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PARALLEL PRODUCTION LINES WITH SEQUENCE-DEPENDENT SETUP TIMES AND SIDE CONSTRAINTS

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ABSTRACT

We consider a scheduling problem observed in a soft-drink production facility with multiple production lines and sequence-dependent setup times. The primary objective is to obtain a weekly schedule that minimizes the total weighted unsatisfied demand. As a secondary objective we aim to minimize the total production and setup times. The number of molds and the number of shifts at any day are limited. We formulate the problem as a Mixed Integer Linear Program and propose several heuristic procedures for its solution. The results of our extensive runs have revealed the satisfactory performance of our heuristic procedures.

Keywords: Scheduling, sequence-dependent setup times, heuristic, integer programming

DİZİYE BAĞLI KURULUM SÜRELERİ VE YAN KISITLARI OLAN PARALEL ÜRETİM HATLARI

ÖZ

Çoklu üretim hatları ve diziye bağlı kurulum süreleri ile bir meşrubat üretim tesisinde gözlemlenen bir çizelgeleme problemini ele alıyoruz. Birincil hedefimiz, toplam ağırlıklı karşılanamayan talebi en aza indirecek haftalık bir program elde etmektir. İkinci bir hedef olarak, toplam üretim ve kurulum sürelerini en aza indirmeyi hedefliyoruz. Herhangi birgün de kalıp sayısı ve vardiya sayısı sınırlıdır. Problemi bir Karma Tamsayılı Doğrusal Program olarak formüle edip, çözüm için birkaç sezgisel prosedür önermekteyiz. Kapsamlı çalışmalarımızın sonucu, sezgisel prosedürlerimizin tatmin edici performansını ortaya koymuştur.

Anahtar Kelimeler: Çizelgeleme, dizi bağımlı kurulum zamanları, sezgisel, tam sayılı programlama

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1. INTRODUCTION

We consider the problem of allocating parallel production lines to multiple products at a soft-drink production plant. A product type is represented by its group (drinks using the same syrup type) and its shape, size, and container type. A setup is required when the product type changes. During this setup time, if the bottle shape changes, the mold at the blowing station is changed. Moreover, if the syrup type differs, the tank feeding the bottling station is cleaned and refilled. For each product, there is a set of eligible production lines and each eligible line has a different throughput rate. We intend to schedule the production lines to meet the requirements of the weekly production plan as far as possible, so that the total weighted unsatisfied demand is minimized. As a secondary concern is to minimize the total time spent in the system. Simultaneous scheduling decisions have to be made among the lines as the number of molds and number of shifts at any day are limited.

Soft drink production process is composed of two stages (see Figure 1).

1. Syrup preparation: Syrup is the most expensive ingredient of the soft drinks. After being prepared in the syrup room, it is transferred to the tanks that feed the production lines. The issues here are the tank capacities and the perishability of the syrup. The shelf life of the syrups is only 24 hours; therefore, they should be prepared just before the production.
2. Bottling: A production line is composed of five stations: blow, fill, label, package, and pallet. Tanks feed the filling station at the second stage.

In the literature, the majority of the soft drink production scheduling studies consider two stages of the production process together. The planning horizon is divided into macro periods of constant length, each of which has a demand for each product type. Each macro period is divided into micro periods of varying length, in which one type of product is produced. This problem type is named as integrated lot sizing and scheduling problem. Drexl and Kimms [8] give an extensive review of the lot sizing and scheduling problems. In lot sizing phase, lot sizes, inventory or backlogging levels at the end of each macro period for each product type are determined. In scheduling phase, the order of each product type at each production line is determined.

Some noteworthy studies on the applications of the

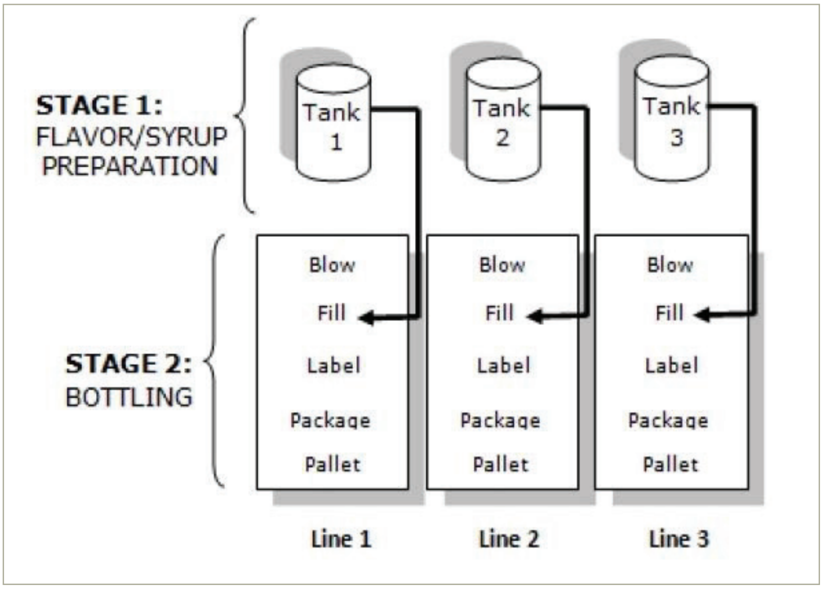

Figure 1. Soft Drink Production Process

lot sizing and scheduling problem in soft-drink industry are due to Toledo et al. [19, 20, 21], Ferriara et al. [9, 10, 11], Baldo et al. [3], and Maldonado et al. [15]. Meyr and Mann [16] give a decomposition approach for lot-sizing and scheduling decisions in parallel production line environments. Clark et al. [6], Araujo and Clark [2], and Gicquel and Minoux [13] study a lot sizing and scheduling problem on a single line for the general setting.

