10	-42,625 (0,46)	-42,738 (0,396)	-32,152 (17,574)	-42,972 (0,417)	5-25	-43,316 (0)
	100	-42,841 (0,22)	-42,899 (0,206)	-35,735 (5,868)	-43,187 (0,186)	10-50	-43,316 (0)
	20	-43,212 (0,141)	-43,315 (0,033)	26,039 (155,447)	-43,316 (0)	2-10	-43,315 (0,014)
	50	-43,116 (0,157)	-43,316 (0)	-42,149 (2,558)	-43,316 (0)	5-25	-43,316 (0)
	100	-43,208 (0,122)	-43,316 (0)	-42,794 (0,477)	-43,316 (0)	10-50	-43,316 (0)
Mc Cormick (0.2926)	20	-1,892 (0,038)	-1,906 (0,017)	1,159 (2,84)	-1,91 (0,006)	2-10	-1,913 (0)
	10	-1,906 (0,011)	-1,911 (0,003)	-0,42 (1,533)	-1,912 (0,001)	5-25	-1,913 (0)
	100	-1,91 (0,004)	-1,912 (0,001)	-0,923 (1,366)	-1,913 (0,001)	10-50	-1,913 (0)
	20	-1,913 (0)	-1,913 (0)	1,062 (2,754)	-1,913 (0)	2-10	-1,913 (0)
	50	-1,913 (0)	-1,913 (0)	-0,806 (1,46)	-1,913 (0)	5-25	-1,913 (0)
	100	-1,913 (0)	-1,913 (0)	-1,095 (1,37)	-1,913 (0)	10-50	-1,913 (0)
Three-Humps Camel Back (0)	20	0,044 (0,058)	0,021 (0,036)	19,682 (86,879)	0 (0)	2-10	0 (0)
	10	0,017 (0,019)	0,006 (0,007)	0,761 (2,029)	0 (0)	5-25	0 (0)
	100	0,006 (0,007)	0,003 (0,003)	0,321 (0,737)	0 (0)	10-50	0 (0)
	20	0,001 (0,001)	0 (0)	25,17 (118,18)	0 (0)	2-10	0 (0)
	50	0,001 (0,001)	0 (0)	0,656 (10,216)	0 (0)	5-25	0 (0)
	100	0 (0)	0 (0)	0,109 (0,86)	0 (0)	10-50	0 (0)
Zettle (-0.0038)	20	0,023 (0,037)	0,007 (0,013)	74,722 (177,25)	-0,001 (0,002)	2-10	-0,004 (0)
	10	0,006 (0,011)	0 (0,004)	3,242 (19,146)	-0,003 (0,001)	5-25	-0,004 (0)
	100	0 (0,005)	-0,002 (0,002)	0,353 (3,278)	-0,003 (0,001)	10-50	-0,004 (0)
	50	-0,003 (0,001)	-0,004 (0)	60,979 (147,379)	-0,004 (0)	2-10	-0,004 (0)
	50	-0,003	-0,004 (0)	0,723	-0,004	5-25	-0,004 (0)

		(0,001)		(7,639)	(0)				
	100	-0,004 (0)	-0,004 (0)	0,007 (0,037)	-0,004 (0)	10-50	-0,004 (0)		
Styblinski-Tang (-78.332)	20	-76,262 (3,394)	-78,07 (0,91)	-49,65 (18,417)	-78,331 (0,001)	2-10	-78,332 (0)		
	10	50	-77,797 (0,825)	-78,274 (0,077)	-62,251 (11,248)	-78,324 (0,012)	5-25	-78,332 (0)	
		100	-78,161 (0,24)	-78,307 (0,029)	-66,42 (9,427)	-78,329 (0,003)	10-50	-78,332 (0)	
		20	-78,328 (0,028)	-78,318 (0,447)	-51,558 (17,344)	-78,332 (0)	2-10	-78,332 (0)	
		50	50	-78,327 (0,008)	-78,332 (0)	-66,731 (10,254)	-78,332 (0)	5-25	-78,332 (0)
			100	-78,331 (0,001)	-78,332 (0)	-69,884 (9,05)	-78,332 (0)	10-50	-78,332 (0)
Bukin (-124.75)	20	-124,323 (2,259)	-122,872 (1,458)	29,652 (138,853)	-124,745 (0,017)	2-10	-124,75 (0)		
	10	50	-124,74 (0,038)	-123,834 (0,682)	-59,3 (65,374)	-124,75 (0,001)	5-25	-124,75 (0)	
		100	-124,749 (0,004)	-124,213 (0,38)	-90,311 (50,032)	-124,75 (0)	10-50	-124,75 (0)	
		20	-124,75 (0)	-124,75 (0)	28,075 (147,68)	-124,75 (0)	2-10	-124,75 (0)	
		50	50	-124,75 (0)	-124,75 (0)	-96,45 (52,503)	-124,75 (0)	5-25	-124,75 (0)
			100	-124,75 (0)	-124,75 (0)	-121,241 (19,511)	-124,75 (0)	10-50	-124,75 (0)
Leon (0)	20	0,131 (0,194)	0,054 (0,075)	14,485 (31,803)	0,02 (0,024)	2-10	0 (0)		
	10	50	0,043 (0,061)	0,01 (0,015)	1,559 (3,669)	0,001 (0,001)	5-25	0 (0)	
		100	0,016 (0,02)	0,004 (0,005)	0,874 (1,229)	0,002 (0,001)	10-50	0 (0)	
		20	0,003 (0,012)	0 (0)	14,989 (35,826)	0,004 (0,003)	2-10	0 (0)	
		50	50	0,001 (0,002)	0 (0)	1,107 (5,934)	0,001 (0,001)	5-25	0 (0)
			100	0 (0)	0 (0)	0,372 (0,659)	0 (0)	10-50	0 (0)
Giunta (0.0645)	20	0,066 (0,003)	0,065 (0)	0,228 (0,114)	0,065 (0)	2-10	0,064 (0)		
	10	50	0,065 (0)	0,065 (0)	0,135 (0,085)	0,064 (0)	5-25	0,064 (0)	
		100	0,065 (0)	0,065 (0)	0,102 (0,061)	0,064 (0)	10-50	0,064 (0)	
		20	0,064 (0)	0,064 (0)	0,213 (0,116)	0,064 (0)	2-10	0,064 (0)	
		50	50	0,064 (0)	0,064 (0)	0,112 (0,077)	0,064 (0)	5-25	0,064 (0)
			100	0,064 (0)	0,064 (0)	0,087 (0,053)	0,064 (0)	10-50	0,064 (0)
Schaffer (0)	10	20	0,107	0,096	0,426	0 (0)	2-10	0 (0)	

