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Stability Analysis of Principal Parametric Resonance of Viscoelastic Pipes by Using Multiple Time Scales

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Abstract

This study investigates the transverse vibrations taking place tensioned viscoelastic pipes conveying fluid with time-dependent velocity taking into account simple supports condition. The governing equation is derived from Newton's second law, Boltzmann's superposition principle, and the stress-strain relation given for Maxwell viscoelastic model. The time-dependent velocity is assumed to vary harmonically about mean velocity. This system experiences a Coriolis acceleration component which renders such systems gyroscopic. The equation of motion is solved using the multiple time scale method. Principal parametric resonance is investigated. Stability boundaries are determined analytically. It is demonstrated that instabilities occur when the frequency of velocity fluctuations is close to two times the natural frequency of the system with constant velocity or when the frequency is close to the sum of any two natural frequencies.

1. INTRODUCTION

The dynamics of axially moving continua has been extensively studied due to its technological importance. There are many engineering designs which involve axially moving continua structures such as high speed magnetic tapes, band-saws, textile and composite fibers, pipes and beams conveying fluid, etc. Over a certain critical moving speed causes structural failures on such axially moving structures because of severe vibrations and dynamical instabilities. To explore stable working conditions for such structural systems, the dynamic responses and stability of such systems have been studied extensively. In literature, string behavior [1], beam behavior [1,2] and transition behavior from string to beam [1] have been used to model axially moving structures with respect to elastic behavior. However, with the advancement of material technologies, new widely used engineering materials such as plastics, metallic or ceramic reinforced composite materials and polymeric materials exhibit viscoelastic behavior. Reduced noise and vibrations can be achieved with the viscoelastic behavior of the materials in the accessory systems. The viscoelastic theory of materials allows to model the creeping and damping characteristics of the materials. The differential operator and the hereditary integrals methods are utilized to describe the stress–strain relations of viscoelastic materials because they do not obey Hook's law.

When a viscoelastic model is expressed in a differential operator method, the equation of motion describing the viscoelastic behavior of system as a beam/pipe or string is modeled in a partial differential equation form. In the differential operator method, the stress–strain relationship is expressed as $P\sigma_{ij} = Qe_{ij}$, where P and Q are differential operators; σ_{ij} and e_{ij} are the stress and elongation. The equation of motion of the system under consideration is written in a form including a set of stress terms. The equation is then multiplied by the differential operator, P. In the latest equation, Qe_{ij} is written in place of $P\sigma_{ij}$ to define a specific equation of motion for the viscoelastic model desired. Fung et al. [3] first applied this methodology to axially moving string problem and developed an equation of motion for a 3-parameter viscoelastic model referred to as standard linear solid model. In a similar way, the equations of motion were obtained for a 4-paremeter viscoelastic model known as Burger model [4] and Zener viscoelastic model based on three parameters [5] for axially moving beam. Zhang and Zu [6] and Hou and Zu [7] examined the dynamic analysis of axially moving string problem using the relation, $\sigma_{ij} = Q / Pe_{ij}$.

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However, when a viscoelastic model of hereditary integral type is adopted, the equation motion is modeled as a partial differential–integral equation form. The integral method is not useful for time dependent constitutive models due to complicated formulations that are less suitable for numerical calculations. Various methods have been presented for the vibration analysis of structures composed of viscoelastic materials. Fung et al. [8] used Galerkin method and a finite difference numerical integration procedure to obtain the transient responses. Chen et al. [9] gives a good solution by Galerkin method to axially moving viscoelastic beam modelled by integral method. Yang and Chen [10] examined parametric resonance case of linear axially moving beam modelled Boltzmann integral constitutive low by the multiple time scale method.

Several viscoelastic models are analyzed in literature [11]. Pipes composed of Kelvin-Voight material are extensively studied. However, Kruijer, et al., [12] showed experimentally that a pipe materials can obey different viscoelastic models. The dynamic of pipes conveying fluid is a popular problem for axially moving subject. Therefore, the dynamic analyses of pipes have been investigated by several researchers. An extensive review is given by Païdoussis and Li [13]. Païdoussis [14,15] are also published two well recognized books on fluid structure interactions.