Toledo et al. [19, 20, 21] consider the two-stage lot sizing and scheduling problem with sequence-dependent setup times on unrelated parallel machines. The first study proposes a mixed integer linear model, the second study presents a multi-population genetic algorithm for hierarchically structured populations, and the final study suggests a genetic algorithm embedded into mathematical programming. Ferriara et al. [9] consider a two-stage problem, and propose a mathematical model that links the stages via continuous variables.

Ferriara et al. [9] consider a single-stage and single machine lot-sizing and scheduling problem with sequence-dependent setup costs. They aim to minimize the sum of the inventory, backorder, and machine changeover costs.

In Ferriara et al. [11], the first stage of the soft drink production process is embedded into the second stage by modifying the setup times in the second stage. The setup time is taken as the maximum of the bottling line product setup time, and the respective tank syrup setup time. The objective is to minimize the total holding and backlogging costs.

We model the second stage of production since its optimal schedule forms the basis for the organization of the first stage. We include the first stage in our model by putting the constraint that a limited number of syrup types can be prepared at a given time.

Our model covers many aspects of scheduling problems that have been extensively studied in the literature. Basically, it is a parallel line scheduling problem, in which lot splitting is allowed, setup times are sequence dependent and the number of molds is limited (related to tool constraints in the literature).

Yalaoui and Chu [22] consider an identical parallel machine scheduling problem with sequence-dependent setup times and lot splitting, and minimize the makespan. They propose a heuristic procedure, by first reducing the problem into a single machine scheduling problem, whose solution is used as an initial feasible solution. Following this, they apply improvement steps considering lot splitting and setup times. Tahar et al. [18] consider the same problem and state that the considered problem is NP-hard. They suggest a heuristic algorithm and show the satisfactory performance of their heuristic. Chen and Wu [5] and Shim and Kim [17] consider an unrelated machine scheduling with machine-and sequence-dependent setup times and tool constraints and aim to minimize total tardiness. Chen and Wu [5] propose a heuristic using threshold-accepting methods, tabu lists, and improvement steps. Shim and Kim [17] develop a branch and bound algorithm along with several dominance properties and lower bounds.

Dhaenens-Flipo [7] also consider unrelated parallel machine scheduling with machine- and sequence-dependent setup times. Their model has limited time to process all jobs and the objective is to minimize the total cost of production, distribution, and switching. Boudhar and Haned [4] consider an identical machine scheduling problem where preemption is allowed and makespan is minimized. They show that the problem is NP-hard and present heuristics and lower bounds to approximate the optimal solution. Freeman et al. [12] consider an unrelated parallel machine scheduling problem with sequence-dependent setup times and their objective is to minimize the total waste and overtime costs. They formulate the problem as a Mixed Integer Program and propose a decomposition heuristic from its solution. Kaya and Sarac [14] study an identical parallel machine scheduling problem with sequence-dependent setup times. Their objectives are minimizing the makespan and total tardiness. They use goal programming to solve a real-life instance from a plastic product manufacturing plant.

Our model differs from the previously reported models in many aspects. First it is different in terms of its objective function. Our objective is to minimize the weighted sum of unsatisfied demand at the end of the planning horizon. When there is a sufficient production time to satisfy all demand, the secondary objective of minimizing the total processing and setup times becomes effective. The secondary objective reflects the production cost. Different from all other studies we have time limitations and shift considerations. The time to complete all production, and our planning horizon is 6 days. The number of workers is limited and each day they are distributed to the production lines by shifts. Suppose that the number of workers is enough to cover six shifts a day and we have three production lines. We also decide which shifts are to be covered at each line. There can be a feasible solution, where lines 1 and 2 cover three shifts, and line 3 stays idle.

We start with formulating a specific model and proposing efficient solution procedures for finding approximate solutions. In Section 2, we define our problem and give a Mixed Integer Linear Programming model. In Section 3, we present our heuristic procedures, and in Section 4 we discuss their performance. In Section 5, we conclude the study by pointing out our main findings and suggestions for future research.

2. PROBLEM DEFINITION AND THE MODEL

The plant makes weekly production plans. Therefore, the preset planning horizon is for six days. We discretize the planning horizon as follows. A day is composed of three 8-hour shifts. A day has a number of intervals each α hours long. We denote a time period by its day and interval number. *d* represents the day index, *k* is the shift index, *t* is the interval index. Therefore, a (*d*,*t*) pair represents the decision time. When $\alpha = 4$ we have the case in Figure 2.

We make the following further assumptions.

- The bottling (second) stage of production is considered as its optimal schedule forms the basis for the organization of the syrup preparation (first) stage.
- At most one product type is assigned to each period, to avoid frequent setups.
- The setup for the first day is carried out the previous night, therefore the setup time for the first period of the planning horizon is zero.
- Each product belongs to a single group but each group may have more than one product.

