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The New Exact and Approximate Solution for the Nonlinear Fractional Diffusive Predator-Prey System

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Original Article

Abstract — In this article two methods, q-Homotopy analysis Method (q-HAM) and Sine-Gordon expansion method are proposed for solving fractional Diffusive Predator-Prey system. The fractional derivative is considered in the conformable sense. The obtained solutions using the suggested methods are in good agreement with the existing ones and show that these approaches can be used for solving various conformable time fractional partial differential equations arising in different branches of science.

Keywords — *Sine-Gordon Expansion Method, Fractional Diffusive Predator-Prey system, q-Homotopy Analysis Method, Conformable Fractional Derivative.*

1. Introduction

Fractional calculus has a very long history. However, this field lagged behind classic analysis. There is an increasing interest to study of the fractional differential equations because of their various applications such as in viscoelasticity, anomalous diffusion, mechanics, biology, chemistry, acoustics, control theory, etc. A great deal of effort has also been expanded in attempting to find robust and stable numerical and analytical methods for solving fractional differential equations of physical interest. In this paper, we have applied a numerical method called Homotopy analysis method and an analytical method called Sine-Gordon expansion method to obtain solutions of Fractional Diffusive Predator-Prey system. The homotopy analysis method (HAM) was first introduced by Liao [1], who employed the basic ideas of the homotopy in topology to propose a general analytic method for nonlinear problems. El-Tawil and Huseen [2] proposed a modified namely q-homotopy analysis method (q-HAM) which is a more general method of HAM. This method is applied to solve many nonlinear problems [3, 4, 5, 6].

The Sine-Gordon expansion method is an efficient and powerful technique for solving differential equations. This method is firstly proposed by the Chinese mathematician Yan [7]. The Sine-Gordon expansion method is based on the explicit linearization of differential equations for traveling waves which leads to a second-order differential equation with constant coefficients. Moreover, the solutions obtained by this method are of general nature and a number of specific solutions can be deduced by putting conditions on arbitrary constants present in the general solutions [8, 9, 10].

In this paper, we applied q-homotopy analysis and Sine-Gordon expansion methods for solving fractional Diffusive Predator-Prey system. This work is organized as follows: In section 2 we provide

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some preliminaries of conformable fractional derivative. Section 3 introduces the concept of Sine-Gordon expansion method, while section 4 gives to solutions of fractional Diffusive Predator-Prey system. The q-Homotopy analysis method (q-HAM) is analyzed in section 5. Graphics of the numerical examples are provided in section 6. The conclusions are given in section 7.

2. Governing equations

One of the most popular fractional predator-prey system in nonlinear fractional evolution equations can be expressed as follows (for $\alpha = 1$, see [11, 12])

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} - \beta u + (1+\beta)u^2 - u^3 - uv, \tag{1}$$

$$\frac{\partial^\alpha v}{\partial t^\alpha} = \frac{\partial^2 v}{\partial x^2} + \kappa uv - mv - \delta v^3, \tag{2}$$

where κ, δ and β are positive parameters, and where $\frac{\partial^\alpha}{\partial t^\alpha}$ is conformable deravative operator of order $\alpha \in (0, 1)$ in the $t > 0$ can be defined as follows [14]

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \lim_{\varepsilon \to 0} \frac{u(t + \varepsilon t^{1-\alpha}) - u(t)}{\varepsilon}.$$

Later on, many useful methods for obtaining exact solutions of several nonlinear fractional evolution equations by using this fractional derivative have been reported [15-33].
In this paper, we investigate a fractional order prey-predator interaction with following relations between the parameters

$$m = \beta, \qquad \kappa + \frac{1}{\sqrt{\delta}} = \beta + 1.$$

Based on these assumptions, Eqs. (2.1) and (2.2) are established by the following

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} - \beta u + \left(\kappa + \frac{1}{\sqrt{\delta}}\right) u^2 - u^3 - uv \tag{3}$$

$$\frac{\partial^\alpha v}{\partial t^\alpha} = \frac{\partial^2 v}{\partial x^2} + \kappa uv - \beta v - \delta v^3. \tag{4}$$

The fractional prey-predator system incorporating diffusion is of profound interest because it involves the heterogeneity of both the populations the environment. Formation of the spatial distribution pattern with the diffusion models even in the absence of environmental heterogeneity is another interesting event [13]. For better understand about the processes involved, existences of exact solutions are needed.

