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Confidence Interval based Quality Improvement for Non-normal Responses

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Abstract

Robust parameter design is an effective tool to determine the optimal operating conditions of a system. Because of its practicability and usefulness, the widespread applications of robust design techniques provide major quality improvements. The usual assumptions of robust parameter design are that normally distributed experimental data and no contamination due to outliers. Optimizing an objective function under the normality assumption for a skewed data in dual-response modeling may result in misleading fit and operating conditions located far from the optimal values. This creates a chain of degradation in the production phase, e.g., poor quality products. This paper focuses on skewed experimental data. The proposed approach is constructed on the confidence interval of the process mean which makes the system median unbiased for the mean using the skewness information of the data. The response modeling of the midpoint of the interval is proposed as a location performance response. The main advantages of the proposed approach are that it gives a robust solution due to the skewed structure of the experimental data distribution and does not need any transformation which causes any loss of information in estimation of the mean response. The procedure and the validity of the proposed approach are illustrated on a popular example, the printing process study.

Keywords: Confidence interval, Dual response surface, Non-normal data, Robust parameter design.

1. Introduction

Robust parameter design (RPD) was introduced by Taguchi [1] to make the system as robust as possible to undesirable fluctuations in the system's performance. RPD, along with Taguchi's philosophy, has received considerable attention for more than thirty years in different industrial fields. However, his experimental methodology and analysis techniques have been exposed to a lot of criticism from the statistical community – e.g., Box [2], Vining and Myers [3]. Consequently, new methodologies have been proposed based on these criticisms. Vining and Myers [3] conducted one of the earliest research attempts to develop an alternative tool for off-line quality, and discussed a procedure constructed by combining response surface methodology (RSM) and the some effective properties of Taguchi's method. RSM, first presented by Box and Wilson [4], is an effective procedure for modeling a possible process relationship between a quality characteristic and design factors to determine the optimal operating conditions.

Vining and Myers [3] introduced the dual response surface (DRS) which is configured by separately fitting mean and variance response surfaces. Thereby, DRS achieve the primary goal of RPD by optimizing primary response subject to a pre-defined value of secondary response. This novel approach to RSM has become sound and is widely quoted in the current literature. Further improvements for DRS problem were carried out by Del Castillo and Montgomery [5]'s study which proposed using standard nonlinear programming techniques based on inequality constraints. Lin and Tu [6] focused on the process bias along with the variability and proposed minimizing the mean squared error (MSE) criterion for the DRS problem. A slightly different version of the MSE criterion, based on considering how far the mean can be located from its target, is discussed by Copeland and Nelson [7]. Further work in the area of DRS has been conducted by Köksoy and Doganaksoy [8]. They proposed an alternative formulation based on joint optimization of the mean and standard deviation responses under no constraints or minimally constrained. Following these articles, several approaches have been proposed for the DRS problem by [9-12].

A robust design approach aims to determine the optimal levels of the design factors by optimizing the estimated model from the experimental data. However, since experimental data are limited, the distribution and model parameters cannot be estimated precisely, so they can involve estimation error due to insufficient experimental data or unknown random effects. Elsayed and Chen [13] showed that the error existing in the estimated parameters causes the problem such as the estimated model varies from the true model, thus optimal operating conditions may be located quite far from the true values. Especially, in robust parameter design which aims to optimize an objective function obtained by point estimate, ignoring these uncertainties may result in misleading fit and large variance in the system performance; see, Tan and Ng [14]. Recently, several approaches based on confidence regions are developed to reduce the effect of these uncertainties in model parameters on robust design. Ouyang et al. [15] adopted the worst and best mean squared error in their approach by using the midpoint and radius of the interval as the model performances from the perspective of frequentist approach. On the other hand, Bayesian approaches have been quite discussed in the literature; see, Miro-Quesada et al. [16].