We have the following operating constraints.

1. **Syrup room constraint:** Syrups are perishable and therefore are prepared immediately before produc-

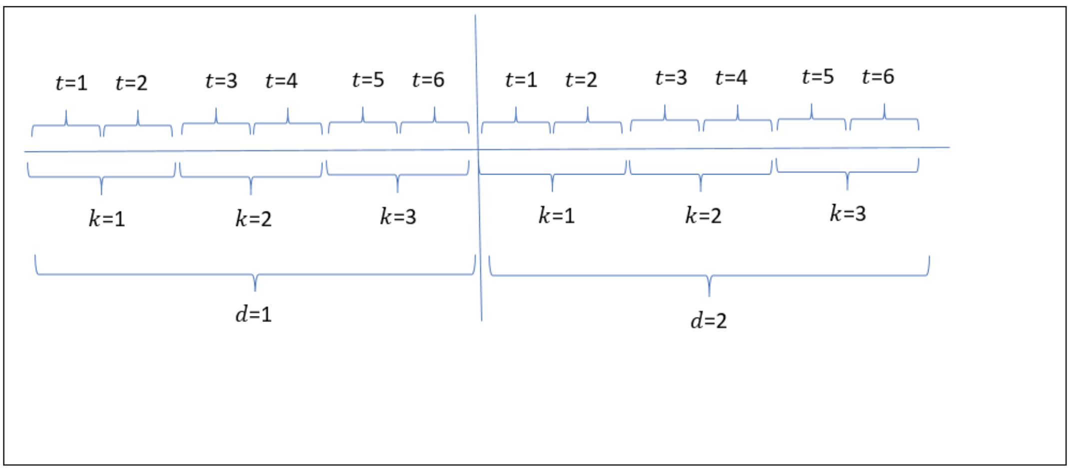

Figure 2. Periods For Two Days When $\alpha = 4$

tion. Since there are limited number of tanks for preparation, the number of different syrups that can be used at a given time is also limited.

2. **Mold availability constraint:** Each product requires a specific mold type. There are limited number of molds of each type and this restricts the number of lines that can simultaneously produce each product type using the same type of mold.
3. **Line eligibility constraint:** The technological capabilities of the lines are different; hence each product can be produced on a specified subset of the production lines.
4. **Number of shifts available in a day:** There are a limited number of workers available during the day, which determines the number of shifts available in a day.
5. **Same shift on all days:** The company aims to run the same shifts at each line for each day throughout the planning horizon to ease the production control.

The preferences that form the objective function are listed below.

- **Demand satisfaction and product priorities:** For each product, we keep track of the fraction of the demand satisfied by the production schedule. Some product types have prespecified priorities. The penalty for failing to satisfy a unit of a product type depends on its priority level.
- **Production time and setup time reduction:** The total production time and time used by setups are as small as possible.

The parameters of the problem are defined below.

α: length of a planning period in hours. It should be more than the maximum setup time between product types and an integer that divides 8. The possible values are 1, 2, 4, and 8.

W: Number of shifts available per day

A: Number of different types of syrups that can be prepared at any time

i: Product type index, $i = 1, \ldots, N$

d: Production day index, $d = 1, \ldots, D$

k: Shift index in a day, $k = 1,2,3$

t: Period index in a day, $t = 1,\ldots,24/\alpha$.

l: Line index, $l = 1,\ldots,L$

m: Mold type index, $m = 1,\ldots,M$

E_l: Set of eligible drinks for line l

q_i: Demand for drink i in units

v_i: Penalty of not satisfying one unit of demand for product type i

B_m: Available number of mold type m

T_k: Set of periods in shift $k, T_k = \{\frac{8}{\alpha}(k-1)+1, \ldots, \frac{8}{\alpha}k\}$

O_{im}: 1 if drink i requires mold m, 0 otherwise

RT_{il}: Rate of production for product type i on line l in units per hour

G_{ig}: 1 if drink i belongs to drink group g, 0 otherwise

f_{ij}: Setup time from product type i to j in hours

$\in_1$: Weight of the second objective (a positive value)

Our decision variables are

x_{idtl}: Fraction of demand for product type i that is satisfied in period (d,t) at line l

y_{idtl}: 1 if the line l is setup for product type i at the beginning of period (d,t), otherwise 0

z_{idtl}: 1 if there is a production of product type i at line l in period (d,t), otherwise 0

st_{dtl}: Setup time spent in period (d,t) at line l

β_{dtg}: 1 if a product type from group g is produced in period (d,t), otherwise 0

λ_{kl}: 1 if shift k is utilized for line l throughout the planning horizon, otherwise 0

γ_i: Fraction of the unsatisfied demand of product type i at the end of the horizon.