It is observed that there is no investigation on dynamics of viscoelastic pipes constituted by the viscoelastic constitutive law of an integral type. This paper attempts to address the lack of research in the literature, by investigating the stability of principal parametric resonances of viscoelastic pipe constituted by Boltzmann's superposition principle. The equations of motion for viscoelastic models are obtained by Newton's second law of motion. The pipe material is modeled using Maxwell viscoelastic models. The fluid velocity is assumed to vary harmonically about a constant mean velocity. The multiple time scale method is used to determine boundary line for principal parametric resonance. The numerical results of viscoelastic pipes conveying fluid are calculated for simply supported ends condition.

2. GOVERNING EQUATION OF MOTION

A uniform viscoelastic pipe is considered with density ρ_p, cross-sectional area A_p, distance L, moment of inertial I and initial tension P_0. The fluid velocity, $V(T)$, is the time dependent and ρ_f and A_f are the density and cross-sectional area of the fluid, respectively. The bending vibration of the pipe described by the transverse displacement $U(X,T)$ is considered. Newton's second low of motion yields [10]

$$\left(\rho_p A_p\right)\frac{\partial^2 U}{\partial T^2}+\left(\rho_f A_f\right)\left(\frac{\partial^2 U}{\partial T^2}+2V\frac{\partial^2 U}{\partial X\,\partial T}+\frac{dV}{dT}\frac{\partial U}{\partial X}+V^2\frac{\partial^2 U}{\partial X^2}\right)=P_0\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 M}{\partial X^2} \tag{1}$$

where T, X is the time and the axial coordinate respectively, and M is the bending moment given by

$$M\left(X,T\right)=-\int_A Z\sigma\left(X,Z,T\right)dA \tag{2}$$

where Z, X plane is the principal plane of bending, and $\sigma(X,Z,T)$ is the disturbed normal stress. The one dimensional constitutive equation of an integral type material which is given by the Boltzmann superposition principle is adopted as [8,10,16]

$$\sigma\left(X,Z,T\right)=e\left(X,Z,T\right)E\left(0\right)+\int_0^T \dot{E}\left(T-T'\right)e\left(X,Z,T'\right)dT' \tag{3}$$

where $E(T)$ is the relaxation modules and $e(X, Z, T)$ is the axial strain. For small deflection, the strain-displacement relation is

$$e\left(X,Z,T\right)=-Z\frac{\partial^2 U}{\partial X^2} \tag{4}$$

For linear viscoelastic models such as Maxwell, the extensional relaxation function is given as follows;

$$E\left(T\right)=E+F\,e^{-\eta T} \tag{5}$$

where E, F and viscosity coefficient, η, are varying coefficients determined by Laplace transform of constitutive relation of the viscoelastic model. The relaxation modules and constitutive relation for Maxwell viscoelastic model [16] is given in Table 1. Where E_1 and E_2 are spring coefficients and C is dashpot coefficient. The viscoelastic coefficient, η, is assumed is very small then the Eq. (5) is transformed into following form [10].

Table1. Maxwell viscoelastic model, with constitutive relation and relaxation modules

Viscoelastic Model	Illustration of Viscoelastic Model	Constitutive Relation	Relaxation Module $E(T)=E+Fe^{-\varepsilon\eta T}$
Maxwell		$\sigma+\frac{C}{E_1}\dot{\sigma}=C\dot{\varepsilon}$	$E(T)=E_1e^{-\frac{E_1}{C}T}$

$$E(T)=E+F\,e^{-\varepsilon\eta T} \tag{6}$$

where the bookkeeping device ε is a small dimensionless parameter. Substitution of Eqs.(4) and (6) into Eq.(3)

$$\sigma(X,Z,T)=-Z\frac{\partial^2U}{\partial X^2}(E+F)+\varepsilon\eta F\int_0^T e^{-\varepsilon\eta(T-T')}\,Z\frac{\partial^2U}{\partial X^2}dT' \tag{7}$$

and substitution of the above equation into Eq. (2) leads to

$$M(X,T)=(E+F)I\frac{\partial^2U}{\partial X^2}-\varepsilon\eta FI\int_0^T e^{-\varepsilon\eta(T-T')}\frac{\partial^2U}{\partial X^2}dT' \tag{8}$$