3. Sine-Gordon expansion method

In this section we describe the first step of the Sine-Gordon expansion method for finding exact solutions of nonlinear conformable fractional partial differential equations (PDEs).
We consider the following time conformable fractional nonlinear partial differential equation in two variables and a dependent variable u

$$F\left(u, \frac{\partial^\alpha u}{\partial t^\alpha}, \frac{\partial u}{\partial x}, \frac{\partial^{2\alpha} u}{\partial t^{2\alpha}}, \frac{\partial^2 u}{\partial x^2}, \ldots\right) = 0, \tag{5}$$

where F is a polynomial in u and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved and $\frac{\partial^{2\alpha} u}{\partial t^{2\alpha}}$ means two times conformable fractional derivative of function $u(x, t)$. To solve Eq.(5), we take the traveling wave transformation

$$u(x, t) = U(\xi), \qquad \xi = x - c\frac{t^\alpha}{\alpha}, \tag{6}$$

where $c \neq 0$ is a constant to be determined later. This enables us to use the following changes

$$\frac{\partial^{\alpha}(.)}{\partial t^{\alpha}} = -c\frac{d(.)}{d\xi}, \qquad \frac{\partial(.)}{\partial x} = \frac{d(.)}{d\xi}, \qquad \frac{\partial^{2\alpha}(.)}{\partial t^{2\alpha}} = -c\frac{d^2(.)}{d\xi^2}, \ldots.$$

Substituting Eq.(6) in Eq. (5) yields a nonlinear ordinary differential equation as following

$$G(U, U', U'', U''', \ldots) = 0, \tag{7}$$

where $U = U(\xi), U' = \frac{dU}{d\xi}, U'' = \frac{d^2U}{d\xi^2}, \ldots$ and so on.

Now lets describe the procedure of Sine-Gordon expansion method.This method established on the Sine-Gordon equation and wave transform. The Sine-Gordon equation which is presented as a model field theory;

$$u_{xx} - u_{tt} = \tau^2 sin(u), \tag{8}$$

where τ is a real constant and $u = u(x,t)$. Considering the wave transformation $\xi = \mu(x - ct)$ over the Eqn. (8) the function $u = u(x,t)$, turns into $U(\xi)$, then we have the following nonlinear differential equation,

$$U'' = \frac{\tau^2}{\mu^2(1-c^2)}\sin(U). \tag{9}$$

By simplifying the Eq. (9),

$$\left[\left(\frac{U}{2}\right)'\right]^2 = \frac{\tau^2}{\mu^2(1-c^2)}\sin^2\left(\frac{U}{2}\right) + K, \tag{10}$$

where K is integration constant. Supposing $K = 0$, $\Phi(\xi) = \frac{U}{2}$, $\varrho^2 = \frac{\tau^2}{\mu^2(1-c^2)}$ and subrogating into Eqn. (10),

$$\Phi' = \varrho\sin(\Phi), \tag{11}$$

regarding $\varrho = 1$ in Eqn. (11), led to

$$\Phi' = \sin(\Phi). \tag{12}$$

Evaluating the solution of (12) by using separation of variables method, we attain the following equations,

$$\sin(\Phi) = \sin(\Phi(\xi)) = \frac{2\zeta e^{\xi}}{\zeta^2 e^{2\xi}+1}|_{\zeta=1} = \text{sech}(\xi), \tag{13}$$

$$\cos(\Phi) = \sin(\Phi(\xi)) = \frac{\zeta^2 e^{2\xi}-1}{\zeta^2 e^{2\xi}+1}|_{\zeta=1} = \tanh(\xi), \tag{14}$$

where ζ is integration constant. To obtain the solution of nonlinear conformable PDE (5);

$$G(u, D_t^{\alpha}u, D_x u, D_{xx}u, D_t^{\alpha}D_t^{\alpha}u, \ldots), \tag{15}$$

we design,

$$U(\xi) = \sum_{i=1}^{n}\tanh^{i-1}(\xi)\left[B_i\,\text{sech}(\xi) + A_i\tanh(\xi)\right] + A_0, \tag{16}$$

due to to Eqns. (13) and (14), Eqn. (16) can be regulated as

$$U(\Phi) = \sum_{i=1}^{n}\cos^{i-1}(\Phi)\left[B_i\sin(\Phi) + A_i\cos(\Phi)\right] + A_0. \tag{17}$$

The parameter n can be determined balancing the degrees between the highest order linear term and nonlinear term in Eq.(7). Next equating all the coefficients of $\cos^i(\Phi)$ and $\sin^i(\Phi)$ to be zero yields an equation system. Solving system using an computer software such as Maple the values of A_i, B_i, μ and c can be derived. Lastly subrogating the values of A_i, B_i, μ and c in Eqn. (16), we can express the traveling wave solutions.