		(0,079)	(0,063)	(0,1)				
	50	0,048 (0,046)	0,052 (0,036)	0,348 (0,142)	0 (0)	5-25	0 (0)	
	100	0,022 (0,025)	0,033 (0,023)	0,28 (0,147)	0 (0)	10-50	0 (0)	
	20	0,012 (0,017)	0,002 (0,003)	0,397 (0,134)	0 (0)	2-10	0 (0)	
	50	0,003 (0,006)	0 (0,001)	0,241 (0,183)	0 (0)	5-25	0 (0)	
	100	0,001 (0,002)	0 (0)	0,107 (0,146)	0 (0)	10-50	0 (0)	
Schwefel (-837,966)	20	-698,646 (90,85)	-771,261 (62,431)	-303,619 (178,511)	-814,183 (27,664)	2-10	-837,729 (7,483)	
	10	50	-744,772 (73,514)	-813,67 (34,524)	-458,757 (162,125)	-836,981 (0,543)	5-25	-837,966 (0)
		100	-789,51 (52,019)	-830,362 (12,963)	-521,811 (148,818)	-837,516 (0,263)	10-50	-837,966 (0)
	20	-811,898 (52,733)	-836,803 (11,215)	-337,601 (189,937)	-837,963 (0,034)	2-10	-837,966 (0)	
	50	50	-817,097 (46,394)	-837,964 (0,008)	-542,363 (156,527)	-837,966 (0)	5-25	-837,966 (0)
		100	-834,756 (15,099)	-837,966 (0,001)	-626,766 (145,336)	-837,966 (0)	10-50	-837,966 (0)
Ackley (0)	20	2,364 (1,545)	2,715 (1,155)	15,196 (5,157)	0 (0)	2-10	0 (0)	
	10	50	1,247 (0,903)	1,946 (0,935)	9,794 (5,176)	0 (0)	5-25	0 (0)
		100	0,597 (0,475)	1,38 (0,837)	7,414 (3,91)	0 (0)	10-50	0 (0)
	20	0,023 (0,151)	0 (0)	13,906 (6,247)	0 (0)	2-10	0 (0)	
	50	50	0,016 (0,013)	0 (0)	4,196 (6,155)	0 (0)	5-25	0 (0)
		100	0,006 (0,005)	0 (0)	0,867 (2,605)	0 (0)	10-50	0 (0)
Bohachevsky (0)	20	0,662 (0,411)	0,467 (0,283)	71,355 (84,115)	0 (0)	2-10	0 (0)	
	10	50	0,507 (0,276)	0,29 (0,195)	10,7 (25,031)	0 (0)	5-25	0 (0)
		100	0,358 (0,225)	0,174 (0,141)	3,705 (6,584)	0 (0)	10-50	0 (0)
	20	0,023 (0,038)	0 (0)	56,403 (77,762)	0 (0)	2-10	0 (0)	
	50	50	0,029 (0,037)	0 (0)	2,602 (13,553)	0 (0)	5-25	0 (0)
		100	0,011 (0,017)	0 (0)	0,159 (0,711)	0 (0)	10-50	0 (0)
Griewank (0)	20	0,122 (0,076)	0,12 (0,075)	1,054 (0,735)	0 (0)	2-10	0 (0)	
	10	50	0,076 (0,044)	0,074 (0,041)	0,546 (0,346)	0 (0)	5-25	0 (0)
		100	0,055	0,05	0,389	0 (0)	10-50	0 (0)

		(0,031)	(0,027)	(0,244)			
	20	0,023 (0,017)	0,017 (0,012)	0,836 (0,729)	0 (0)	2-10	0 (0)
50	50	0,016 (0,01)	0,009 (0,006)	0,265 (0,298)	0 (0)	5-25	0 (0)
	100	0,01 (0,007)	0,007 (0,004)	0,129 (0,149)	0 (0)	10-50	0 (0)

**Table3: Simulation Results for 2-vars (Continued)**

Function Name	Iter	Pop	GA	DE	PSO	ABC	M-F	$\alpha$ -MGA	
Holzman (0)	20	50	0,034 (0,119)	0,002 (0,007)	1144,584 (2416,603)	0 (0)	2-10	0 (0)	
			0,007 (0,02)	0 (0,001)	59,988 (360,288)	0 (0)	5-25	0 (0)	
			0,001 (0,004)	0 (0)	4,381 (30,146)	0 (0)	10-50	0 (0)	
	50	50	0 (0)	0 (0)	1011,876 (2196,517)	0 (0)	2-10	0 (0)	
			0 (0)	0 (0)	11,948 (176,309)	0 (0)	5-25	0 (0)	
			0 (0)	0 (0)	0,829 (26,213)	0 (0)	10-50	0 (0)	
	Hyperellipsoid (0)	20	50	0,024 (0,035)	0,01 (0,014)	7,873 (9,834)	0 (0)	2-10	0 (0)
				0,011 (0,014)	0,003 (0,004)	1,401 (3,745)	0 (0)	5-25	0 (0)
				0,005 (0,006)	0,002 (0,002)	0,401 (0,855)	0 (0)	10-50	0 (0)
50		50	0 (0)	0 (0)	6,627 (8,947)	0 (0)	2-10	0 (0)	
			0 (0)	0 (0)	0,275 (1,529)	0 (0)	5-25	0 (0)	
			0 (0)	0 (0)	0,032 (0,675)	0 (0)	10-50	0 (0)	
Levy (0)		20	50	0,033 (0,048)	0,01 (0,018)	3,753 (5,212)	0,001 (0,002)	2-10	0 (0)
				0,016 (0,022)	0,003 (0,004)	0,917 (1,566)	0 (0)	5-25	0 (0)
				0,007 (0,008)	0,001 (0,002)	0,366 (0,652)	0 (0)	10-50	0 (0)
	50	50	0 (0)	0 (0)	3,528 (5,123)	0 (0)	2-10	0 (0)	
			0 (0)	0 (0)	0,599 (1,355)	0 (0)	5-25	0 (0)	
			0 (0)	0 (0)	0,096 (0,511)	0 (0)	10-50	0 (0)	
	Maxmod (0)	20	50	1,302 (0,913)	1,551 (0,95)	33,915 (22,938)	0 (0)	2-10	0 (0)
				0,87 (0,53)	0,884 (0,52)	11,662 (10,909)	0 (0)	5-25	0 (0)
				0,582 (0,359)	0,612 (0,332)	6,849 (5,327)	0 (0)	10-50	0 (0)
50		50	0,058 (0,053)	0 (0)	28,639 (22,637)	0 (0)	2-10	0 (0)	
			0,083 (0,056)	0 (0)	2,604 (7,021)	0 (0)	5-25	0 (0)	
			0,037 (0,029)	0 (0)	0,19 (0,746)	0 (0)	10-50	0 (0)	
Multimod (0)		10	20	0,014	0,001	28,851 (110,77)	0 (0)	2-10	0 (0)