Finally, substituting Eq.(8) into Eq.(1), the equation of motioned is obtained

$$\begin{aligned}(\rho_pA_p)\frac{\partial^2U}{\partial T^2}+(\rho_fA_f)\left(\frac{\partial^2U}{\partial T^2}+2V\frac{\partial^2U}{\partial X\partial T}+\frac{dV}{dT}\frac{\partial U}{\partial X}+V^2\frac{\partial^2U}{\partial X^2}\right)\\ -P_0\frac{\partial^2U}{\partial X^2}+(E+F)I\frac{\partial^4U}{\partial X^4}-\varepsilon\eta IF\int_0^T e^{-\varepsilon\eta(T-T')}\frac{\partial^4U}{\partial X^4}dT'=0\end{aligned} \tag{9}$$

Because U and V are dependent, and X and T independent variables in Eq. (9), they can be non-dimensionalized. U and X can be non-dimensionalized by substituting

$$u=\frac{U}{L},\quad x=\frac{X}{L} \tag{10}$$

V and T can be non-dimensionalized using;

$$t=\frac{T}{L^2}\sqrt{\frac{E_1I}{m}},\quad v=VL\sqrt{\frac{\rho_fA_f}{E_1I}} \tag{11}$$

where $m=\rho_fA_f+\rho_pA_p$. The equation of motions can then be written in dimensionless form for all viscoelastic models:

$$\frac{\partial^2 u}{\partial t^2}+2v\sqrt{\beta}\frac{\partial^2 u}{\partial x\partial t}+\sqrt{\beta}\frac{dv}{dt}\frac{\partial u}{\partial x}+\left(v^2-\gamma\right)\frac{\partial^2 u}{\partial x^2}+\frac{\partial^4 u}{\partial x^4}=\varepsilon\mu q s \tag{12}$$

where

$$\beta=\frac{\rho_f A_f}{m},\qquad s=\int_0^t e^{-\varepsilon\mu(t-t')}\frac{\partial^4 u}{\partial x^4}dt' \tag{13}$$

$$\mu=\frac{E_1}{C}\sqrt{\frac{mL^4}{E_1 I}} \tag{14}$$

q=1.0. The time derivative of the last term of dimensionless equation motion is as follows [10]:

$$\dot{s}=-\varepsilon\mu s+\frac{\partial^4 u}{\partial x^4} \tag{15}$$

The velocity of fluid is assumed to be a small simple harmonic variation, with the frequency ω and the amplitude εv_1 about the mean speed v_0

$$v(t)=v_0+\varepsilon v_1\sin(\omega t) \tag{16}$$

Here ε is used to show the fact that the fluctuation amplitude is small, with the same order as the dimensionless viscosity coefficient. Substitution of Eq. (16) into Eq. (12) and neglect higher orders ε terms in the resulting equation yield

$$\begin{gathered}\frac{\partial^2 u}{\partial t^2}+2v_0\sqrt{\beta}\frac{\partial^2 u}{\partial x\partial t}+\left({v_0}^2-\gamma\right)\frac{\partial^2 u}{\partial x^2}+\frac{\partial^4 u}{\partial x^4}=\\ \varepsilon\left(\mu q s-2v_1\sqrt{\beta}\sin(\omega t)\frac{\partial^2 u}{\partial x\partial t}-v_1\omega\sqrt{\beta}\cos(\omega t)\frac{\partial u}{\partial x}-2v_0v_1\sin(\omega t)\frac{\partial^2 u}{\partial x^2}\right)\end{gathered} \tag{17}$$

3. SOLUTION OF THE EQUATION

The Multiple Time Scale method is employed in search of approximate solution of the dimensionless equation of motion. The following expansion is assumed

$$u(x,t;\varepsilon)=u_0(x,T_0,T_1)+\varepsilon u_1(x,T_0,T_1)+.... \tag{18}$$

and a zero order to s is

$$s(x,t;\varepsilon)=s_1(x,T_0,T_1;\varepsilon) \tag{19}$$

where $T_0=t$ is the fast time scale and $T_1=\varepsilon t$ is the slow time scale. Time derivatives are defined as

$$\frac{d}{dt}=D_0+\varepsilon D_1+...\qquad\qquad \frac{d^2}{dt^2}=D_0^2+2\varepsilon D_0D_1+... \tag{20}$$

where $D_n=\frac{\partial}{\partial T_n}$. Substituting Eqs. (18)-(20) into Eq. (17) and separating terms at each order of ε, equations are obtained