4. Application of the Sine-Gordon expansion method to Fractional Diffusive Predator-Prey system

We suppose that

$$u(x,t) = U(\xi), \qquad v(x,t) = V(\xi), \qquad \xi = x + \nu\frac{t^\alpha}{\alpha}, \tag{18}$$

where ν is constant. Using the conformable chain rule and on substituting these into Eq.(3), we have

$$\begin{aligned} &U'' - \nu U' - \beta U + \left(\kappa + \frac{1}{\sqrt{\delta}}\right) U^2 - U^3 - UV = 0, \\ &V'' - \nu V + \kappa UV - \beta UV - \delta V^3 = 0. \end{aligned} \tag{19}$$

In order to solve system (19), let us consider the following transformation

$$V = \frac{U}{\sqrt{\delta}}. \tag{20}$$

Substituting the transformation (20) into (19),we get

$$U'' - \nu U' - \beta U + \kappa + U^2 - U^3 = 0. \tag{21}$$

Due to procedure of Sine-Gordon expansion method, assume that U can be written in the form

$$U(\Phi) = \sum_{i=1}^{n} \cos^{i-1}(\Phi)\left[B_i \sin(\Phi) + A_i \cos(\Phi)\right] + A_0. \tag{22}$$

Balancing the terms U'' and U^3 led to $n = 1$, thus

$$U = B\sin\Phi + A\cos\Phi + C, \tag{23}$$

and

$$U'' = -B(\sin\Phi)^3 + B(\cos\Phi)^2\sin\Phi - 2A(\sin\Phi)^2\cos\Phi. \tag{24}$$

Replacing the equations (23) and (24) into (21), using some trigonometric identities and setting all the coefficients of $\cos^i\Phi$ and $\sin^i\Phi$ produces the following algebraic equation system

$$\begin{aligned} &A^3 + 3B^2A + 2A = 0, \\ &2B + B^3 - 3BA^2 = 0, \\ &\kappa A^2 + 3B^2C - 3A^2C - \kappa B^2 + \nu A = 0, \\ &\nu B + 2\kappa BA - 6BAC = 0, \\ &2\kappa AC - \beta A - 3B^2A - 3AC^2 - 2A = 0, \\ &-\beta B + 2\kappa BC - B - B^3 - 3BC^2 = 0, \\ &\kappa B^2 - 3B^2C - \beta C + \kappa C^2 - C^3 - \nu A = 0. \end{aligned} \tag{25}$$

Solving the system with the aid of Maple, we obtain the following solution sets,

$$A = \mp\sqrt{2}, B = 0, C = \mp\frac{\nu}{\sqrt{2}}, \beta = -2 + \frac{\nu^2}{2}, \kappa = \mp\sqrt{2}\nu,$$

$$A = \pm\sqrt{2}, B = 0, C = \mp\sqrt{2}, \beta = 4 + 2\nu, \kappa = \mp\frac{\sqrt{2}(6+\nu)}{2},$$

$$A = \mp\sqrt{2}, B = 0, C = \mp\sqrt{2}, \beta = 4 - 2\nu, \kappa = \pm\frac{\sqrt{2}(\nu - 6)}{2},$$

$$A = \pm\frac{\sqrt{2}}{2}, B = \mp\frac{i\sqrt{2}}{2}, C = \mp\frac{\sqrt{2}}{2}, \beta = \nu + 1, \kappa = \mp\frac{\sqrt{2}(3+\nu)}{2},$$

$$A = \mp\frac{\sqrt{2}}{2}, B = \mp\frac{i\sqrt{2}}{2}, C = \mp\frac{\sqrt{2}}{2}, \beta = 1 - \nu, \kappa = \pm\frac{\sqrt{2}(-3+\nu)}{2},$$

$$A = \mp\frac{\sqrt{2}}{2}, B = \mp\frac{i\sqrt{2}}{2}, C = \mp\frac{\nu\sqrt{2}}{2}, \beta = \frac{\nu^2 - 1}{2}, \kappa = \mp\sqrt{2}\nu.$$

