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On Strong Pre*-I-Open Sets in Ideal topological Spaces

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Abstract — The aim of the present paper is to introduce the class of *strong* $pre^* - I - open$ *sets* which is strictly placed between the class of all $pre - I - open$ and the class of all $pre^* - I - open$ *subsets* of X. Relationships with some other types of sets were given. Furthermore, by using the new notion, we defined the *strong* $pre^* - I - Interior$ and *strong* $pre^* I - closure$ operators and established their various properties.

Keywords — *Local functions, ideal topological spaces, strong pre*-I-open sets, strong pre*-I-closed sets*

1. Introduction and Preliminaries

Kuratowski [1] defined the concept of ideals in topological spaces. Jankovic and Hamlett [2] introduced the notion of I-open sets in topological spaces. Several kinds of $I - openness$ have been initiated. Abd El-Monsef et al. [3] investigated further properties of $I - open$ sets and $I - continuous$ functions. Dontchev [4] introduced the notion of $pre-I-open$ sets and obtained a decomposition of $I-continuity$. In 2002, Hatir and Noiri [5] presented the concept of $semi - I - open$ sets in ideal topological spaces. Recently, Ekici tntrodused the notions of $pre^* - I - open$ [6]. In this paper, we define the notions of $strong\ pre*-I-open$ sets and $strong\ pre*-I-closed$ sets. Several characteristics and properties are studied. Throughout the present paper, (X, τ) will denote topological spaces on which no separation property is assumed unless explicity stated. In topological space (X, τ), the *closure* and the *interior* of any subset A of X will be denoted by $cl(A)$ and $int(A)$, respectively. An ideal I on X is defined as a nonempty collection of subsets of X satisfying the following two conditions: (1) If $A \in I$ and $B \subset A$, then $B \in I$. (2) If $A \in I$ and $B \in I$, then $A \cup B \in I$. Let (X, τ) be a topological space and I an ideal on X. An ideal topological space is a topological space (X, τ) with an ideal I on X and denoted by (X, τ, I). For a subset $A \subset X$, $A^*(I, \tau) = x \in X | U \cap A \notin I foreachneighborhoodUofx$ is called the local function of A with respect to I and τ [2]. It is obvious that $(.)^* : (X) \to (X)$ is a set operator. Throughout this paper, we use A^* instead of $A^*(I, \tau)$. Besides, in [7], authors introduced a new Kuratowski closure operator $cl^*(.)$ defined by $cl^*(A) = A \cup A^*$ and obtained a new topology on X which is called *-topology. This topology is denoted by τ^* which is finer than τ. We start with recalling some lemmas and definitions which are necessary for this study in the sequel.

Lemma 1.1. [2] Let (X, τ) be a topological space and I an ideal on X. For every subset A of X, the following property holds: $A^* \subset cl(A)$.

Definition 1.2. A subset A of an ideal topological space (X, τ, I) is called:

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1. $pre-open$, if $A \subset int(cl(A))$ [8];
2. $pre-I-open$, if $A \subset int(cl^*(A))$ [4];
3. $pre^*-I-open$, if $A \subset itn^*(cl(A))$ [6];
4. $\alpha-I-open$, if $A \subset int(cl^*(int(A)))$ [5];
5. $semi-I-open$, if $A \subset cl^*(int(A))$ [5];
6. $pre-I-regular$, if A is $pre-I-open$ and $pre-I-closed$ [9];
7. $strong\ semi^*-I-open$, if $A \subset cl^*(int^*(A))$ [10];
8. $\beta^*-I-open$, if $A \subset cl(int^*(cl(A)))$ [6] ;
9. $strong\beta-I-open$, if $A \subset cl^*(int(cl^*(A)))$ [11];
10. $\beta-I-open$, if $A \subset cl(int(cl^*(A)))$ [5] ;
11. $\beta-open$, if $A \subset cl(int(cl(A)))$ [12] ;
12. $weakly\ semi-I-open$, if $A \subset cl^*(int(cl(A)))$ [13];
13. $I-open$, if $A \subset int(A^*)$ [3];
14. $almost\ strong-I-open$, if $A \subset cl^*(int(A^*))$ [14];
15. $*-perfect$, if $A = A^*$ [15];
16. $C^*-I-set$, if $A = L \cap M$, where $L \in \tau$ and M is $pre-I-regular$ [9];
17. $S-I-set$, if $int(A) = cl^*(int(A))$ [13].