O(1): $$D_0^2u_0+2v_0\sqrt{\beta}D_0u_0'+\left(v_0^2-\gamma\right)u_0''+u_0^{iv}=0 \tag{21}$$

O(ε):
$$\begin{aligned}&D_0^2u_1+2v_0\sqrt{\beta}D_0u_1'+\left(v_0^2-\gamma\right)u_1''+u_1^{iv}=\mu q s_1-2D_0D_1u_0\\&-2v_0\sqrt{\beta}D_1u_0'-2v_1\sin\left(\omega T_0\right)\left(\sqrt{\beta}D_0u_0'+v_0u_0''\right)-v_1\omega\sqrt{\beta}\cos\left(\omega T_0\right)u_0'\end{aligned} \tag{22}$$

The solution at order 1 can be written as

$$u_0\left(x,T_0,T_1;\varepsilon\right)=A_n\left(T_1\right)e^{i\omega_nT_0}Y_n\left(x\right)+c.c. \tag{23}$$

where *c.c.* stands for the complex conjugate of all preceding terms on the right hand of an equation. s_1 is given as follows [10]

$$s_1\left(x,T_0,T_1;\varepsilon\right)=A_n\left(T_1\right)e^{i\omega_nT_0}\frac{1}{i\omega_n}Y_n^{iv}\left(x\right)+c.c. \tag{24}$$

The spatial functions $Y_n(x)$ with boundary conditions satisfy the equation

$$Y_n^{iv}+v^2Y_n''+2\sqrt{\beta}vi\omega_nY_n'-\omega_n^2Y_n=0 \tag{25}$$

The solution is

$$Y_n\left(x\right)=c_{1n}\left(e^{i\alpha_{1n}x}+c_{2n}e^{i\alpha_{2n}x}+c_{3n}e^{i\alpha_{3n}x}+c_{4n}e^{i\alpha_{4n}x}\right) \tag{26}$$

where c_{in} are arbitrary coefficients. The α_{in} satisfy the dispersive relation

$$\alpha_{in}^4-v^2\alpha_{in}^2-2\sqrt{\beta}v\omega_n\alpha_{in}-\omega_n^2=0 \qquad i=1,2,3,4 \quad n=1,2,... \tag{27}$$

c_{in} are determined by applying boundary conditions. The boundary conditions of the pipe with simple supports in dimensionless form are

$$Y_n\left(0\right)=Y_n\left(1\right)=Y_n''(0)=Y_n''(1)=0 \tag{28}$$

The mode shapes of that boundary conditions were calculated previously [17]

$$\begin{aligned}Y_n\left(x\right)=&e^{i\alpha_{1n}x}-\frac{\left(\alpha_{4n}^2-\alpha_{1n}^2\right)\left(e^{i\alpha_{3n}}-e^{i\alpha_{1n}}\right)}{\left(\alpha_{4n}^2-\alpha_{2n}^2\right)\left(e^{i\alpha_{3n}}-e^{i\alpha_{2n}}\right)}e^{i\alpha_{2n}x}-\frac{\left(\alpha_{4n}^2-\alpha_{1n}^2\right)\left(e^{i\alpha_{2n}}-e^{i\alpha_{1n}}\right)}{\left(\alpha_{4n}^2-\alpha_{3n}^2\right)\left(e^{i\alpha_{2n}}-e^{i\alpha_{3n}}\right)}e^{i\alpha_{3n}x}\\&+\left(-1+\frac{\left(\alpha_{4n}^2-\alpha_{1n}^2\right)\left(e^{i\alpha_{3n}}-e^{i\alpha_{1n}}\right)}{\left(\alpha_{4n}^2-\alpha_{2n}^2\right)\left(e^{i\alpha_{3n}}-e^{i\alpha_{2n}}\right)}+\frac{\left(\alpha_{4n}^2-\alpha_{1n}^2\right)\left(e^{i\alpha_{2n}}-e^{i\alpha_{1n}}\right)}{\left(\alpha_{4n}^2-\alpha_{3n}^2\right)\left(e^{i\alpha_{2n}}-e^{i\alpha_{3n}}\right)}\right)e^{i\alpha_{4n}x}\end{aligned} \tag{29}$$

where α_{in} are eigenvalues of simple supported case.

4. PRINCIPLE PARAMETRIC RESONANCES

In this section, it is assumed that one dominant mode of vibration exists. Depending on the numerical values of frequency of the pipe, the case with ω close to $2\omega_n$ is investigated.