$$\text{Minimize} \sum_i v_i q_i \gamma_i + \epsilon_1 (\sum_i \sum_d \sum_t \sum_l \frac{q_i}{RT_{il}} x_{idtl} + \sum_d \sum_t \sum_l st_{dtl})$$

subject to

$$y_{idtl} = 0 \qquad \forall d, t, l, i \notin E_l \qquad (1)$$

$$\sum_i y_{idtl} = 1 \qquad \forall d, t, l \qquad (2)$$

$$x_{idtl} \leq z_{idtl} \qquad \forall i, d, t, l \qquad (3)$$

$$\sum_d \sum_t \sum_l x_{idtl} + \gamma_i \geq 1 \qquad \forall i \qquad (4)$$

$$x_{idtl} \leq y_{idtl} \qquad \forall i, d, t, l \qquad (5)$$

$$st_{11l} = 0 \qquad \forall l \qquad (6)$$

$$st_{d1l} \geq f_{ij}(y_{i,d-1,\frac{24}{\alpha},l} + y_{jd1l} - 1) \qquad \forall d \geq 2, l, i, j \qquad (7)$$

$$st_{dtl} \geq f_{ij}(y_{i,d,t-1,l} + y_{jdtl} - 1) \qquad \forall d, t \geq 2, l, i, j \qquad (8)$$

$$\alpha \sum_i z_{idtl} \leq \sum_i \frac{q_i}{RT_{il}} x_{idtl} + st_{dtl} \leq \alpha \qquad \forall d, t, l \qquad (9)$$

$$\sum_k \sum_l \lambda_{kl} \leq W \qquad (10)$$

$$\lambda_{kl} \geq \lambda_{k+1,l} \qquad \forall k < 3, l \qquad (11)$$

$$\sum_i \sum_{t \in T_k} z_{idtl} \leq \frac{8}{\alpha} \lambda_{kl} \qquad \forall d, k, l \qquad (12)$$

$$\sum_i \sum_l O_{im} z_{idtl} \leq B_m \qquad \forall d, t, m \qquad (13)$$

$$\sum_i \sum_l G_{ig} z_{idtl} \leq L\beta_{dtg} \qquad \forall d, t, g \qquad (14)$$

$$\sum_g \beta_{dtg} \leq A \qquad \forall d, t \qquad (15)$$

$$y_{idtl}, z_{idtl} \in \{0, 1\} \qquad \forall i, d, t, l \qquad (16)$$

$$\beta_{dtg} \in \{0, 1\} \qquad \forall d, t, g \qquad (17)$$

$$\lambda_{kl} \in \{0, 1\} \qquad \forall k, l \qquad (18)$$

$$x_{idtl} \geq 0 \qquad \forall i, d, t, l \qquad (19)$$

$$\gamma_i \geq 0 \qquad \forall i \qquad (20)$$

Now, we can present our Mixed Integer Linear Programming (MILP) model.

The objective is composed of two components, given in the order of importance. The first part is the total weighted penalty of the unsatisfied demand at the end of the horizon.

$$\sum_i v_i q_i \gamma_i$$

Second term is the total time spent on production and setup.

$$\sum_i \sum_d \sum_t \sum_l \frac{q_i}{RT_{il}} x_{idtl} + \sum_d \sum_t \sum_l st_{dtl}$$

Constraint set (1) ensures that products are assigned to their eligible lines. Constraint set (2) keeps track of the product type which is currently setup on a line for each period (d,t). Constraint set (3) ensures that for a given time period and line, if there is a production, then the related z variable takes value 1. Constraint set (4) calculates the fraction of unsatisfied demand for each product type at the end of the horizon.

Constraint set (5) ensures that for each line and time period, production can occur if there is a related setup. Constraint sets (7) and (8) calculate the setup time if there is a product change from one period to the next. We assume that setup for the first day is carried out the previous night, therefore the setup time for the first period of the planning horizon is zero. This assumption is ensured by Constraint set (6).

Constraint set (9) ensures that if a period is used, then the whole time is spent for production and setup, the idle time is not allowed. Constraint set (10) allocates the available shifts during the day to the lines. This schedule is applied during the planning horizon. Constraint set (11) ensures that earlier shifts are preferred to later shifts. Constraint set (12) ensures that a period can be used if the related shift is used.

Based on constraint set (13), it is impossible to use more than the available number of molds for each type at any time during the planning horizon. Constraint set (14) checks whether any product type for each product group is produced for each period in the planning horizon. Constraint set (15) ensures that no more than A types of groups are produced at the same time.

Constraint sets (16, 17, and 18) define the binary variables and constraint sets (19) and (20) ensure the nonnegativity.

3. SOLUTION PROCEDURES

Our initial experiments revealed that the MILP model cannot be solved within a reasonable time even for small sized problem instances. To find high quality solutions in a reasonable time, we develop three heuristic procedures, based on a number of relaxation schemes designed to provide lower bounds on the optimal objective function value.

3.1 Relaxation Schemes

We present two relaxation schemes in this section. In the first, all integer decision variables are relaxed, and in the second only the setup related decision variables are relaxed. These solutions are used as constraints in our heuristic algorithms. We first define these relaxation schemes, and then discuss the details of our heuristic procedures.

Pure Linear Programming Relaxation (LP)

We obtain the pure linear programming relaxation of the model by relaxing all integer decision variables, i.e., the setup variables, shift variables and group variables, (y_{idtl}, z_{idtl}, λ_{kl}, and β_{dtg}). The binary requirement on decision variables are simply removed and replaced by the constraint forcing them to be between 0 and 1. Our first lower bound is the optimal objective function value of the model LP.

Partial Linear Programming Relaxation (PLP)

We obtain the partial Linear Programming relaxation of the MILP by relaxing only the binary requirements for the setup and production variables, y_{idtl} and z_{idtl}. Our second lower bound is the optimal objective function value of the PLP model.