		(0,033)	(0,003)				
	50	0,004 (0,007)	0 (0,001)	1,659 (4,464)	0 (0)	5-25	0 (0)
	100	0,001 (0,002)	0 (0)	0,775 (2,752)	0 (0)	10-50	0 (0)
	20	0 (0)	0 (0)	21,35 (104,755)	0 (0)	2-10	0 (0)
	50	0 (0)	0 (0)	0,056 (1,435)	0 (0)	5-25	0 (0)
	100	0 (0)	0 (0)	0,002 (0,022)	0 (0)	10-50	0 (0)
Rastrigin (0)	20	1,983 (1,35)	1,876 (1,121)	15,261 (9,574)	0 (0)	2-10	0 (0)
	10	1,232 (0,736)	1,077 (0,644)	9,118 (6,156)	0 (0)	5-25	0 (0)
	100	0,846 (0,592)	0,74 (0,492)	6,588 (4,38)	0 (0)	10-50	0 (0)
	20	0,178 (0,341)	0,149 (0,263)	12,461 (9,686)	0 (0)	2-10	0 (0)
	50	0,08 (0,181)	0,025 (0,052)	4,221 (4,673)	0 (0)	5-25	0 (0)
	100	0,028 (0,067)	0,008 (0,011)	2,243 (2,734)	0 (0)	10-50	0 (0)
Rosenbrock (0)	20	1,703 (2,307)	1,034 (1,251)	17720,27 (66454,04)	0,212 (0,208)	2-10	0 (0)
	10	0,752 (0,906)	0,316 (0,406)	217,524 (2195,584)	0,049 (0,058)	5-25	0 (0)
	100	0,332 (0,373)	0,122 (0,137)	32,155 (167,565)	0,02 (0,024)	10-50	0 (0)
	20	0,087 (0,157)	0,01 (0,047)	19664,01 (81922,65)	0,046 (0,046)	2-10	0 (0)
	50	0,076 (0,112)	0 (0,001)	30,562 (219,707)	0,009 (0,009)	5-25	0 (0)
	100	0,032 (0,047)	0 (0)	9,086 (118,357)	0,003 (0,003)	10-50	0 (0)
Sine Envelope (0)	20	0,14 (0,093)	0,103 (0,064)	0,421 (0,112)	0 (0)	2-10	0,003 (0,01)
	10	0,084 (0,06)	0,058 (0,037)	0,346 (0,143)	0 (0)	5-25	0 (0,002)
	100	0,045 (0,033)	0,037 (0,024)	0,274 (0,146)	0 (0)	10-50	0 (0)
	20	0,03 (0,033)	0,01 (0,002)	0,399 (0,129)	0 (0)	2-10	0,001 (0,007)
	50	0,032 (0,031)	0,009 (0,002)	0,255 (0,18)	0 (0)	5-25	0 (0)
	100	0,012 (0,009)	0,008 (0,003)	0,124 (0,153)	0 (0)	10-50	0 (0)
Sphere (0)	20	0,014 (0,02)	0,007 (0,01)	5,333 (6,202)	0 (0)	2-10	0 (0)
	10	0,007 (0,009)	0,002 (0,003)	0,848 (1,975)	0 (0)	5-25	0 (0)
	100	0,004 (0,004)	0,001 (0,001)	0,244 (0,629)	0 (0)	10-50	0 (0)
	50	0 (0)	0 (0)	4,707 (5,929)	0 (0)	2-10	0 (0)

		50	0 (0)	0 (0)	0,18 (0,983)	0 (0)	5-25	0 (0)
		100	0 (0)	0 (0)	0,005 (0,121)	0 (0)	10-50	0 (0)
Sumsquares (0)	10	20	0,081 (0,118)	0,038 (0,053)	29,539 (37,683)	0 (0)	2-10	0 (0)
		50	0,045 (0,053)	0,013 (0,015)	4,693 (11,137)	0 (0)	5-25	0 (0)
		100	0,019 (0,024)	0,006 (0,006)	1,27 (2,747)	0 (0)	10-50	0 (0)
	50	20	0,001 (0,001)	0 (0)	26,949 (36,544)	0 (0)	2-10	0 (0)
		50	0,001 (0,001)	0 (0)	1,354 (8,694)	0 (0)	5-25	0 (0)
		100	0 (0,001)	0 (0)	0,047 (0,761)	0 (0)	10-50	0 (0)

When the number of parameters is increased, the performance distinction is more revealed. Table 4 summarizes the reports for 10-variables cases. It is shown in Table 4 that *ABC* and  *$\alpha$ -MGA* clearly have better performances than *GA*, *DE*, and *PSO*.  *$\alpha$ -MGA* and *ABC* have nearly equal performance except three functions. For Levy and Rosenbrock functions  *$\alpha$ -MGA* performs better than *ABC*. However, for Sine Envelope function, *ABC* performs better. Overall,  *$\alpha$ -MGA* outperforms the rest except for Sine Envelope function.

**Table4:** Simulation Results for 10-vars

Function Name	Iter	Pop	GA	DE	PSO	ABC	M-F	$\alpha$ -MGA
Ackley (0)	10	20	13,664 (1,856)	17,642 (1,103)	19,68 (0,98)	0 (0)	2-10	0 (0)
		50	10,276 (1,621)	16,772 (1,134)	18,686 (1,443)	0 (0)	5-25	0 (0)
		100	8,419 (1,206)	16,103 (1,053)	17,82 (1,48)	0 (0)	10-50	0 (0)
	50	20	8,25 (1,965)	6,935 (1,103)	19,313 (1,122)	0 (0)	2-10	0 (0)
		50	4,321 (1,082)	6,194 (0,776)	17,061 (2,326)	0 (0)	5-25	0 (0)
		100	2,462 (0,668)	5,654 (0,669)	13,676 (3,185)	0 (0)	10-50	0 (0)
Bohachevsky (0)	10	20	147,479 (69,627)	362,909 (111,571)	939,113 (374,452)	0 (0)	2-10	0 (0)
		50	78,586 (32,106)	277,018 (81,691)	588,271 (270,352)	0 (0)	5-25	0 (0)
		100	54,039 (18,95)	227,112 (62,437)	422,896 (197,751)	0 (0)	10-50	0 (0)
	50	20	32,623 (19,941)	15,184 (4,422)	885,218 (363,058)	0 (0)	2-10	0 (0)
		50	10,877 (4,009)	11,988 (2,477)	339,369 (222,862)	0 (0)	5-25	0 (0)
		100	7,778 (1,836)	10,317 (1,783)	139,899 (123,145)	0 (0)	10-50	0 (0)
Griewank (0)	10	20	1,569 (0,278)	2,506 (0,444)	4,765 (1,501)	0 (0)	2-10	0 (0)

	50	1,296 (0,137)	2,145 (0,341)	3,46 (1,088)	0 (0)	5-25	0 (0)
	100	1,19 (0,091)	1,952 (0,268)	2,749 (0,827)	0 (0)	10-50	0 (0)
	20	0,973 (0,181)	0,975 (0,094)	4,616 (1,475)	0 (0)	2-10	0 (0)
50	50	0,846 (0,116)	0,905 (0,104)	2,41 (0,974)	0 (0)	5-25	0 (0)
	100	0,819 (0,108)	0,848 (0,108)	1,513 (0,509)	0 (0)	10-50	0 (0)
Holzman (0)	20	972,231 (1008,855)	4495,354 (2753,344)	31790,38 (24119,01)	0 (0)	2-10	0 (0)
10	50	314,529 (275,631)	2513,16 (1436,615)	12437,99 (11562,48)	0 (0)	5-25	0 (0)
	100	145,016 (118,433)	1733,758 (986,383)	6835,87 (7309,596)	0 (0)	10-50	0 (0)
	20	75,959 (127,9)	4,973 (5,741)	28679,41 (23996,48)	0 (0)	2-10	0 (0)
50	50	10,306 (15,282)	2,076 (1,636)	5352,36 (8347,281)	0 (0)	5-25	0 (0)
	100	4,838 (6,031)	1,188 (0,808)	1186,418 (2619,582)	0 (0)	10-50	0 (0)
Hyperellipsoid (0)	20	29,656 (15,228)	75,98 (24,914)	215 (90,073)	0 (0)	2-10	0 (0)
10	50	15,738 (7,009)	55,657 (16,354)	128,897 (62,882)	0 (0)	5-25	0 (0)
	100	10,442 (4,269)	45,851 (13,581)	90,271 (44,727)	0 (0)	10-50	0 (0)
	20	5,799 (4,79)	1,945 (0,956)	206,624 (97,834)	0 (0)	2-10	0 (0)
50	50	1,179 (0,759)	1,286 (0,483)	78,076 (57,863)	0 (0)	5-25	0 (0)
	100	0,563 (0,317)	0,992 (0,329)	27,119 (25,638)	0 (0)	10-50	0 (0)
Levy (0)	20	7,245 (3,652)	14,956 (5,087)	44,649 (22,728)	0,46 3 (0,106)	2-10	0 (0)
10	50	4,03 (2,051)	11,443 (3,566)	27,83 (14,099)	0,00 2 (0) 0,03	5-25	0 (0)
	100	2,863 (1,257)	9,431 (2,947)	20,209 (10,428)	2 (0,005)	10-50	0 (0)
50	20	1,409 (1,001)	0,7 (0,318)	38,385 (20,41)	0,02 7 (0,022)	2-10	0 (0)
	50	0,498 (0,293)	0,476 (0,149)	14,968 (10,226)	0,00 1 (0)	5-25	0 (0)