The probability distribution of an experimental data plays also an important role in robust design. The traditional assumption behind the response surface modeling is that the data are normally distributed and contain no contamination. However, these assumptions may not always reflect the reality, especially in real-world industrial problems. In many engineering problems, responses have Poisson (count data), exponential/gamma (time-to-failure data), or Bernoulli (defective/non-defective data) distributions. Therefore, since the fitted process mean and variance responses are very sensitive to the data distribution, using traditional normal theory for a non-normal response modeling often overshadows the reliability of quality improvement techniques. There are some options to study with non-normal responses to robust design. Bisgaard and Fuller [17] and Box and Fung [18] propose data transformation. However, several disadvantages of such transformations are highlighted in the literature, such that they often do not fully normalize the data and transformed metrics of quality can be difficult to interpret. An another option is using generalized linear models (GLMs), where normality and constancy of variance are not required; see, Nelder and Lee [19], and Myers et al. [20]. However, GLMs do have certain limitations, such as lack of discriminatory power, restrictions in the response distribution, and sometimes have poor interpretability. On the other hand, when any departures from the normality and contamination are the case, the usage of robust estimators in the mean and variance response modeling has become a popular preference in recent years. Boylan and Cho [21] discussed the effectiveness of the robust estimators which are more efficient and resistant in the presence of outliers. On the other hand, Zeybek and Köksoy [22] studied the effects of gamma noise factors on the distribution of the experimental data and presented a probability density function which has a skewed nature.

This paper focuses on skewed experimental data. To make an improvement of the robustness performance of the estimations of the responses, the proposed approach is constructed on a confidence interval of the process mean which makes the system median unbiased for the mean using skewness information of the experimental data. In addition, the midpoint of the upper and lower endpoints is used as a measure of location performance. Additionally, the proposed approach does not need any transformation, and it therefore does not allow any loss of information in estimation of responses.

The remainder of this manuscript is divided into three sections as follows: In the next section, the proposed approach is presented. The validity of the proposed approach is illustrated on the basis of a popular example printing process study, before the paper finally ends with a conclusion.

2. Materials and Methods

Consider a quality characteristic y depends on k controllable factors, $x_1, x_2, \ldots, x_k$. Suppose that the interested design has n-design points, each replicated r times, where y_{ij} represents the j^{th} response at the i^{th} design point, $j = 1, \ldots, r$ and $i = 1, \ldots, n$. Note that,

$$\bar{y}_i = \frac{1}{r}\sum_{j=1}^{r} y_{ij} \qquad (1)$$

and

$$s_i^2 = \frac{1}{r-1}\sum_{j=1}^{r}\left(y_{ij} - \bar{y}_i\right)^2 \qquad (2)$$

are sample means and variances of each design point.

Suppose that y follows a skewed distribution. Since y is not normally distributed, as noted by Johnson [23], the robustness of the sample mean (and associated tests and confidence intervals) to the non-normality of the population has been raised suspicion. This affects the dual-response modeling results in misleading fit and operating conditions located far from the optimal values.

This paper presents an alternative approach to cope with non-normal responses and error from the fitted model in terms of constructing confidence intervals for the system mean. In this context, $(1-\alpha)\%$ confidence interval for the mean, which is proposed by Johnson [23] for the non-normal data, is constructed in the following form for the i^{th} design point,

$$\left(\bar{y}_i + \frac{\hat{\mu}_{3,i}}{6n\, s_i^2}\right) \mp t_{\frac{\alpha}{2}} s_i/\sqrt{n} \qquad (3)$$

where $\hat{\mu}_{3,i}$ is the third central moment estimated by the sample quantities, and is used to define the skewness of the designed data. This confidence interval corrects the difference between the median and the mean due to a skewed population and makes the median unbiased for the mean.

According to the theory of interval analysis – see; Boukezzoula et al. [24], and Ouyang et al. [15] – the midpoint of an interval can be used as a measure of the location performance. Then, considering Equations (3), a new location performance, $\bar{y}_i^*$, can be defined as follows,

$$\bar{y}_i^* = \bar{y}_i + \frac{\hat{\mu}_{3,i}}{6n\, s_i^2} \quad (4)$$

The proposed fitted mean response surface is constructed using Equation (4). Thus, the mean and standard deviation response surfaces are as follows,

$$\hat{\mu}(x) = \hat{\gamma}_0 + \sum_{i=1}^{k} \hat{\gamma}_i\, x_i + \sum_{i=1}^{k} \hat{\gamma}_{ii}\, x_i^2 + \sum_{i<t}^{k} \sum \hat{\gamma}_{it} x_i x_t \quad (5)$$