3.2 Decomposition Heuristic

Our first heuristic procedure is based on the concept of decomposing the problem into smaller subproblems, each of which is solved to optimality by an optimization software. Each subproblem has the same flavor as the original problem, except that it considers fewer periods. Once the problem is decomposed into subproblems of *p* days long, it is assumed that all subproblems, except the last one, have exactly *p* days. The last subproblem considers the remaining days.

Two consecutive subproblems *u* and *u* + 1 are related in the sense that the product type produced in the last period of subproblem *u* will be the reflected as the product type setup in the first period of the subproblem *u* + 1. Therefore, the product types produced in the last period of each line for a subproblem are taken as constraints for the next subproblem.

Each subproblem aims to maximize the weighted sum of the satisfied demand. In doing so, as much work as possible is concentrated to the initial periods, favoring our the concern of producing at earlier periods. Such a solution may be essential when there is incomplete information about the product types and demand values. For those uncertain environments, it is more logical to use the initial periods with known demands, and assign the slack capacity to the new product arrivals or extra production requests. We give the stepwise description of our first heuristic procedure below.

Heuristic 1(p)

Step 0. Divide the problem into *U* subproblems, where $U = \lceil D/p \rceil$

Let *u* = 1, solve the first subproblem by considering the first *p* periods.

Step 1. If $u < U$ then let $u = u + 1$ else go to Step 3

Solve subproblem *u* considering the days $(u - 1)\ p + 1$ through $\min\{D, u*p\}$ by fixing the shift decisions taken in subproblem 1,

the updated unsatisfied demand values after subproblem $u - 1$, the setups done at each line in the last shift in subproblem $u - 1$.

Step 2. Update the following parameters of the problem

- the unsatisfied demand amount for each product type using the production in subproblem *u*
- the production types setup at each line by the setups valid in the last period of subproblem *u*
- the current objective function value by adding the second part of the objective from subproblem *u*.

Go to Step 1

Step 3. Stop, all subproblems are solved. The total weighted unsatisfied demand is added to the objective value.

This heuristic uses the maximum number of days considered for a subproblem, *p*, as the main parameter. As *p* increases, the solution quality of the heuristic improves at the expense of increasing the solution time. At one extreme *p* = *D*, hence no decomposition is done, and at another extreme *p* = 1, hence each day forms its own problem, and *D* subproblems are solved. The latter problem is the easiest to solve and produces poor quality solutions.

3.3 Linear Programming Relaxation Based Heuristic Procedures

Our preliminary experiments on small sized problem instances have revealed the satisfactory behavior of the Partial Linear Programming Relaxation (PLP). Based on the instances that are solved to optimality, we find that most of the products that are fully unsatisfied are same for the optimal solution of the original model (MILP) and the PLP. Therefore, we decided to use the optimal solution of the PLP in developing two heuristic procedures.

The first LP-based heuristic procedure, Heuristic 2, uses the shift decisions given by PLP, and finds the values of other decisions by solving MILP, by taking shift decisions which are parameters, rather than decision variables.

Moreover, we observe that the majority of the product types that are not included in the solution of PLP (i.e. the ones with zero satisfied demand values) are also not included in the optimal MILP solution. In other words, if we obtain $\gamma_i^{PLP} = 1$, then it is very likely that $\gamma_i^* = 1$. Following this observation, we will not consider product type *i* if $\gamma_i^{PLP} = 1$. Therefore, we reduce the problem size.

Below is the stepwise description of our first partial linear programming relaxation based heuristic procedure.

Heuristic 2

Step 0. Relax the binary constraint on setup and production variables.

Include constraints, $0 \leq y_{idtl} \leq 1$ and $0 \leq z_{idtl} \leq 1$.

Step 1. Solve the resulting linear programming relaxation problem.

Suppose that λ^{PLP}_{kl} and γ_i^{PLP} are the optimal solution values for the partial relaxation.

Step 2. Reduce the problem size by removing each product type i if $\gamma_i^{PLP} = 1$

Solve the reduced MILP model using the λ_{kl}^{PLP} values as parameters and replacing the constraint set $xidtl \leq yidtl$ by $zidtl \leq yidtl$.

The second partial Linear Programming Relaxation based heuristic procedure, Heuristic 3, uses the production decisions given by (PLP), i.e., x_{idtl} values, in addition to the shift decisions. As in heuristic 2, we first reduce the problem size by removing the products with $\gamma_i^{PLP} = 1$. Moreover, on each day, we only consider the product types produced in the optimal solution of (PLP). We set $\sum_t \sum_l y_{idtl} = 0$ if $\sum_t \sum_l x_{idtl}^{PLP} = 0$.

We define S_d as the set of products that are produced in day d in the optimal solution of (PLP), i.e.,

$$S_d = \{i | \sum_t \sum_l x_{idtl}^{PLP} > 0\}.$$

Table 1. Parameters

W	α	L	D	M	A	
6	4	3	6	6	2	1

Formally, we incorporate the following constraint for each d.

$$\sum_t \sum_l \sum_{i \notin S_d} y_{idtl} = 0 \text{ or } \sum_t \sum_l \sum_{i | x_{idtl}^{PLP} = 0} y_{idtl} = 0 \quad (21)$$

Note that according to the above constraint set, the heuristic selects among the products in set S_d. Below is the stepwise description of our third heuristic.