	100	0,317 (0,135)	0,377 (0,096)	7,352 (5,294)	0,00 4 (0,00 2)	10-50	0 (0)	
Maxmod (0)	20	25,844 (6,771)	50,005 (7,923)	67,736 (13,411)	0 (0)	2-10	0 (0)	
	10	50	17,888 (4,061)	43,974 (6,633)	54,175 (11,942)	0 (0)	5-25	0 (0)
	100	14,096 (3,029)	39,945 (5,771)	46,693 (11,093)	0 (0)	10-50	0 (0)	
	20	17,438 (6,098)	21,968 (4,55)	66,049 (15,157)	0 (0)	2-10	0 (0)	
	50	50	6,006 (2,359)	18,527 (3,31)	43,812 (13,071)	0 (0)	5-25	0 (0)
	100	2,683 (0,924)	16,688 (2,624)	31,32 (10,123)	0 (0)	10-50	0 (0)	
Multimod (0)	20	12,383 (92,505)	101,737 (357,865)	787727,8 (9142498)	0 (0)	2-10	0 (0)	
	10	50	1,422 (5,098)	9,178 (25,199)	29396,3 (414184,7)	0 (0)	5-25	0 (0)
	100	0,213 (0,858)	1,783 (3,985)	2642,988 (13607,49)	0 (0)	10-50	0 (0)	
	20	0,012 (0,133)	0 (0)	4120,023 (26255,66)	0 (0)	2-10	0 (0)	
	50	50	0,046 (0,403)	0 (0)	76,866 (1143,41)	0 (0)	5-25	0 (0)
	100	0,005 (0,029)	0 (0)	6,843 (106,578)	0 (0)	10-50	0 (0)	
Rastrigin (0)	20	65,23 (11,076)	81,226 (11,56)	117,713 (23,615)	0 (0)	2-10	0 (0)	
	10	50	59,214 (9,314)	72,368 (9,658)	103,49 (19,032)	0 (0)	5-25	0 (0)
	100	54,427 (8,316)	66,862 (8,774)	95,714 (16,243)	0 (0)	10-50	0 (0)	
	20	23,744 (8,333)	30,663 (5,713)	104,94 (24,675)	0 (0)	2-10	0 (0)	
	50	50	27,932 (10,143)	26,375 (4,443)	68,697 (19,379)	0 (0)	5-25	0 (0)
	100	34,278 (7,398)	23,662 (3,948)	57,079 (16,709)	0 (0)	10-50	0 (0)	
Rosenbrock (0)	20	17042,88 (16001,3)	23140,21 (15034,07)	400170,1 (328178,6)	9 (0)	2-10	0 (0)	
	10	50	6414,64 (5259,97 9)	13605,17 (7501,357)	143787,2 (147094,7)	9 (0)	5-25	0 (0)
	100	3334,193 (2275,22 7)	8956,315 (4807,962)	68520,52 (67955,59)	3,55 1 (0,22 2)	10-50	0 (0)	
	50	20	1402,844	243,277	394071,8	8,97	2-10	0 (0)

		(1925,824)	(114,312)	(318823,7)	7 (0,187) 7,50			
	50	376,119 (362,377)	161,01 (61,437)	60474,99 (94959,98)	6 (1,787)	5-25	0 (0)	
	100	258,205 (170,224)	122,794 (40,909)	13290,32 (22382,68)	2,64 1 (0,774)	10-50	0 (0)	
Sine Envelope (0)	20	3,696 (0,254)	3,707 (0,229)	4,159 (0,235)	0 (0)	2-10	0,128 (0,193)	
	10	50	3,54 (0,245)	3,559 (0,203)	4,031 (0,235)	0 (0)	5-25	0,044 (0,027)
		100	3,428 (0,239)	3,45 (0,19)	3,951 (0,218)	0 (0)	10-50	0,029 (0,021)
	50	20	2,726 (0,424)	2,716 (0,269)	4,021 (0,294)	0 (0)	2-10	0,022 (0,048)
		50	2,848 (0,464)	2,522 (0,243)	3,743 (0,313)	0 (0)	5-25	0,005 (0,011)
		100	2,765 (0,413)	2,382 (0,232)	3,576 (0,302)	0 (0)	10-50	0,001 (0,004)
Sphere (0)	20	6,051 (2,911)	15,672 (5,053)	41,221 (16,183)	0 (0)	2-10	0 (0)	
	10	50	3,216 (1,446)	11,885 (3,525)	25,417 (11,229)	0 (0)	5-25	0 (0)
		100	2,091 (0,815)	9,975 (2,796)	18,202 (8,186)	0 (0)	10-50	0 (0)
	50	20	1,072 (0,739)	0,417 (0,194)	38,197 (15,33)	0 (0)	2-10	0 (0)
		50	0,231 (0,152)	0,268 (0,101)	14,273 (9,089)	0 (0)	5-25	0 (0)
		100	0,11 (0,067)	0,206 (0,069)	5,808 (5,383)	0 (0)	10-50	0 (0)
Sumsquares (0)	20	114,106 (55,213)	291,724 (95,993)	817,292 (359,308)	0 (0)	2-10	0 (0)	
	10	50	59,741 (26,435)	216,137 (62,926)	497,445 (249,402)	0 (0)	5-25	0 (0)
		100	39,19 (16,176)	176,605 (51,026)	344,278 (169,925)	0 (0)	10-50	0 (0)
	50	20	21,352 (16,761)	7,473 (3,429)	767,07 (367,142)	0 (0)	2-10	0 (0)
		50	4,293 (2,909)	4,956 (1,832)	277,628 (192,593)	0 (0)	5-25	0 (0)
		100	2,095 (1,193)	3,735 (1,212)	114,339 (110,063)	0 (0)	10-50	0 (0)

Table 5 presents some additional information. It is shown in Table 5 that performances of *GA*, *DE*, and *PSO* are slightly decreased as the number of parameters is increased. The performances of these algorithms drastically decreased for some functions. However, success of *ABC* and  $\alpha$ -*MGA* still remains

same. As in shown in 4, *ABC* has the same failure for Levy and Rosenbrock. Similarly,  $\alpha$ -MGA failures for Sine Envelope for  $p = 25$  as same in case of  $p = 10$ .