In this case, to represent the nearness of velocity variation frequency to two times one of the natural frequencies

$$\omega=2\omega_n+\varepsilon\sigma \tag{30}$$

The solvability condition requires

$$D_1A_n+v_1k_0\bar{A}_ne^{i\sigma T_1}+\mu k_1A_n=0 \tag{31}$$

where k_0 and k_1 are

$$k_0 = \frac{\frac{1}{2}\omega\sqrt{\beta}\int_0^1 \bar{Y}_n'\bar{Y}_n dx \quad -\sqrt{\beta}\omega_n\int_0^1 \bar{Y}_n'\bar{Y}_n dx - iv_0\int_0^1 \bar{Y}_n''\bar{Y}_n dx}{2i\omega_n\int_0^1 \bar{Y}_n Y_n dx + 2v_0\sqrt{\beta}\int_0^1 \bar{Y}_n Y_n' dx} \tag{32}$$

$$k_1 = \frac{\frac{iq}{\omega_n}\int_0^1 \bar{Y}_n Y_n^{iv} dx}{2i\omega_n\int_0^1 \bar{Y}_n Y_n dx + 2v_0\sqrt{\beta}\int_0^1 \bar{Y}_n Y_n' dx} \tag{33}$$

To perform a stability analysis, the following transformation is used

$$A_n = B_n e^{-i(\sigma/2)T_1} \tag{34}$$

$$D_1 B_n + \left(i\frac{\sigma}{2} + \mu k_1\right)B_n + v_1 k_0 \bar{B}_n = 0 \tag{35}$$

Eq. (35) takes the same form as Eq. (19) in the paper of Öz et al. [1]. The coefficients used in the analytical expression of the stability boundaries of parametric resonance in Eq. (36) are different from the coefficients given by Öz et al. [1]. The stability boundaries are determined by

$$\sigma = 2\mu k_1^I \mp 2v_1\sqrt{k_0^{R2} + k_0^{I2}} \tag{36}$$

where superscripts R and I denote the real and imaginary parts of coefficient, respectively. Inserting σ further into Eq. (30) gives two different values of ω. The two values denote the stability boundaries for small ε. Numerical solutions will be given in numerical results section. Note that for constant velocities, v_1=0 and hence the second term of right hand side in Eq. (36) is disappeared.

5. NUMERICAL RESULTS

The natural frequencies and stability boundaries given analytically in the previous sections of the viscoelastic pipe conveying fluid are investigated. The numerical results are obtained for the following parameters: the fluid-mass ratio, β=0.8; dimensionless mean fluid velocity, v_0=5.0; initial tension parameter, γ=25; and the small non-dimensional parameter, ε=0.1. The parameter q is 1.0 for Maxwell viscoelastic model.

The natural frequencies for the first, second and third modes are plotted depending on mean velocity. Stability analysis is done for the principal parametric resonance case. In principal parametric resonances analysis, when the velocity fluctuation frequency is close to zero, no instabilities are detected up to the first order of perturbation. When the fluctuation frequency is away from zero and twice the natural frequency, the solutions are bounded and no instabilities observed. Instable regions occur when the frequency of fluctuation is close to two times natural frequencies of the constant velocity system.

Numerical values of the natural frequencies can be computed by using Eq.(27). The principal parametric instabilities are plotted using Eq. (36). Eqs.(32) and (33) are used on calculations of k_0 and k_1. For the computation of the coefficients k_0 and k_1, Eq.(29) is used as the shape function, $Y_n(x)$. As can be seen from Figure 1 the natural frequencies of the system decreases when the mean velocity increases. As the frequency values vanish, the divergence instability occurs. The velocity for which the natural frequency is lost is named the critical fluid velocity. The natural frequency and critical velocity values increases at higher modes. When v=0.0, the natural frequency is 18,55 for the first mode, 50.45 for the second mode and 100.55 for the third mode. The velocity for which the frequency values vanish cannot be determined exactly due to numerical procedures. However, it can be stated that the velocity is about 5.8 for first mode, 8.2 for second mode and 11.3 for third mode.