Using the above values of A, B, C, β, κ and (20), the solutions of $u(x,t)$ and $v(x,t)$ can be obtained as

$$
\begin{aligned}
u_1(x,t) &= \mp\frac{\nu}{\sqrt{2}} \mp \sqrt{2}\tanh\left[x+\frac{t^\alpha \nu}{\alpha}\right],\\
u_2(x,t) &= \mp\left(\sqrt{2}-\sqrt{2}\tanh\left[x+\frac{t^\alpha \nu}{\alpha}\right]\right),\\
u_3(x,t) &= \mp\left(\sqrt{2}+\sqrt{2}\tanh\left[x+\frac{t^\alpha \nu}{\alpha}\right]\right),\\
u_4(x,t) &= \pm\left(-\frac{1}{\sqrt{2}}-\frac{i\text{sech}\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}+\frac{\tanh\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}\right),\\
u_5(x,t) &= \pm\left(-\frac{1}{\sqrt{2}}-\frac{i\text{sech}\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}-\frac{\tanh\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}\right),\\
u_6(x,t) &= \mp\left(\frac{\nu}{\sqrt{2}}+\frac{i\text{sech}\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}+\frac{\tanh\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}\right),
\end{aligned}
$$

$$
\begin{aligned}
v_1(x,t) &= \frac{\mp\frac{\nu}{\sqrt{2}} \mp \sqrt{2}\tanh\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{\delta}},\\
v_2(x,t) &= \frac{\sqrt{2}-\sqrt{2}\tanh\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{\delta}},\\
v_3(x,t) &= \frac{-\sqrt{2}+\sqrt{2}\tanh\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{\delta}},\\
v_4(x,t) &= \pm\left(\frac{-\frac{1}{\sqrt{2}}-\frac{i\text{sech}\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}+\frac{\tanh\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}}{\sqrt{\delta}}\right),\\
v_5(x,t) &= \pm\left(\frac{-\frac{1}{\sqrt{2}}-\frac{i\text{sech}\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}-\frac{\tanh\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}}{\sqrt{\delta}}\right),\\
v_6(x,t) &= \mp\left(\frac{\frac{\nu}{\sqrt{2}}+\frac{i\text{sech}\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}+\frac{\tanh\left[x+\frac{t^\alpha\nu}{\alpha}\right]}{\sqrt{2}}}{\sqrt{\delta}}\right).
\end{aligned}
$$

5. Numerical Solution of Fractional Diffusive Predator-Prey system

In this section, we implement q-homotopy analysis method (q-HAM) which is generalized version of homotopy analysis method (HAM) [34] to obtain the numerical solution of diffusive predator-prey system. q-HAM involves the parameter h which is used for adjusting and controlling the convergence of solution series.(See [35, 36]) Regard the nonlinear system of equations in with the following initial conditions

$$
\begin{aligned}
u(x,0) &= 1+\sqrt{2}\tanh[x],\\
v(x,0) &= 2+2\sqrt{2}\tanh[x].
\end{aligned}
\tag{26}
$$

We consider the coefficients $\nu=\sqrt{2}, \delta=\frac{1}{4}, \kappa=2, \beta=2$ for both calculations in the rest of article. We can chose the linear operators To obtain the series solutions of system of equations in (3) with initial conditions 26) as follows

$$
\begin{aligned}
\mathcal{L}_1\left[\varphi_1(x,t;q)\right] &= D_t^\alpha \varphi_1(x,t;q),\\
\mathcal{L}_2\left[\varphi_2(x,t;q)\right] &= D_t^\alpha \varphi_2(x,t;q),
\end{aligned}
$$

where the linear operators satisfies the condition $\mathcal{L}_j\,[m] = 0$ for each $j \in \{1,2\}$, where m is constant. The non-linear operators can be defined from the system (3) such as

$$
\begin{aligned}
\mathcal{N}_1\left[\varphi_1(x,t;q)\right] &= \frac{\partial^\alpha \varphi_1(x,t;q)}{\partial t^\alpha} - \frac{\partial^2 \varphi_1(x,t;q)}{\partial x^2} + \beta\varphi_1(x,t;q),\\
&- \left(\kappa + \frac{1}{\sqrt{\delta}}\right)\varphi_1(x,t;q)^2 - \varphi_1(x,t;q)^3 + \varphi_1(x,t;q)\varphi_2(x,t;q),\\
\mathcal{N}_2\left[\varphi(x,t;q)\right] &= \frac{\partial^\alpha \varphi_2(x,t;q)}{\partial t^\alpha} - \frac{\partial^2 \varphi_2(x,t;q)}{\partial x^2} - \kappa\varphi_1(x,t;q)\varphi_2(x,t;q) + \beta\varphi_2(x,t;q) + \delta\varphi_2(x,t;q)^3.
\end{aligned}
$$