**Table5:** Simulation results for 25-vars

Function Name	Iter	Pop	GA	DE	PSO	ABC	M-F	$\alpha$ -MGA
Ackley (0)	10	20	15,591 (1,116)	20,141 (0,294)	20,124 (0,459)	0 (0)	2-10	0 (0)
		50	12,712 (1,039)	19,919 (0,31)	19,61 (0,636)	0 (0)	5-25	0 (0)
		100	10,819 (0,885)	19,73 (0,305)	19,256 (0,661)	0 (0)	10-50	0 (0)
	50	20	13,114 (1,332)	16,843 (0,662)	19,872 (0,527)	0 (0)	2-10	0 (0)
		50	9,242 (1,113)	16,207 (0,64)	18,94 (0,926)	0 (0)	5-25	0 (0)
		100	6,666 (0,838)	15,797 (0,598)	17,794 (1,189)	0 (0)	10-50	0 (0)
Bohachevsky (0)	10	20	596,049 (171,753)	2555,948 (360,233)	2663,682 (645,92)	0 (0)	2-10	0 (0)
		50	348,977 (97,386)	2267,685 (297,989)	2079,327 (527,234)	0 (0)	5-25	0 (0)
		100	245,535 (59,148)	2108,463 (268,957)	1761,264 (439,633)	0 (0)	10-50	0 (0)
	50	20	297,013 (100,067)	672,875 (132,4)	2596,184 (633,267)	0 (0)	2-10	0 (0)
		50	121,054 (37,452)	572,092 (95,564)	1533,057 (486,283)	0 (0)	5-25	0 (0)
		100	69,175 (19,407)	500,888 (77,811)	989,931 (349,042)	0 (0)	10-50	0 (0)
Griewank (0)	10	20	3,295 (0,681)	10,895 (1,396)	11,238 (2,435)	0 (0)	2-10	0 (0)
		50	2,305 (0,373)	9,8 (1,082)	8,995 (2,003)	0 (0)	5-25	0 (0)
		100	1,883 (0,239)	9,08 (1,028)	7,805 (1,72)	0 (0)	10-50	0 (0)
	50	20	2,118 (0,418)	3,581 (0,506)	10,849 (2,433)	0 (0)	2-10	0 (0)
		50	1,416 (0,15)	3,165 (0,376)	6,957 (1,866)	0 (0)	5-25	0 (0)
		100	1,201 (0,072)	2,898 (0,316)	4,899 (1,402)	0 (0)	10-50	0 (0)
Holzman (0)	10	20	12730,57 (8239,852)	170395,1 (48367,99)	202353,5 (93126,63)	0 (0)	2-10	0 (0)
		50	4988,124 (3079,53)	136220,5 (34913,89)	128031 (70142,38)	0 (0)	5-25	0 (0)
		100	2575,84 (1540,132)	113060,6 (28568,24)	91709,15 (49673)	0 (0)	10-50	0 (0)
	50	20	4019,249 (2961,206)	14552,4 (6130,233)	187778,2 (91848,68)	0 (0)	2-10	0 (0)
		50	926,4	9842,23	82009,01	0 (0)	5-25	0 (0)

			(683,06)	(3588,24)	(56733,65)			
		100	373,321 (283,523)	7540,364 (2524,816)	36054,46 (28341,8)	0 (0)	10-50	0 (0)
Hyperellipsoid (0)		20	292,814 (96,646)	1237,22 (196,461)	1396,96 (377,215)	0 (0)	2-10	0 (0)
	10	50	165,578 (50,289)	1082,355 (155,653)	1018,812 (291,165)	0 (0)	5-25	0 (0)
		100	111,387 (32,599)	975,771 (136,795)	861,945 (243,886)	0 (0)	10-50	0 (0)
		20	145,466 (58,716)	305,826 (65,239)	1324,351 (365,588)	0 (0)	2-10	0 (0)
	50	50	49,812 (18,168)	256,33 (47,921)	733,985 (254,159)	0 (0)	5-25	0 (0)
		100	23,6 (8,578)	222,357 (38,123)	465,168 (184,265)	0 (0)	10-50	0 (0)
Levy (0)		20	34,236 (11,462)	117,111 (21,333)	138,133 (38,479)	1,39 (0,112)	2-10	0 (0)
	10	50	22,347 (7,027)	101,432 (16,371)	106,036 (34,202)	0,005 (0)	5-25	0 (0)
		100	16,106 (5,404)	91,576 (14,114)	89,07 (26,28)	0,09 (0,005)	10-50	0 (0)
		20	14,386 (5,832)	30,443 (6,189)	126,745 (39,444)	0,53 (0,132)	2-10	0 (0)
	50	50	6,209 (2,718)	25,253 (4,746)	69,083 (26,987)	0,005 (0,001)	5-25	0 (0)
		100	3,029 (1,314)	22,035 (4,124)	44,766 (16,847)	0,051 (0,009)	10-50	0 (0)
Maxmod (0)		20	38,464 (6,675)	79,928 (5,152)	79,305 (9,284)	0 (0)	2-10	0 (0)
	10	50	27,873 (4,252)	76,119 (4,749)	71,203 (9,917)	0 (0)	5-25	0 (0)
		100	22,514 (3,132)	73,377 (4,542)	67,495 (9,325)	0 (0)	10-50	0 (0)
		20	32,844 (6,364)	63,13 (5,335)	78,116 (10,069)	0 (0)	2-10	0 (0)
	50	50	17,844 (3,453)	59,107 (4,705)	64,634 (10,31)	0 (0)	5-25	0 (0)
		100	11,254 (2,044)	56,299 (4,139)	55,567 (9,701)	0 (0)	10-50	0 (0)
Multimod (0)		20	116179,1 (938043,3)	7,5e9 (3.6e10)	1.7e13 (2.1e14)	0 (0)	2-10	0 (0)
	10	50	2368,921 (25954,74)	3.2e8 (1.5e9)	7.2e10 (7.99e11)	0 (0)	5-25	0 (0)
		100	24,123 (326,568)	3.4e7 (1.3e8)	1.75e10 (4.2e11)	0 (0)	10-50	0 (0)
		20	167,382 (4147,982)	1,45 (6,939)	3.3e11 (8e12)	0 (0)	2-10	0 (0)
	50	50	37,647 (866,935)	0,057 (0,553)	3.79e6 (4.25e8)	0 (0)	5-25	0 (0)
		100	0,054 (0,99)	0,005 (0,02)	363447,8 (7526011)	0 (0)	10-50	0 (0)
Rastrigin (0)	10	20	213	306,379	323,735	0 (0)	2-10	0 (0)