and

$$\hat{\sigma}(x) = \hat{\delta}_0 + \sum_{i=1}^{k} \hat{\delta}_i\, x_i + \sum_{i=1}^{k} \hat{\delta}_{ii}\, x_i^2 + \sum_{i<t}^{k} \sum \hat{\delta}_{it} x_i x_t \quad (6)$$

where $\hat{\gamma} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{w}_{\mu}$ and $\hat{\delta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{w}_{\sigma}$. The vectors of the sample mean and standard deviation $\mathbf{w}_{\mu} = (\bar{y}_1^*, \bar{y}_2^*, \dots, \bar{y}_n^*)'$ and $\mathbf{w}_{\sigma} = (s_1, s_2, \dots, s_n)'$ are obtained from Equations (4) and (2), respectively. And $\mathbf{X}$ denotes the design matrix.

Finally, the following proposed optimization approaches are constructed by adapting to the three fundamental situations of RPD — target-is-best (NTB), larger-the-better (LTB) and smaller-the-better (STB). The optimization phase is conducted based on the MSE criterion which gives a fairly general method to solve the DRS problem; see, Lin and Tu [6] and Köksoy [25]. By adopting Köksoy [25] and Ding et al. [26]'s optimization schemes, the optimization phases based on the MSE response functions are conducted as follows:

For the NTB case,

$$Min \quad MSE = (\hat{\mu}(x) - \tau)^2 + \hat{\sigma}^2(x)$$
$$s.t. \quad \boldsymbol{x}^* \in R$$

For the LTB case,

$$Min \quad -\left(\hat{\mu}(x)\right)^2 + \hat{\sigma}^2(x)$$
$$s.t. \quad \boldsymbol{x}^* \in R$$

For the STB case,

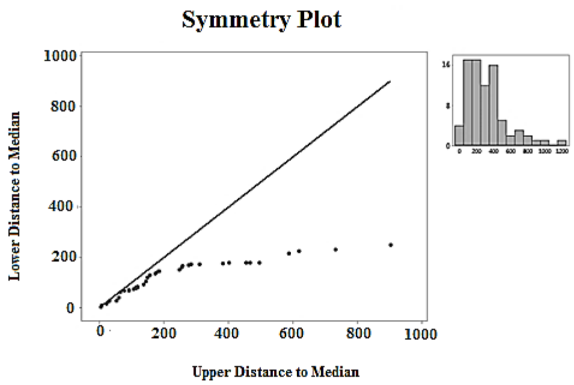
$$Min \quad \left(\hat{\mu}(x)\right)^2 + \hat{\sigma}^2(x)$$
$$s.t. \quad \boldsymbol{x}^* \in R$$

Here, τ is the target for process mean. The experimental region can be defined as $-1 \le x_i \le 1,\ i = 1, \dots, k$ for cuboidal designs and $\mathbf{x}'\mathbf{x} \le \rho^2$ for spherical designs, where ρ is the design radius.

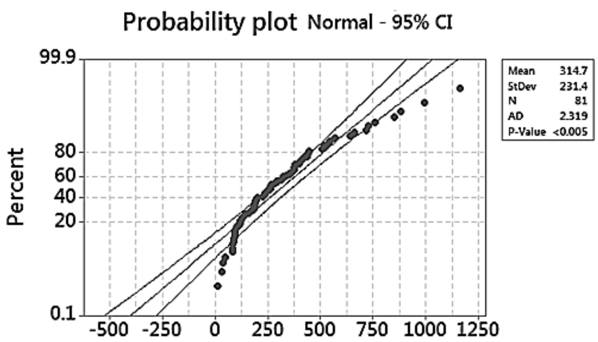
3. Results and Discussion

In this section, the proposed procedure is illustrated by a well-known *printing process study* example. This experiment was originally presented as an exercise in Box and Draper [27], and has been analyzed by many authors such that Vining and Myers [3], Copeland and Nelson [7], DelCastillo and Montgomery [5], Köksoy and Doganaksoy [8]. As indicated in Table 1, 3^3 factorial design with three replicates is performed to examine the effect of speed (x_1), pressure (x_2), and distance (x_3) on the ability of a printing machine (y) to apply colored inks to package labels. The aim of the experiment is to improve the quality of the printing process.