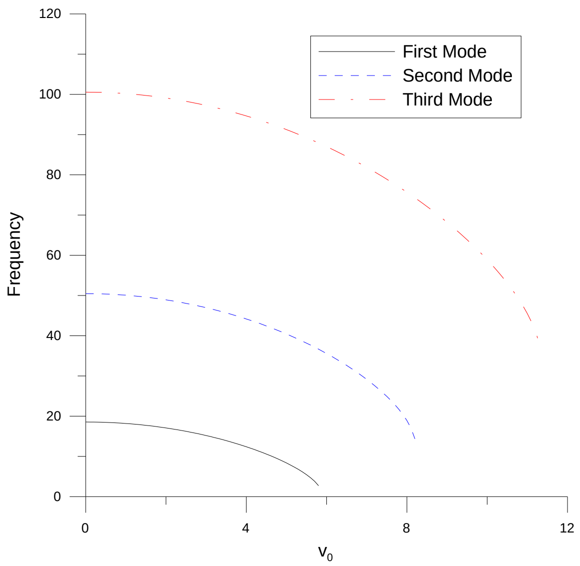

Figure 1. Natural frequencies of the pipe versus mean velocity for different modes with simply supports

In Figures 2, the stable and unstable regions are plotted for the principal parametric resonance case for different amplitude of fluctuation velocity and viscosity coefficient.