			(20,334)	(22,324)	(37,242)			
	50		199,327 (16,79)	290,143 (19,087)	298,567 (30,551)	0 (0)	5-25	0 (0)
	100		191,584 (14,645)	277,861 (18,168)	285,816 (28,529)	0 (0)	10-50	0 (0)
	20		123,685 (22,096)	181,071 (14,75)	299,926 (39,794)	0 (0)	2-10	0 (0)
	50	50	120,475 (24,468)	170,48 (13,23)	235,241 (34,072)	0 (0)	5-25	0 (0)
	100		137,999 (20,882)	161,588 (11,984)	217,121 (32,874)	0 (0)	10-50	0 (0)
Rosenbrock (0)	20		149335,1 (70645,16)	628979,1 (220802,8)	1220033 (566082,1)	24 (0)	2-10	0 (0)
	10	50	84214,46 (35776,88)	463505,1 (153314,5)	726755,4 (371050,5)	24 (0)	5-25	0 (0)
	100		53482,74 (20751,93)	366797,6 (112093,1)	527091,6 (268808,5)	9,911 (0,134)	10-50	0 (0)
	20		48965,88 (28333,92)	47580,36 (18805,54)	1147589 (518185,2)	24 (0)	2-10	0 (0)
	50	50	22540,67 (12972,13)	31791,56 (11112,14)	462267,1 (295798,9)	24 (0)	5-25	0 (0)
	100		13398,31 (7238,126)	24822,7 (7984,633)	198604,2 (146506,3)	9,674 (0,453)	10-50	0 (0)
Sine Envelope (0)	20		10,787 (0,369)	11,02 (0,241)	11,429 (0,308)	0 (0)	2-10	0,459 (0,642)
	10	50	10,597 (0,352)	10,832 (0,233)	11,264 (0,317)	0 (0)	5-25	0,166 (0,083)
	100		10,409 (0,327)	10,705 (0,235)	11,166 (0,299)	0 (0)	10-50	0,125 (0,053)
	20		9,524 (0,628)	9,891 (0,316)	11,153 (0,384)	0 (0)	2-10	0,063 (0,195)
	50	50	9,578 (0,622)	9,659 (0,291)	10,849 (0,413)	0 (0)	5-25	0,009 (0,018)
	100		9,372 (0,61)	9,476 (0,293)	10,662 (0,386)	0 (0)	10-50	0,002 (0,006)
Sphere (0)	20		23,946 (7,464)	102,913 (14,281)	108,344 (25,436)	0 (0)	2-10	0 (0)
	10	50	13,419 (3,737)	91,873 (12,115)	83,823 (20,927)	0 (0)	5-25	0 (0)
	100		9,405 (2,589)	85,209 (10,245)	70,264 (17,634)	0 (0)	10-50	0 (0)
	20		11,523 (4,197)	27,231 (5,27)	103,017 (25,319)	0 (0)	2-10	0 (0)
	50	50	4,414 (1,581)	22,313 (3,759)	62,555 (19,236)	0 (0)	5-25	0 (0)
	100		2,153 (0,765)	19,815 (3,164)	39,895 (14,161)	0 (0)	10-50	0 (0)
Sumsquares (0)	20		1125,485 (386,555)	4731,712 (743,823)	5280,613 (1373,774)	0 (0)	2-10	0 (0)
	10	50	636,441 (192,483)	4138,663 (603,23)	3980,108 (1146,182)	0 (0)	5-25	0 (0)
	100		427,158 (122,255)	3748,475 (541,73)	3250,859 (930,374)	0 (0)	10-50	0 (0)

	20	532,576 (205,717)	1197,032 (259,949)	5048,675 (1441,534)	0 (0)	2-10	0 (0)
	50	189,056 (71,022)	971,884 (182,146)	2895,587 (1038,595)	0 (0)	5-25	0 (0)
	100	89,528 (32,12)	853,336 (145,365)	1762,823 (699,497)	0 (0)	10-50	0 (0)

The simulation results show that *ABC* and  *$\alpha$ -MGA* outperform other methods in many cases. We applied a 2-samples *Wilcoxon Test (Mann-Whitney)* for independent samples to test equality of location parameters of two populations formed by results obtained by algorithms. Small p-values obtained by the test indicate that we can safely reject the null-hypothesis of  $H_0: \mu_1 = \mu_2$  in contrast to  $H_a: \mu_1 \neq \mu_2$  where  $\mu_1$  and  $\mu_2$  are location parameters of distributions related to the corresponding objective values. Table 6 reports the test results which have p-values are greater than 0.01.

**Table 6:** Cases of have that p-value greater than 0.01

Algorithm	Test Function	Variable Count	Iteration	Population Size	M-F	p-value
ABC	Carrom Table	2	50	100	10/50	0.1574
DE	Cross in Tray	2	50	100	10/50	0.02523
GA	Cross Leg Table	2	10	100	10/50	0.08321
ABC	Cross Leg Table	2	10	100	10/50	0.08321
ABC	Cross Leg Table	2	50	20	2/10	0.04539
DE	Pen Holder	2	50	100	10/50	0.31779
ABC	Modified Schaffer 1	2	10	50	5/25	0.04539
ABC	Chichinadze	2	50	50	5/25	0.31779
ABC	Mc Cormick	2	50	20	2/10	0.31779
ABC	Zettle	2	50	100	10/50	0.31779
DE	Styblinski-Tang	2	50	20	2/10	0.31779
DE	Bukin	2	50	50	5/25	0.08321
GA	Bukin	2	50	100	10/50	0.31779
DE	Leon	2	50	100	10/50	0.31779
GA	Giunta	2	50	100	10/50	0.0142
DE	Multimod	10	50	20	2/10	0.02523

It is shown in Table 6 that performance equality of *ABC* and  *$\alpha$ -MGA* can not be rejected for Carrom Table, Cross Leg Table, Modified Schaffer 1, Chichinadze, Mc Cormick, and Zettle functions in the case of 2 parameters. Performance difference of *DE* and  *$\alpha$ -MGA* is insignificant for 2-parameters versions of Cross In Tray, Pen Holder, Styblinski-Tang, Bukin, and Leon functions. Similarly, *GA* has nearly equal performance with  *$\alpha$ -MGA* for 2-parameters versions of Cross Leg Table, Bukin, and Giunta. Performance of *PSO* is almost always different than the performance of  *$\alpha$ -MGA*. Note that these comparisons include only the successfully calculated test cases that report the p-value larger than 0.01. In some cases, mostly as seen between *ABC* and  *$\alpha$ -MGA*, algorithms reported the optimum precisely and the corresponding p-values are reported as NAs. These cases are not reported.

## 5. CONCLUSION

$\alpha$ -MGA is a modern, efficient, sexist, and population based evolutionary algorithm that mimic the sexual selection of animal swarms. Since only the general principals are given in the original paper (Drezner & Drezner, 2018), we specialize some steps of the algorithm for floating-point optimization of functions. We suggest to use the *Linear Crossover* and *Hooke-Jeeves* method for recombination and local fine-tuning operators, respectively. According to the previous simulation studies, It is proven that the *Linear Crossover* operator outperforms its counterparts. *Hooke-Jeeves* algorithm is a successful method for local search and it is very useful when it is coupled with an evolutionary algorithm as a hybridization tool. We perform a simulation study on a rich set of test functions to reveal the performance differences of the specialized  $\alpha$ -MGA and the some set of evolutionary optimization methods. The simulation study involves comparisons with Genetic Algorithms (GAs) with floating-point encoding scheme, Differential Evolution (DE), Particle Swarm Optimization (PSO), and Artificial Bee Colony algorithm (ABC). Simulation results show that ABC and the specialized  $\alpha$ -MGA have superior performance by means of obtaining global optimum of test functions. Dispite having similar results, the specialized algorithm outperforms the ABC method in a small subset of simulation configuration cases.