When the distribution of this data is examined, it is obvious that the printing process data has a right-skewed shape; see, Figure 1. Moreover, normality tests results (Shapiro Wilk test, W = 0.891, p-value < 0.000 and Anderson-Darling test, AD = 2.319, p-value < 0.005) are verified that this experimental data do not follow a normal distribution. Note that the *p-value*, (i.e., *p-value* > 0.25), for the Anderson-Darling test (null hypothesis that the data follows a gamma distribution) indicates that the printing process data does not deviate significantly from a gamma distribution. In fact, this result actually supports Oyeyemi [28]'s arguments about the true distribution of the printing process data.



(a)



(b)

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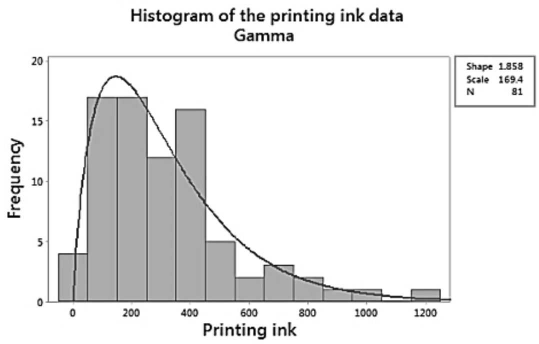
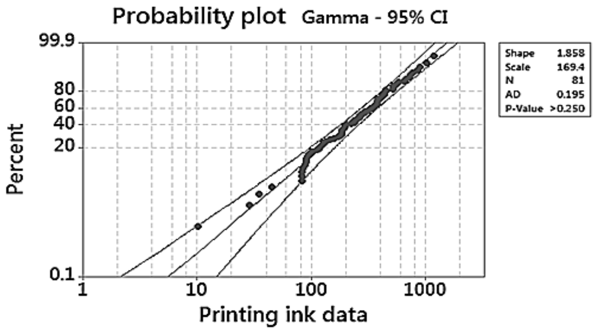


(c)



(d)

Figure 1. Plots of the printing process data; (a) symmetry plot, (b) probability plot for normal distribution, (c) histogram, (d) probability plot for gamma distribution.

Based on this result about the distribution of the data, the proposed approach will be examined through the printing process data.

For the illustrative purposes, $\alpha = 0.05$ is chosen and the estimates of the lower (L.E.) and upper (U.E.) confidence limits for each design point is obtained using Equation (3), see Table 1. And, the proposed location performance, $\bar{y}^*$, is determined for each design point using Equation (4).

Finally, the fitted mean and standard deviation response surfaces for the printing process data are modeled as follows,

$$\hat{\mu}(x) = 326.8 + 177x_1 + 109.4x_2 + 132x_3 + 32{x_1}^2 - 21.8{x_2}^2 - 28.3{x_3}^2 + 66.2x_1x_2 + 75.5x_1x_3 + 43.9x_2x_3 \quad (7)$$

$$\hat{\sigma}(x) = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 4.2{x_1}^2 - 1.3{x_2}^2 + 16.8{x_3}^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3 \quad (8)$$

According the information obtained from previous studies, the requirements of this printing example are as $\tau = 500$ and the desired standard deviation is less than 60. In the optimization phase, the spherical ($\rho^2 = 1$) region is considered for the NTB case. Table 2 compares the results obtained by the proposed modeling and the existing approaches.

Table 1. The printing process study data.