So the zero-order deformation equations can be constituted as:

$$
\begin{aligned}
(1-nq)\mathcal{L}_1\left[\varphi_1(x,t;q) - u_0(x,t)\right] &= qh_1\mathcal{N}\left[\varphi_1(x,t;q)\right],\\
(1-nq)\mathcal{L}_2\left[\varphi_2(x,t;q) - v_0(x,t)\right] &= qh_2\mathcal{N}\left[\varphi_2(x,t;q)\right].
\end{aligned}
$$

When $H_j(x,t) = 1$ chosen properly [35], for each $j \in \{1,2\}$, the mth-order deformation equation is

$$
u_m(x,t) = \chi_m^* u_{m-1}(x,t) + h_1\mathcal{L}_1^{-1}\left[R_{1,m}\left(\mathbf{u}_{m-1}\right)\right], \tag{27}
$$
$$
v_m(x,t) = \chi_m^* v_{m-1}(x,t) + h_2\mathcal{L}_2^{-1}\left[R_{2,m}\left(\mathbf{v}_{m-1}\right)\right], \tag{28}
$$

where χ_m^*

$$
\chi_m^* = \begin{cases} 0 & m \leqslant 1,\\ n & otherwise. \end{cases} \tag{29}
$$

Finally using using Equations (27) and (28) with initial conditions given by (26), we respectively obtain the approximate analytical solutions

$$
\begin{aligned}
u_0(x,t) &= 1 + \sqrt{2}\tanh[x],\\
v_0(x,t) &= 2 + 2\sqrt{2}\tanh[x],\\
u_1(x,t) &= \frac{ht^\alpha \text{sech}[x]^2\left(-1 + 3\text{cosh}[2x] + 3\sqrt{2}\text{sinh}[2x]\right)}{2\alpha},\\
v_1(x,t) &= \frac{ht^\alpha \text{sech}[x]^2\left(-1 + 3\text{cosh}[2x] + 3\sqrt{2}\text{sinh}[2x]\right)}{\alpha},\\
u_2(x,t) &= \frac{h^2t^\alpha \text{sech}[x]^3\left(\left(-45t^\alpha + 2\alpha\right)\cosh[x] + \left(33t^\alpha + 6\alpha\right)\cosh[3x]\right)}{8\alpha^2}\\
&+ \frac{2\sqrt{2}\left(12\alpha\text{cosh}[x]^2 + t^\alpha(-5 + 27\text{cosh}[2x])\right)\sinh[x]}{8\alpha^2}\\
&+ \frac{hnt^\alpha \text{sech}[x]^2\left(-1 + 3\text{cosh}[2x] + 3\sqrt{2}\text{sinh}[2x]\right)}{2\alpha},\\
v_2(x,t) &= \frac{h^2t^\alpha \text{Sech}[x]^3\left(2\sqrt{2}\left(12\alpha\text{Cosh}[x]^2 + t^\alpha(-5 + 27\text{Cosh}[2x])\right)\text{Sinh}[x]\right)}{4\alpha^2}\\
&\quad \frac{h^2t^\alpha \text{Sech}[x]^3\left(\left(-45t^\alpha + 2\alpha\right)\text{Cosh}[x] + \left(33t^\alpha + 6\alpha\right)\text{Cosh}[3x] + 2\sqrt{2}\left(12\alpha\text{Cosh}[x]^2\right)\right)}{4\alpha^2}\\
&\quad + \frac{h^2t^{2\alpha}\text{Sinh}[x]\text{Sech}[x]^3(-5 + 27\text{Cosh}[2x])}{4\alpha^2}\\
&\quad + \frac{hnt^\alpha \text{Sech}[x]^2\left(-1 + 3\text{Cosh}[2x] + 3\sqrt{2}\text{Sinh}[2x]\right)}{\alpha},\\
&\ \vdots
\end{aligned}
$$

We can obtain $u_m(x,t)$, $v_m(x,t)$, for $m = 3,4,5,\cdots$, following the same approach, using Mathematica, Maple or MATLAB.