## REFERENCES

- Allenson, R. (1992). Genetic algorithms with gender for multi-function optimisation. Edinburgh Parallel Computing Centre, Edinburgh, Scotland, Tech. Rep. EPCC-SS92-01.
- Ansotegui, C., Sellmann, M., & Tierney, K. (2009). A gender-based genetic algorithm for the automatic conguration of algorithms. *International Conference on Principles and Practice of Constraint Programming*, 142-157.
- Drezner, T. & Drezner, Z. (2006). Gender-specic genetic algorithms. *INFOR: Information Systems and Operational Research*, 44(2), 117-127.
- Drezner, Z. (2008). Extensive experiments with hybrid genetic algorithms for the solution of the quadratic assignment problem. *Computers & Operations Research*, 35(3), 717-736.
- Drezner, Z. & Drezner, T. D. (2018). The alpha male genetic algorithm. *IMA Journal of Management Mathematics*.
- Eberhart, R. & Kennedy, J. (1995). A new optimizer using particle swarm theory. In *Micro Machine and Human Science, 1995. MHS'95, Proceedings of the Sixth International Symposium on*, 39-43.
- Esquivel, S. C., Leiva, H. A., & Gallard, R. H. (1999). Multiplicity in genetic algorithms to face multicriteria optimization. *Evolutionary Computation, 1999. CEC 99. Proceedings of the 1999 Congress on*, 1, 85-90.
- Goldberg, D. (1989). *Genetic algorithms: search and optimization algorithms*.

- Herrera, F., Lozano, M., & Sanchez, A. M. (2003). A taxonomy for the crossover operator for real-coded genetic algorithms: An experimental study. *International Journal of Intelligent Systems*, 18(3), 309-338.
- Holland, J. H. (1975). *Adaptation in natural and artificial systems. An introductory analysis with application to biology, control, and artificial intelligence*. Ann Arbor, MI: University of Michigan Press, 439-444.
- Sanchez-Velazco, J. & Bullinaria, J. A., (2003). Sexual selection with competitive/co-operative operators for genetic algorithms. *Neural Networks and Computational Intelligence*, 191-196.
- Karaboga, D. & Basturk, B. (2007). A powerful and efficient algorithm for numerical function optimization: artificial bee colony (abc) algorithm. *Journal of Global Optimization*, 39(3), 459-471.
- Lis, J. & Eiben, A. E. (1997). A multi-sexual genetic algorithm for multiobjective optimization. *Evolutionary Computation, 1997., IEEE International Conference on*, 59-64.
- Mishra, S. K. (2006). Some new test functions for global optimization and performance of repulsive particle swarm method.
- Moser, I. (2009). Hooke-jeeves revisited. *Evolutionary Computation, 2009. CEC'09. IEEE Congress on*, 2670-2676.
- Rejeb, J. & AbuElhajj, M. (2000). New gender genetic algorithm for solving graph partitioning problems. *Circuits and Systems, 2000. Proceedings of the 43rd IEEE Midwest Symposium on*, 444-446.
- Satman, M. H. (2015). Hybridization of floating-point genetic algorithms using hooke-jeeves algorithm as an intelligent mutation operator. *Journal of Mathematical and Computational Science*, 5(3), 320.
- Satman, M. H. & Akadal, E. (2017). Machine-coded genetic operators and their performances in floating-point genetic algorithms. *International Journal of Advanced Mathematical Sciences*, 5(1), 8-19.
- Storn, R. & Price, K. (1997). Differential evolution-a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*, 11(4), 341-359.
- Wagner, S. & Affenzeller, M. (2005). Sexualga: Gender-specific selection for genetic algorithms. *Proceedings of the 9th World Multi-Conference on Systemics, Cybernetics and Informatics (WMSCI)*, 4, 76-81.
- Vrajitoru, D. (2002). Simulating gender separation with genetic algorithms. *Proceedings of the 4th Annual Conference on Genetic and Evolutionary Computation*. 634-641. Morgan Kaufmann Publishers Inc., 2002.



**APPENDIX 1**

Test tube holder function:

$$f(x_1, x_2) = -4|\sin(x_1)\cos(x_2)e^{|\cos(\frac{x_1^2+x_2^2}{200})|}$$

for  $-10 \leq x_1, x_2 \leq 10$  and  $f^* = -10.8723$  is the global minimum.

Holder table function:

$$f(x_1, x_2) = -|\cos(x_1)\cos(x_2)e^{-(x_1^2+x_2^2)^{0.5}/\pi}|$$

for  $-10 \leq x_1, x_2 \leq 10$  and  $f^* = 26.92$  is the global minimum.

Carrom table function:

$$f(x_1, x_2) = -[\cos(x_1)\cos(x_2)e^{1-(x_1^2+x_2^2)^{0.5}/\pi}]^2/30$$

for  $-10 \leq x_1, x_2 \leq 10$  and  $f^* = -24.1568155$  is the global minimum.

Cross in tray function:

$$f(x_1, x_2) = -0.0001[|\sin(x_1)\sin(x_2)e^{100-(x_1^2+x_2^2)^{0.5}/\pi}| + 1]^{0.1}$$

for  $-10 \leq x_1, x_2 \leq 10$  and  $f^* = -2.06261218$  is the global minimum.

Crowned cross function:

$$f(x_1, x_2) = 0.0001[|\sin(x_1)\sin(x_2)e^{100-(x_1^2+x_2^2)^{0.5}/\pi}| + 1]^{0.1}$$

for  $-10 \leq x_1, x_2 \leq 10$  and  $f^* = 0$  is the global minimum.

Cross function:

$$f(x_1, x_2) = [|\sin(x_1)\sin(x_2)e^{100-(x_1^2+x_2^2)^{0.5}/\pi}| + 1]^{-0.1}$$

for  $-10 \leq x_1, x_2 \leq 10$  and  $f^* = 0$  is the global minimum.

Cross-leg table function:

$$f(x_1, x_2) = -[|\sin(x_1)\sin(x_2)e^{100-(x_1^2+x_2^2)^{0.5}/\pi}| + 1]^{-0.1}$$

for  $-10 \leq x_1, x_2 \leq 10$  and  $f^* = -1$  is the global minimum.

Pen holder function:

$$f(x_1, x_2) = -\exp[-|\cos(x_1)\cos(x_2)e^{1-(x_1^2+x_2^2)^{0.5}/\pi}|^{-1}]$$

for  $-11 \leq x_1, x_2 \leq 11$  and  $f^* = -0.96354$  is the global minimum.

Bird function:

$$f(x_1, x_2) = \sin(x_1)e^{[1-\cos(x_2)]^2} + \cos(x_2)e^{[1-\sin(x_1)]^2} + (x_1 - x_2)^2$$

for  $-2\pi \leq x_1, x_2 \leq 2\pi$  and  $f^* = -106.764537$  is the global minimum.

Modified Schaffer function #1:

$$f(x_1, x_2) = 0.5 + \frac{\sin^2(x_1^2 + x_2^2) - 0.5}{[1 + 0.001(x_1^2 + x_2^2)]^2}$$

for  $-100 \leq x_1, x_2 \leq 100$  and  $f^* = 0$  is the global minimum.