Definition 1.3. [16] In ideal topological space (X,τ,I), I is said to be codence if $\tau \cap I = \phi$.

Lemma 1.4. ([17]) Let (X,τ,I) be an ideal space, where I is codence, then the following hold:

1. $cl(A) = cl^*(A)$, for every $*-open$ set A;
2. $int(A) = in^*(A)$, for every $*-closed$ set A.

Lemma 1.5. [18] For a subset A of an ideal topological space (X,τ,I), the following are hold:

1. $pIcl(A) = A \cup cl(int^*(A))$;
2. $pIint(A) = A \cap int(cl^*(A))$;
3. $sIcl(A) = A \cup int^*(cl(A))$;
4. $sIint(A) = A \cap cl^*(int(A))$.

Lemma 1.6. [2] For two subsets, A and B of a space (X,τ,I), the following are hold:

1. If $A \subset B$, then $A^* \subset B^*$;
2. If $U \in \tau$, then $(U \cap A^*) \subset (U \cap A)^*$.

Lemma 1.7. [14] Let A be a subset of an ideal topological space (X,τ,I) and U be an open set. Then, $U \cap cl^*(A) \subset cl^*(U \cap A)$.

Lemma 1.8. [17] Let (X,τ,I) be an ideal space and A be a $*-dense$ in $itself$ subset of X. Then $A^* = cl(A^*) = cl(A) = cl^*(A)$.

Definition 1.9. [7] An ideal topological space (X,τ,I) is said to be I-extremally disconnected if $cl^*(A) \in \tau$ for each $A \in \tau$.

Lemma 1.10. [19] A subset A of an ideal topological space (X,τ,I) is weakly I-local closed if and only if there exists an open set U such that $A = U \cap cl^*A$.

Lemma 1.11. [20] An ideal topological space (X,τ,I) is I-extremally disconnected if and only if $cl^*(int(A)) \subset int(cl^*(A))$, for every subset A of X.

2. Strong Pre*-I-Open Sets

Definition 2.1. A subset A of an ideal topological space (X, τ, I) is said to be *strong* $pre^* - I - open$ (briefly $S.P^* - I - open$) if $A \subset int^*(cl^*(A))$. We denote that all $S.P^* - I - open$ by $S.P^* - I - O(X)$.

Lemma 2.2. Let (X, τ, I) be an ideal topological space, the followings hold, for any subset A of X:

1. Every $pre - I - open$ set is a $S.P^* - I - open$.
2. Every $S.P^* - I - open$ set is a $pre^* - I - open$.

The following diagram holds for any subset A of an ideal topological space (X, τ, I).

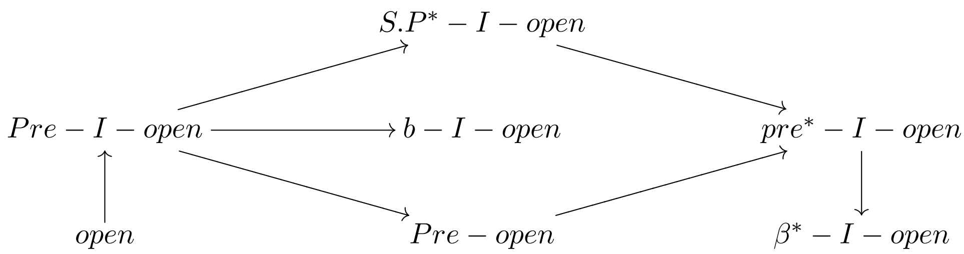

Figure 1. The implication between some generalizations of open sets

Remark 2.3. The converses of these implications in Diagram 1 are not true in general as shown in the following examples:

Example 2.4. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $I = \{\phi, \{a\}, \{d\}, \{a, d\}\}$. Then $A = \{c, d\}$ is a $S.P^* - I - open$, but it is not $pre - I - open$.

Example 2.5. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{c\}, \{a, b, d\}\}$ and $I = \{\phi, \{a\}\}$. Then $A = \{a\}$ is a $pre^* - I - open$ set, but it is not $S.P^* - I - open$.

Remark 2.6. The strong $pre^* - I - open$ sets and $b - I - open$ sets are independent notions, we show that from the next examples:

Example 2.7. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $I = \{\phi, \{a\}, \{d\}, \{a, d\}\}$, if we take $A = \{b, d\}$, then we get A is not $b - I - open$ but it is $S.P^* - I - open$.