As a result series solution expression by q-HAM can be written in the form

$$u(x,t,n,h) = 1+\sqrt{2}\tanh[x]+\sum_{i=1}^{\infty} u_i(x,t;n;h)\left(\frac{1}{n}\right)^i, \tag{30}$$

$$v(x,t,n,h) = 2+2\sqrt{2}\tanh[x]+\sum_{i=1}^{\infty} v_i(x,t;n;h)\left(\frac{1}{n}\right)^i. \tag{31}$$

6. Graphical Comparisons

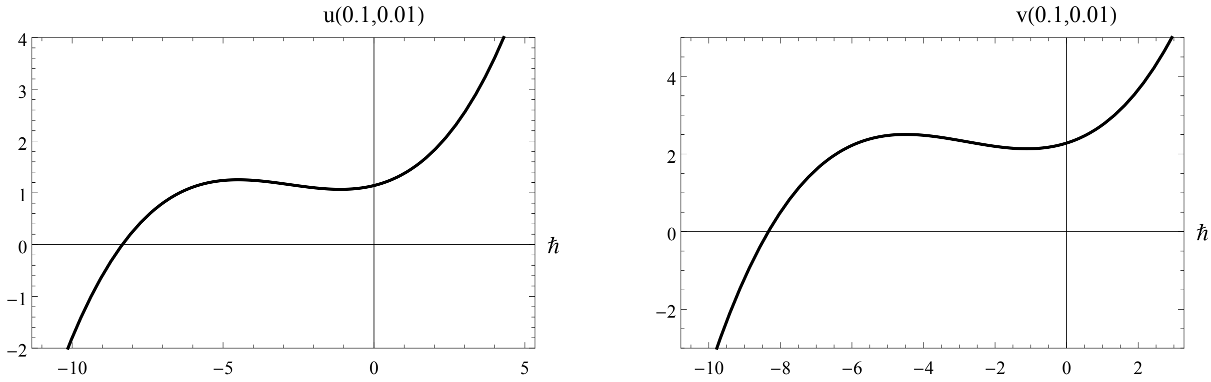

Fig. 1. The h-curves of $u(x,t)$ and $v(x,t)$ for $x = 0.1, t = 0.01, \alpha = 0.7$ respectively.

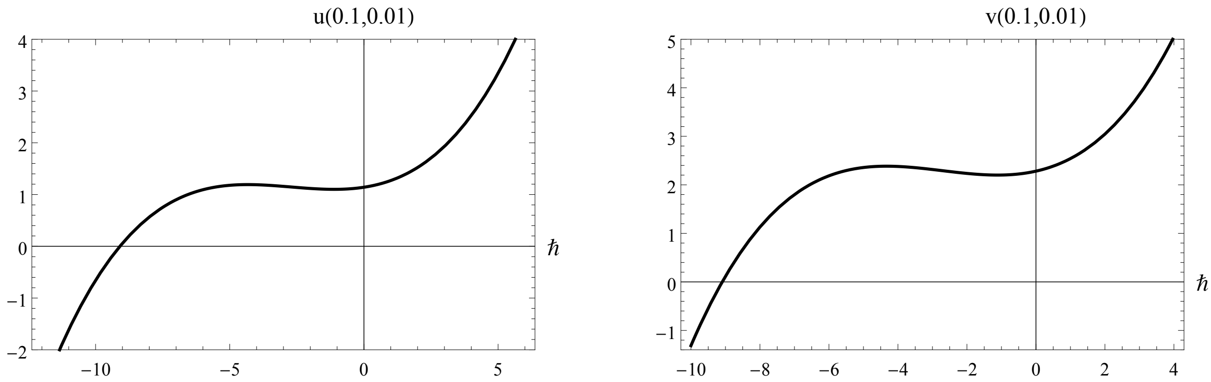

Fig. 2. The h-curves of $u(x,t)$ and $v(x,t)$ for $x = 0.1, t = 0.01, \alpha = 0.8$ respectively.