Heuristic 3

Step 0. Relax the integer constraints on z_{idtl} and y_{idtl}.

Step 1. Solve PLP.

Let x^{PLP}_{jdtl}, λ^{PLP}_{kl}, and γ_j^{PLP} be the optimal solution values.

Step 2. Reduce the problem size by removing the products with $\gamma_i^{PLP} = 1$.

Compute S_d for each d.

Solve the reduced MILP model with constraint set (21).

The idea of fixing some variables based on the optimal values coming from relaxations is also used by Ferreira et al. (9, 10).

4. COMPUTATIONAL RESULTS

We evaluate the performance of our heuristics and MILP on various parameter settings. The algorithms are coded in *C#* programming language, and mixed integer linear models are solved via CPLEX 12.6 and run on 16 GB Dual channel RAM laptops with i7-5600U processor. The company needs an immediate solution, therefore we put a time limit on all our runs. Time limit for MILP is 3600 seconds and the run stops when the optimality gap becomes less than or equal to 1%. The time limit for all heuristics is 900 seconds.

We generate 12 different combinations of the problem parameters. We randomly generate three parameters (demand, production rate, and setup time) from different intervals using uniform distribution. We consider three values of number of product types, $N = 5,10,20$. Some parameters have fixed values, given in Table 1.

- **Demand and setup times:** Demand and setup times are randomly generated based on uniform distribution using the intervals in Table 2. We determine the lower and upper bounds of the uniform distribution that represents the real case at the soft drink production plant.

Note that all setup times are smaller than our defined period length of ($\alpha = 4$) hours, hence no period will be occupied by only setup operations.

Table 2. Demand and Setup Time Distributions

	High (H)	Low (L)	Constant(C)
Demand	$U[800000/N,1500000/N]$	$U[200000/N,1000000/N]$	
Setup time	$U[1.8,3.6]$	$U[0.9,1.8]$	1

Table 3. Production Rate Distributions

	R (Random)		LD (Line dependent)	
	Rate	Probability of eligibility	Rate	Probability of eligibility
Line 1	$U[500,3500]$	1	$U[1000,1500]$	1
Line 2	$U[500,3500]$	0.3	$U[1000,2500]$	0.8
Line 3	$U[500,3500]$	0.4	$U[1000,3500]$	0.6

- **Line eligibility and production rates:** Each of our three production lines can process a subset of all products, hence have different production capabilities. Based on the real case, we assume that the first line is capable of producing all product types.

  For the other two production lines, for each product type we generate a uniform random number between 0 and 1. If the generated number is below the probability of eligibility, we include that product into the eligible set of the line.

  There are two alternatives for generating production rates. In random case (R), the production rate of a product type is uniformly generated from the same interval for each line, however if the production rates depend on the line (LD), they are generated from line-dependent intervals. Line-dependent set is in line with current operation of the company, such that for a given product type, production rate of line 1 < production rate of line 2 < production rate of line 3. We generate data using the values reported in Table 3.

- **Mold availability:** Based on the practical case, we take the number of available molds for a given type as 1 or 2 with respective probabilities of 0.7 and 0.3. Note that we expect that about 70 percent of the mold types has only one available unit.

- **Product-mold requirement:** In our data set, each product belongs to a single group and the group of each product is assigned randomly. All groups are equally likely.

- **Product penalties:** For each product type, we uniformly generate an integer in [1, 10].

For each combination of problem parameters, we randomly generate 10 problem instances. To compare the performance of the heuristics, we use the following measures for each instance.

- TS_i: the time spent by heuristic i (in seconds), i = *MILP* represents the original mathematical model
- Ω_i: the objective value obtained by heuristic i at the end of the time limit, i = *MILP* represents the original mathematical model
- *Gap*: the optimality gap of MILP in 3600 seconds
- Gap_i: the relative gap of heuristic i (run for 900 seconds) in terms of the solution obtained by MILP in 3600 seconds

$$Gap_i = \frac{\Omega_i - \Omega_{MILP}}{\Omega_{MILP}} * 100$$

We give an example of an optimal schedule in the following figure, for a five-product case. Only two lines are utilized and the other line is kept idle. The numbers

	day 1						day 2						day 3					
t	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
line 2	1	1	1	1	1	1	1	1	1	S/2	2	2	2	2	2	2	2	2
line 3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

	day 4						day 5						day 6					
t	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
line 2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
line 3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

Figure 3. An Optimal Schedule for a Five-Product Case

in the boxes represent the product type. We have a setup from product 1 to product 2 on day 2 at t=4.

The summary measure is average, which is the average of the all instances with given set of parameters. First of all, we test the performance of MILP. Table 4 gives average solution times and optimality gaps at the termination limit for all problem settings. We observe that the biggest factor that affects the running time is the size of the problem, N.

As N increases, running time increases significantly. In high demand cases, average running time is below one minute, when there are 5 products and above one hour when there are 20 products. For the cases, in which the optimal solution is not obtained in one hour, average gap also increases significantly as the number of product type increases. For high demand rates, average gap is about 0.5%, 2% and 10% in average for $N = 5$, 10, and 20, respectively. The problem becomes harder as N increases, the running time is an exponential function of N.