Modified Schaffer function #2:

$$f(x_1, x_2) = 0.5 + \frac{\sin^2(x_1^2 - x_2^2) - 0.5}{[1 + 0.001(x_1^2 + x_2^2)]^2}$$

for  $-100 \leq x_1, x_2 \leq 100$  and  $f^* = 0$  is the global minimum.

Modified Schaffer function #3:

$$f(x_1, x_2) = 0.5 + \frac{\sin^2[\cos(|x_1^2 - x_2^2|)] - 0.5}{[1 + 0.001(x_1^2 + x_2^2)]^2}$$

for  $-100 \leq x_1, x_2 \leq 100$  and  $f^* = 0.00156685$  is the global minimum.

Modified Schaffer function #4:

$$f(x_1, x_2) = 0.5 + \frac{\cos^2 [\sin [|x_1^2 - x_2^2|]] - 0.5}{[1 + 0.001(x_1^2 + x_2^2)]^2}$$

for  $-100 \leq x_1, x_2 \leq 100$  and  $f^* = 0.292579$  is the global minimum.

Chichinadze function:

$$f(x_1, x_2) = x_1^2 - 12x_1 + 11 + 10\cos(\pi x_1/2) + 8\sin(5\pi x_1) - \frac{1}{\sqrt{5}}e^{-0.5(x_2-0.5)^2}$$

for  $-30 \leq x_1, x_2 \leq 30$  and  $f^* = -43.3159$  is the global minimum.

McCormick function:

$$f(x_1, x_2) = \sin(x_1 + x_2) + (x_1 - x_2)^2 - 1.5x_1 + 2.5x_2 + 1$$

for  $-100 \leq x_1, x_2 \leq 100$  and  $f^* = 0.292579$  is the global minimum.

Three-humps camel back function:

$$f(x_1, x_2) = 2x_1^2 - 1.05x_1^4 + \frac{x_1^6}{6} + x_1x_2 + x_2^2$$

for  $-5 \leq x_1, x_2 \leq 5$  and  $f^* = 0$  is the global minimum.

Zettl function:

$$f(x_1, x_2) = (x_1^2 + x_2^2 - 2x_1)^2 + 0.25x_1$$

for  $-5 \leq x_1, x_2 \leq 5$  and  $f^* = -0.003791$  is the global minimum.

Styblinski-Tang function:

$$f(x_1, x_2) = \frac{1}{2} \sum_{i=1}^2 (x_i^4 - 16x_i^2 + 5x_i)$$

for  $-5 \leq x_1, x_2 \leq 5$  and  $f^* = -78.332$  is the global minimum.

Bukin function:

$$100(x_2 - 0.01 * x_1^2 + 1) + 0.01(x_1 + 10)^2$$

for  $-15 \leq x_1 \leq -5, -3 \leq x_2 \leq 3$ , and  $f^* = -124.75$  is the global minimum.

Leon function:

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

for  $-1.2 \leq x_1, x_2 \leq 1.2$  and  $f^* = 0$  is the global minimum.

Giunta function:

$$f(x_1, x_2) = 0.6 + \sum_{i=1}^2 \left[ \sin\left(\frac{16}{15}x_i - 1\right) + \sin^2\left(\frac{16}{15}x_i - 1\right) + \frac{1}{50} \sin\left(4\left(\frac{16}{15}x_i - 1\right)\right) \right]$$

for  $-1 \leq x_1, x_2 \leq 1$  and  $f^* = 0.06447$  is the global minimum.

**APPENDIX 2**

Egg holder function:

$$f(x) = \sum_{i=1}^{m-1} \left( -(x_{i+1} + 47) \sin(\sqrt{|x_{i+1} + x_i/2 + 47|}) \right. \\ \left. \sin(\sqrt{|x_i - x_{i+1} + 47|})(-x_i) \right)$$

for  $-512 \leq x_i \leq 512$ ,  $i = 2, \dots, n$ , and  $f^* = 959.64$  is the global minimum.

Ackley function:

$$f(x) = 20 \exp \left( -0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2} \right) - \exp \left( \frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i) \right) + 20 + e$$

for  $-30 \leq x_i \leq 30$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Bohachevsky function:

$$f(x) = \sum_{i=1}^n (x_i^2 + 2x_{i+1}^2 - 0.3 \cos(3\pi x_i) - 0.4 \cos(4\pi x_{i+1}) + 0.7)$$

for  $-50 \leq x_i \leq 50$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Griewank function:

$$f(x) = 1 + \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right)$$

for  $-600 \leq x_i \leq 600$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Holzman function:

$$f(x) = \sum_{i=1}^n i x_i^4$$

for  $-10 \leq x_i \leq 10$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Hyperellipsoid function:

$$f(x) = \sum_{i=1}^n i x_i$$

for  $-5.12 \leq x_i \leq 5.12$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Levy function:

$$f(x) = \sin^2(\pi y_1) + \sum_{i=1}^{n-1} (y_i - 1)^2 (1 + 10 \sin^2(\pi y_i + 1)) + (y_n - 1)^2 (1 + \sin^2(2\pi y_n))$$

where  $y_i = 1 + \frac{x_i - 1}{4}$ ,  $-10 \leq x_i \leq 10$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Maxmod function:

$$f(x) = \max(|x_i|)$$

for  $-10 \leq x_i \leq 10$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Multimod function:

$$f(x) = \sum_{i=1}^n |x_i| \prod_{i=1}^n |x_i|$$

for  $-10 \leq x_i \leq 10$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Rastrigin function:

$$f(x) = \sum_{i=1}^n x_i^2 - 10 \cos(2\pi x_i) + 10$$

for  $-5.12 \leq x_i \leq 5.12$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Rosenbrock function:

$$f(x) = \sum_{i=2}^n 100(x_i - x_{i-1}^2)^2 + (1 + x_{i-1})^2$$

for  $-10 \leq x_i \leq 10$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Schaffer function:

$$f(x) = \sum_{i=1}^{m-1} \left[ \frac{\sin^2 [x_{i+1}^2 - x_i^2] - 0.5}{(0.001(x_{i+1}^2 + x_i^2) + 1)^2} + 0.5 \right]$$

for  $-100 \leq x_i \leq 100$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Sine Envelope sine wave function:

$$f(x) = \sum_{i=1}^{m-1} \left[ \frac{\sin^2 [\sqrt{x_{i+1}^2 + x_i^2}] - 0.5}{(0.001(x_{i+1}^2 + x_i^2) + 1)^2} + 0.5 \right]$$

for  $-100 \leq x_i \leq 100$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Schwefel function:

$$f(x) = \sum_{i=1}^n \left\{ \sum_{j=1}^{j<i} x_j \right\}^2$$

for  $-500 \leq x_i \leq 500$ ,  $i = 2, \dots, n$ ,  $f^* = -837.966$  for  $n = 2$ ,  $f^* = -4189.829$  for  $n = 10$ , and  $f^* = -10474.57259$  for  $n = 25$  are the global minimums.

Sphere function:

$$f(x) = \sum_{i=1}^n x_i^2$$

for  $-10 \leq x_i \leq 10$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.

Sumsquares function:

$$f(x) = \sum_{i=1}^n i x_i^2$$

for  $-10 \leq x_i \leq 10$ ,  $i = 2, \dots, n$ , and  $f^* = 0$  is the global minimum.