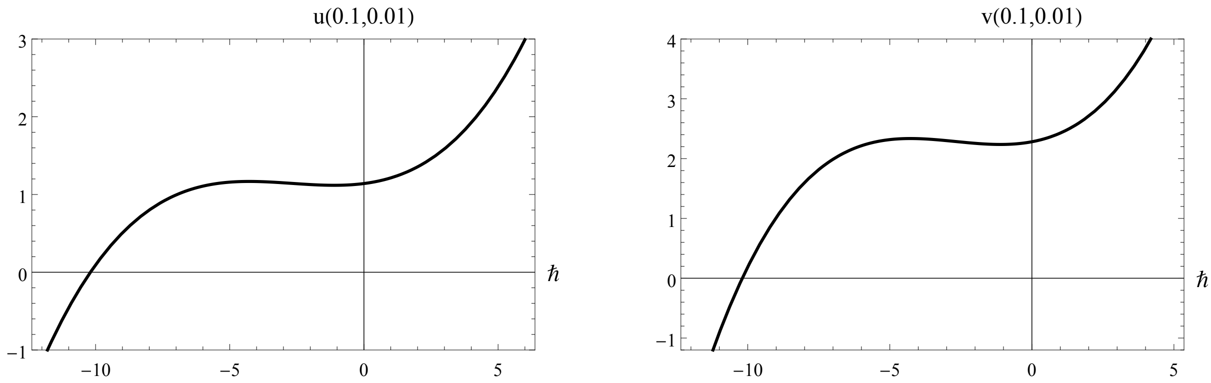

Fig. 3. The h-curves of $u(x,t)$ and $v(x,t)$ for $x = 0.1, t = 0.01, \alpha = 0.9$ respectively.

Figures $1, 2, 3$ show the convergence region of the obtained approximate solutions. By the help of this graphics we can adjust and control the convergence of approximate analytical solution to the exact solution. These graphics helps us for choosing appropriate value of h which is involved in the series solutions (30) and (31). As a consequence of this choice of h the following graphics appears. Both of these graphics shows the obtained numerical solutions are converges properly to exact solutions for different values of α.

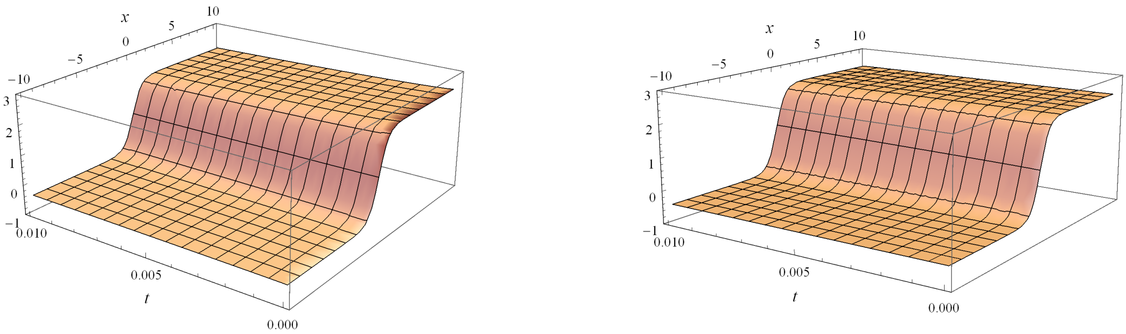

Fig. 4. The graphics of the numerical and exact solutions of $u(x,t)$ for $h=-1, \alpha=0.7$ respectively.

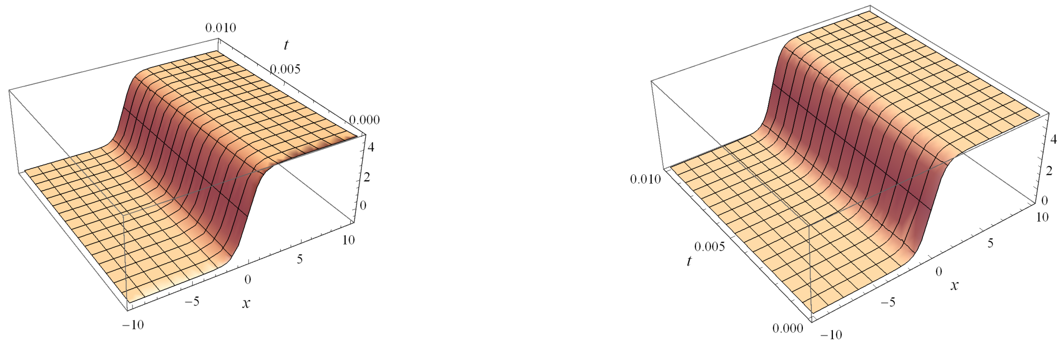

Fig. 5. The graphics of the numerical and exact solutions of $v(x,t)$ for $h=-1, \alpha=0.7$ respectively.