We observe that running time of MILP is also sensitive to the demand rate. The instances with high demand rates are much easier to solve than those with low demand rates. The reason is that MILP with first objective function is much easier to solve than the one with second objective function. Low demand instances leave more room for product to line assignments, therefore this leads to many assignment alternatives. More alternatives, i.e., a bigger feasible region, increase the complexity of the search. When $N = 5$, the average running times are below one minute for high demand instances and about half an hour for low demand instances. In cases, where $N = 10$, almost all of the low demand instances have an optimality gap after one. Whereas, average running times are around 2000 seconds for high demand cases. Optimality gap decreases as demand rates increases. When $N = 5$, and demand rate is high, average gap is around 0.5% and when demand rate is low, average gaps are around 3%. When $N = 10$, average gap is around 2% and 20% for high and low demand instances.

When we have random production rates, the model has to search more to assign products to production lines. We observe that when the setup times are constant, i.e. sequence independent, the instances with line-dependent production rates are easier to solve. For example, for $N = 5$, and when demand rate is high, the running times are 2.57 and 17.07 seconds for line-dependent and random production rates, respectively. When demand rate is low, the running times are 409.95 and 808.41 seconds for line-dependent and random production rates, respectively.

On the other hand, when setup times are sequence-dependent, the running time of MILP is insensitive to dependency of production rates to lines. We next discuss the sensitivity of the running time to the size of setup time. The solution time or gap is smaller when the setup times are lower. When we have high setup times, the objective function value is more sensitive to the sequence of products, i.e. a small change in the sequence may lead to higher deviations in the objective function value.

When $N = 5$, demand rates are high, and production rates are line dependent, average running times are 3.24 and 48.24 seconds, for low and high setup times, respectively. When $N = 10$, demand rates are high, and

Table 4. The Performance of MILP (time limit = 3600 seconds)

N	Demand	Prod Rate	Setup Time	Average TSMILP	Average Gap
5	H	LD	L	3.24	0.56
5	H	LD	H	48.24	0.74
5	H	LD	C	2.57	0.47
5	H	R	L	2.54	0.39
5	H	R	H	20.81	0.38
5	H	R	C	17.07	0.45
5	L	LD	L	1101.58	3.01
5	L	LD	H	2351.77	5.48
5	L	LD	C	409.95	1.35
5	L	R	L	1445.67	2.26
5	L	R	H	1477.64	3.07
5	L	R	C	808.41	1.11
10	H	LD	L	1587.83	1.20
10	H	LD	H	2131.73	1.76
10	H	LD	C	2320.41	2.88
10	H	R	L	1922.40	1.16
10	H	R	H	2911.54	2.43
10	H	R	C	1575.76	1.06
10	L	LD	L	3600.00	18.46
10	L	LD	H	3483.14	15.76
10	L	LD	C	3600.00	11.13
10	L	R	L	3600.00	16.97
10	L	R	H	3600.00	20.58
10	L	R	C	3260.00	17.06
20	H	LD	L	3600.00	9.91
20	H	LD	H	3600.00	19.06
20	H	LD	C	3600.00	7.37
20	H	R	L	3600.00	9.59
20	H	R	H	3600.00	14.64
20	H	R	C	3600.00	7.55

production rates are line dependent, average running times are 1587.83 and 2131.73 seconds, for low and high setup times, respectively.

We now discuss the performance of our heuristic procedures. The performance is measured by average running time or average gap when the time limit is exceeded. We define gap as the deviation of the heuristic solution from MILP solution as the percentage of MILP solution. MILP is considered as the best objective value obtained at the termination limit of one hour.

We report the performance of Heuristic 1 in Table 5. $p = 1$ refers to the case with 6 subproblems, each one day long and $p = 2$ refers to the case with 3 subproblems, each 3 days long. Heuristic 1 uses MILP for each subproblem. Heuristic 1 does not provide satisfactory solutions, since the smallest gap is 778% when $p = 1$, and 304% when $p = 2$. We want to note that both cases are of high demand. As we predicted, the solution quality becomes better as p increases, with an increase in running time, which is less than 2172.95 in average over all instances. We suggest to use it when demand values are not available, namely in uncertain environments.

We evaluate the performance of Heuristic 2 and Heuristic 3 in Table 6. A negative average gap value in tables indicates that on average, heuristic solutions are better than the solutions returned by MILP at the termination limit.

The positive average gap values for Heuristics 2 are 3 are very small (i.e. for heuristic 2, ≤ 8.57 when $N = 5$, ≤ 5.70 when $N = 10$, and ≤ -1.98 when $N = 20$). These observations indicate that their performance (in 15 minutes) is close to performance of MILP (in one hour). The average gap improves as N increases. When $N = 5$, average gaps are between 6% and 9.5% for the hardest combination, and for the other settings almost all average gap values are below 6%.

When $N = 20$ and demand rate is high, all average gaps obtained by Heuristic 2 and Heuristic 3 are negative. When $N = 10$, and demand rate is low, half of the average gaps are negative for both heuristics. Negative gap values indicate that for hard problem settings, both heuristics perform much better than MILP. We observe that Heuristic 2 and Heuristic 3 do not dominate each other, hence they can be used together to achieve higher quality solutions.

We suggest practitioners to use MILP for the cases with $N \leq 10$ with high demand rates, and for all cases, where $N \leq 5$. For $N > 10$ or low demand rates, Heuristic 2 or Heuristic 3 should be used.