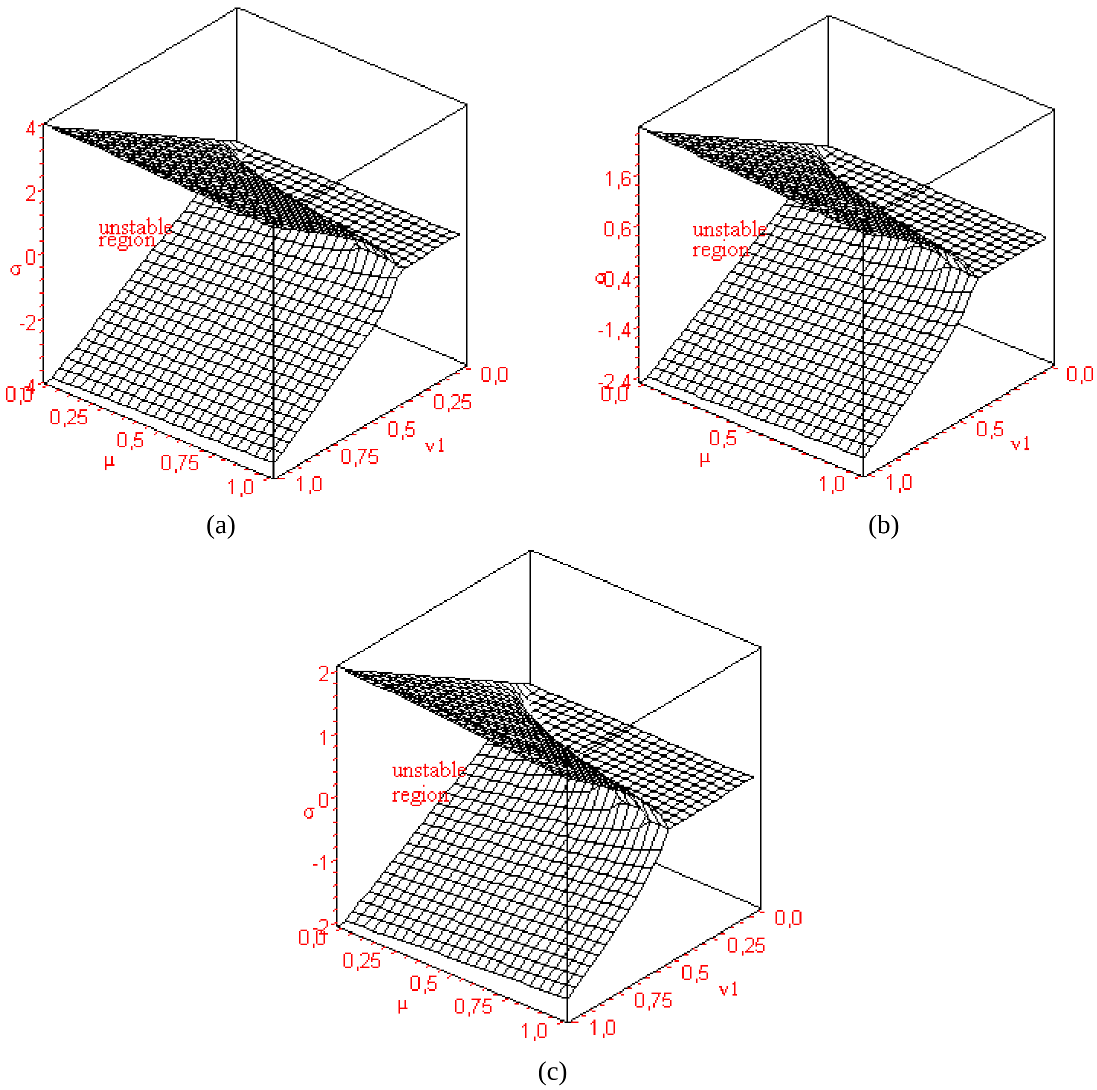

Figure 2. Principal parametric resonances with simply supported end conditions for Maxwell viscoelastic model (a) for the first mode, $2\omega_1$ (b) the second mode, $2\omega_2$ (c) the third mode $2\omega_3$

The stability boundary is nonlinearly dependent on viscosity coefficient and amplitude of fluctuation. The smallest stable area for principal parametric resonances is obtained for the first mode. When the velocity fluctuation amplitude is bigger than 0.14, the system is always unstable for the viscosity coefficient given. The viscosity coefficient, μ, is taken 0.8 in Figures 6.

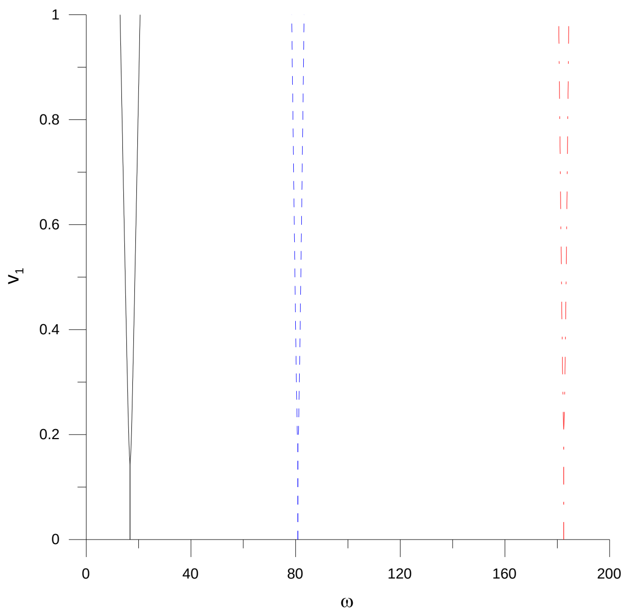

Figure 6. Stable and unstable regions for principal parametric resonances with simply supported end conditions: $\omega=2\omega_1+\varepsilon\sigma$ (——), $\omega=2\omega_2+\varepsilon\sigma$ (- - -), $\omega=2\omega_3+\varepsilon\sigma$ (–·-)

The principal parametric instabilities are plotted in Figure 6 for the first three natural frequencies. The instability region for the first natural frequency is the largest and begins at the minimum amplitude of velocity fluctuation v_1=0.14. The instability region begins at v_1=0.18 for the second mode and at v_1=0.21 for the third mode in Figure 6. The narrowest instability region of principal parametric case is obtained at the third mode.

6. CONCLUSIONS

In this paper, the transverse stability of viscoelastic pipes conveying fluid is studied. The pipe whose material is modeled by Maxwell viscoelastic model is constituted by Boltzmann's superposition principle. The fluid velocity is assumed to be harmonically changing about a mean velocity. The method of multiple time scale is applied to the governing equation. The influence of small fluctuations of fluid velocity on the stability of the pipe is investigated. The boundaries separating stable and unstable regions are derived from the solvability conditions. The pipes with simple supports are numerically investigated. Principal parametric resonances for any two modes are considered in the analyses. A detuning parameter is used to quantify the deviation between the fluctuation frequency of the fluid velocity and the sum of two natural frequency or the multiple of a natural frequency.

The instability region occurs while the fluid velocity fluctuation frequency near twice of any natural frequency. However, for the case of the frequency close to zero, no instabilities are detected up to the first order approximation. The stability boundary is nonlinearly dependent on viscosity coefficient and amplitude of fluctuation.

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