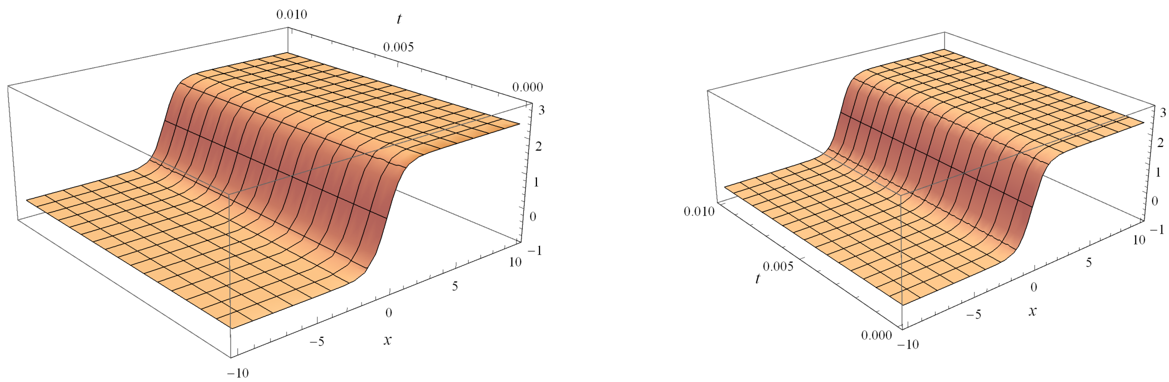

Fig. 6. The graphics of the numerical and exact solutions of $u(x,t)$ for $h=-1, \alpha=0.8$ respectively.

To be more satisfying lets give the numerical comparisons of both exact and approximate analytical solutions over Figures 4,5,6,7 and 8.

7. Conclusion

In this work, we successfully apply the q-homotopy analysis method and Sine-Gordon expansion method to obtain solutions of Fractional Diffusive Predator-Prey system. It may be concluded that the two methods are powerful and efficient techniques for finding exact as well as approximate solutions

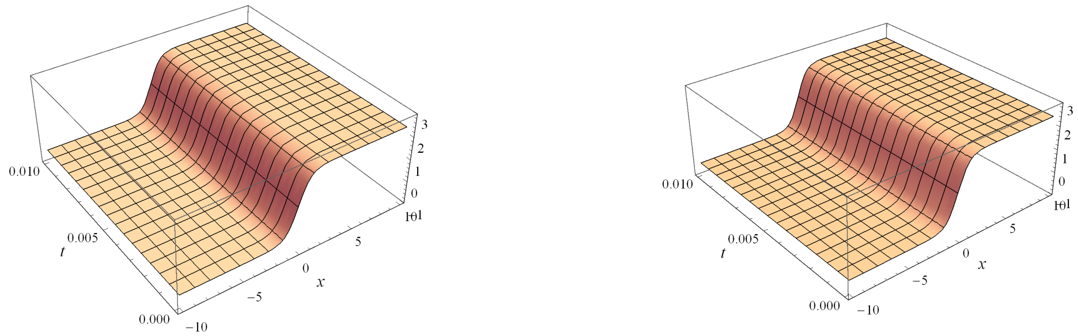

Fig. 7. The graphics of the numerical and exact solutions of $u(x,t)$ for $h=-1, \alpha=0.9$ respectively.

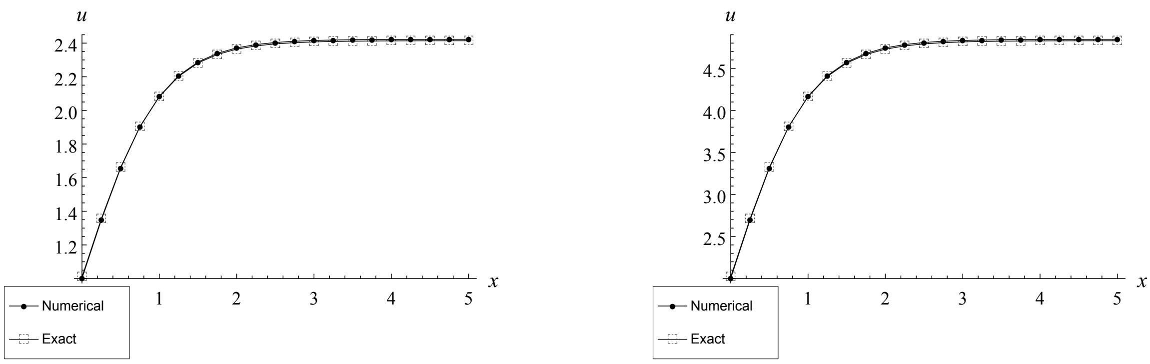

Fig. 8. Comparisons of solutions for $t=0.001, \alpha=0.8, h=-2.7$.

ofhomogeneous fractional partial differential equations. The results reveal that these methods are very effective, convenient and quite accurate to systems of fractional nonlinear equations.

Acknowledgement

All authors contributed equally to this work.

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