5. CONCLUSIONS

In this study, we consider a scheduling problem faced in a soft-drink bottling plant. The problem resides in sequence-dependent setup times, side constraints that stem from the shifts, mold types, and drink groups. Our objective is to minimize the weighted sum of the unsatisfied demand and total production and setup times. We model the problem as a MILP, and show that it is capable of solving instances with up to 5 and 10 products for high and low demand cases, respectively.

We develop three heuristic procedures that use decomposition and Linear Programming

Relaxation ideas. We compare the performances of the heuristics relative to the solutions returned by the MILP model at a specified termination limit of one hour. We observe that the decomposition based heuristics find rapid solutions, but at the expense of decreased solution quality. Linear Programming based heuristics produce higher quality solutions, in a shorter time and could solve the instances with up to 20 products in 15 minutes. We consider that our study has contributed to the scheduling literature, and practices in the soft-drink industry. Future research may lead to the development of exact procedures for the problem under consideration in this study. Moreover, other lower bounds can be developed to assess the quality of our heuristic procedures.

Table 5. The Performance of Heuristic 1, for p = 1,2 (time limit = 900 seconds)

				$p = 1$		$p = 2$	
N	Demand	Prod Rate	Setup Time	Average TS_1	Average Gap_1	Average TS_1	Average Gap_1
5	H	LD	L	0.55	825.40	0.44	328.80
5	H	LD	H	0.34	820.30	0.35	326.58
5	H	LD	C	0.31	778.18	0.26	307.18
5	H	R	L	0.24	861.41	0.25	340.95
5	H	R	H	0.26	887.88	0.30	354.54
5	H	R	C	0.22	977.57	0.27	386.75
5	L	LD	L	0.35	2333.87	0.58	872.48
5	L	LD	H	0.31	2962.92	0.69	1121.50
5	L	LD	C	0.30	3480.86	1.77	1256.20
5	L	R	L	0.31	2164.57	3.24	801.50
5	L	R	H	0.50	1696.45	0.65	665.30
5	L	R	C	0.31	1588.06	0.67	616.00
10	H	LD	L	0.76	850.64	1.51	335.19
10	H	LD	H	0.96	810.91	2.26	323.20
10	H	LD	C	0.99	935.93	1.56	374.41
10	H	R	L	1.02	843.81	1.56	333.94
10	H	R	H	1.13	895.96	3.69	356.06
10	H	R	C	0.91	893.78	2.58	355.85
10	L	LD	L	1.45	1662.27	137.85	643.58
10	L	LD	H	3.66	1446.42	185.42	540.82
10	L	LD	C	1.22	2047.14	57.48	706.91
10	L	R	L	1.93	1678.71	206.78	639.25
10	L	R	H	1.71	1845.22	193.40	727.43
10	L	R	C	1.91	2289.70	283.84	838.71
20	H	LD	L	3.74	867.74	42.66	341.69
20	H	LD	H	14.14	875.54	833.48	330.93
20	H	LD	C	4.19	935.92	100.29	359.00
20	H	R	L	4.44	820.66	169.28	319.30
20	H	R	H	71.62	783.42	2172.95	304.25
20	H	R	C	4.84	874.58	189.54	343.06

Table 6. The Performance of Heuristic 3 and Heuristic 2 (time limit = 900 seconds)

				Heuristic 3		Heuristic 2	
N	Demand	Prod Rate	Setup	Average TS_3	Average Gap_3	Average TS_2	Average Gap_2
5	H	LD	L	2.57	0.14	2.26	0.27
5	H	LD	H	5.12	5.84	5.02	5.55
5	H	LD	C	1.08	1.33	0.89	1.37
5	H	R	L	1.04	4.59	0.90	4.60
5	H	R	H	4.17	3.57	2.32	1.74
5	H	R	C	7.44	3.34	5.10	3.25
5	L	LD	L	255.83	3.50	294.19	3.10
5	L	LD	H	288.01	2.46	414.15	-0.06
5	L	LD	C	95.76	1.39	110.96	1.19
5	L	R	L	224.38	6.70	270.52	6.35
5	L	R	H	179.60	9.35	270.40	8.57
5	L	R	C	155.46	7.26	205.78	7.02
10	H	LD	L	127.60	1.02	241.60	0.86
10	H	LD	H	253.59	1.19	347.48	1.06
10	H	LD	C	479.20	1.57	488.01	1.54
10	H	R	L	296.90	3.45	313.22	3.46
10	H	R	H	354.01	1.79	342.63	1.35
10	H	R	C	192.88	5.29	303.08	5.00
10	L	LD	L	808.90	-2.14	884.42	-2.65
10	L	LD	H	826.16	-0.08	1583.94	-1.35
10	L	LD	C	827.09	0.30	920.84	0.21
10	L	R	L	738.66	4.19	737.26	5.70
10	L	R	H	937.64	-2.55	935.67	-3.70
10	L	R	C	725.54	0.71	911.27	0.52
20	H	LD	L	924.63	-2.30	885.97	-3.61
20	H	LD	H	939.53	-5.85	939.24	-7.26
20	H	LD	C	929.14	-1.72	929.01	-1.98
20	H	R	L	934.21	-2.33	857.78	-2.63
20	H	R	H	929.47	-2.87	928.96	-4.54
20	H	R	C	921.53	-1.67	921.64	-2.15

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