i	x_1	x_2	x_3	y_1	y_2	y_3	$\bar{y}$	s	95% confidence interval $L.E.$	$U.E.$	$\bar{y}^*$
1	-1	-1	-1	34	10	28	24.000	12.490	-7.229	54.830	23.801
2	0	-1	-1	115	116	130	120.333	8.386	99.675	141.345	120.510
3	1	-1	-1	192	186	263	213.667	42.829	108.160	320.964	214.562
4	-1	0	-1	82	88	88	86.000	3.464	77.320	94.532	85.926
5	0	0	-1	44	178	188	136.667	80.407	-64.782	334.736	134.977
6	1	0	-1	322	350	350	340.667	16.166	300.160	380.482	340.321
7	-1	1	-1	141	110	86	112.333	27.574	43.959	180.966	112.462
8	0	1	-1	259	251	259	256.333	4.619	244.760	267.709	256.235
9	1	1	-1	290	280	245	271.667	23.629	212.559	329.964	271.261
10	-1	-1	0	81	81	81	81.000	0.000	*	*	81.000
11	0	-1	0	90	122	93	101.667	17.673	58.127	145.938	102.032
12	1	-1	0	319	376	376	357.000	32.909	274.539	438.053	356.296
13	-1	0	0	180	180	154	171.333	15.011	133.720	208.305	171.012
14	0	0	0	372	372	372	372.000	0.000	*	*	372.000
15	1	0	0	541	568	396	501.667	92.500	270.074	729.679	499.877
16	-1	1	0	288	192	312	264.000	63.498	105.107	420.608	262.857
17	0	1	0	432	336	513	427.000	88.606	206.596	646.850	426.723
18	1	1	0	713	725	754	730.667	21.079	678.591	783.327	730.959
19	-1	-1	1	364	99	199	220.667	133.822	-110.620	554.298	221.839
20	0	-1	1	232	221	266	239.667	23.459	181.767	298.328	240.047
21	1	-1	1	408	415	443	422.000	18.520	376.323	468.344	422.333
22	-1	0	1	182	233	182	199.000	29.445	126.479	272.781	199.630
23	0	0	1	507	515	434	485.333	44.636	373.523	595.303	484.413
24	1	0	1	846	535	640	673.667	158.210	282.406	1068.499	675.452
25	-1	1	1	236	126	168	176.667	55.510	39.231	315.041	177.136
26	0	1	1	660	440	403	501.000	138.935	158.574	848.898	503.736

27	1	1	1	878	991	1161	1010.000	142.454	657.134	1364.939	1011.037

Table 2. A comparative study under the spherical region for the NTB case $(\rho^2 = 1)$.

	$\mathbf{x}'$	$\hat{\boldsymbol{\mu}}$	$\hat{\boldsymbol{\sigma}}$	**MSE**
Proposed approach	$(0.9832, 0.0073, -0.1822)$	494.52	44.74	2031.46
DelCastillo and Montgomery [5]	$(0.9839, 0.0265, -0.1760)$	500.00	45.32	2053.75
Copeland and Nelson [7]	$(0.9809, 0.0036, -0.1829)$	499.00	45.20	2044.04
Koksoy and Doganaksoy [8]	$(0.9835, 0.0073, -0.1822)$	497.52	45.01	2032.05

From Table 2, it is clear that, under the region $(\rho^2 = 1)$, the optimal solution is obtained as $\mathbf{x}' = (0.9832, 0.0073, -0.1822)$ with a confidence level of 95%. Here, the 95% probability relates to the reliability of the estimation procedure. And the corresponding mean (location) performance value is estimated as 494.52 while the standard deviation is $\hat{\sigma}(x) = 44.74$. It is obvious that the proposed approach yields the smallest estimated process standard deviation. And, an effective improvement can be achieved by the proposed approach when compared the MSE values, i.e., the proposed method has the smallest MSE value rather than DelCastillo and Montgomery [5], Copeland and Nelson [7], and Köksoy and Doganaksoy [8]. These results actually provide important information about the printing process example in the sense of correcting the difference between the mean and median due to the skewed structure of the experimental data.

4. Conclusion

The robust design alternatives for non-normal experimental data are necessary studies for quality improvement in different fields. The results obtained under mistaken assumptions about data distribution constitute unreliable and misleading sources of information for quality improvement practitioners.

This paper takes into account the case where the experimental data has a skewed distribution and proposes an alternative to the existing approaches. To make an improvement of the robustness performance of the process design, the proposed approach is constructed on a confidence interval which makes the system median unbiased for the mean using the skewness information of the experimental data. The midpoint of the interval is used as a measure of the location performance of the process. And, in the optimization phase of a DRS problem, using this new location performance response surface is proposed instead of the regular mean response. The main advantage of the proposed approach is that it provides a robust solution due to the skewed structure of the experimental data distribution.

The procedure and the validity of the proposed approach are illustrated on a popular example, the printing process study. This printing process data is a very popular example in the literature and has a skewed distribution. The proposed modeling gives the smallest MSE value and provides the minimum variability for the printing process data, compared with DelCastillo and Montgomery [5], Copeland and Nelson [7], and Köksoy and Doganaksoy [8]. The main reason of this performance is that the proposed approach utilizes the information of the skewness of the data. This study therefore offers a useful reference for practitioners in terms of providing an engineering understanding of the non-normal processes.

Ethics

There are no ethical issues after the publication of this manuscript.

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