Example 2.8. Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ and $I = \{\phi, \{b\}\}$. Then $A = \{a, b\}$ is a $b - I - open$ but it is not $S.P^* - I - open$.

Example 2.9. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{d\}, \{a, c\}, \{a, c, d\}\}$ and $I = \{\phi, \{c\}, \{d\}, \{c, d\}\}$. Then $A = \{c\}$ is $pre - open$ but it is not $S.P^* - I - open$.

Example 2.10. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$. Then $A = \{a, c\}$ is an $S.P^* - I - open$ set, but it is not $pre - open$.

From Examples 2.6 and 2.5, we conclude that the concepts of $pre - open$ sets and $S.P^* - I - open$ sets are independent notions.

Theorem 2.11. Let (X, τ, I) be an ideal topological space then, A is an $S.P^* - I - open$ set if and only if there exists an $S.P^* - I - open$ B such that $A \subset B \subset cl^*(A)$.

PROOF. Let A be a $S.P^* - I - open$, then $A \subset int^*(cl^*(A))$. We put $B = int^*(cl^*(A))$, which is a $* - open$ set. Therefore $B = int^*(B) \subset int^*(cl^*(B))$ be an $S.P^* - I - open$ set Such that $A \subset B = int^*(cl^*(B)) \subset cl^*(A)$.
Conversely, if B is an $S.P^* - I - open$ set such that $A \subset B \subset cl^*(A)$, taking $* - closure$, then $cl^*(A) \subset cl^*(B)$. On the other hand $A \subset B \subset int^*(cl^*(B)) \subset int^*(cl^*(A))$. Which shows that A is $S.P^* - I - open$. □

Corollary 2.12. Let (X, τ, I) be an ideal topological space, then A is a $S.P^* - I - open$ set if and only if there exists an open set $A \subset B \subset cl^*(A)$.

PROOF. Obvious. □

Corollary 2.13. Let (X, τ, I) be an ideal topological space. If A is an $S.P^* - I - open$ set, then $cl^*(A)$ is a $strong\ semi^* - I - open$ set.

PROOF. Let A be $S.P^* - I - open$. Then $A \subset int^*(cl^*(A))$ and $cl^*(A) \subset cl^*(int^*(cl^*(A)))$. This implies $cl^*(A)$ is a $strong\ semi^* - I - open$. □

Corollary 2.14. Let (X, τ, I) be an ideal topological space. If A is a strong $semi^* - I - open$, then $int^*(A)$ is an $S.P^* - I - open$ set.

PROOF. Let A be strong $semi^* - I - open$, then $A \subset cl^*(int^*(A)) \Rightarrow int^*(A) \subset int^*(cl^*(int^*(A)))$. This implies $int^*(A)$ is an $S.P^* - I - open$. □

Theorem 2.15. Let (X, τ, I) be an ideal topological space, A and B are subsets of X. the following are hold:

1. If $U \in SP^*IO(X, \tau)$, for each $\alpha \in \Delta$, then $\bigcup\{U_\alpha : \alpha \in \Delta\} \in SP^*IO(X, \tau)$
2. If $A \in SP^*IO(X, \tau)$, and $B \in \tau$, then $A \cap B \in SP^*IO(X, \tau)$.

PROOF. (1) Since $U_\alpha \in SP^*IO(X, \tau)$, we have $U_\alpha \subset itn^*(cl^*(U_\alpha))$, for each $\alpha \in \Delta$. Then we obtain:

$$\begin{aligned}
\bigcup_{\alpha\in\Delta} U_\alpha &\subset \bigcup_{\alpha\in\Delta} int^*(cl^*(U_\alpha)) \\
&\subset int^*(\bigcup_{\alpha\in\Delta} cl^*(U_\alpha)) \\
&= int^*(\bigcup_{\alpha\in\Delta} (U_\alpha^* \cup U_\alpha)) \\
&= int^*(\bigcup_{\alpha\in\Delta} U_\alpha^* \cup \bigcup_{\alpha\in\Delta} U_\alpha) \\
&\subset int^*((\bigcup_{\alpha\in\Delta} U_\alpha)^* \cup \bigcup_{\alpha\in\Delta} U_\alpha) \\
&= int^*(cl^*(\bigcup_{\alpha\in\Delta} U_\alpha))
\end{aligned}$$

This shows that $\bigcup_{\alpha\in\Delta} U_\alpha \in SP^*IO(X, \tau)$.

(2) Let $A \in SP^*IO(X, \tau)$ and $B \in \tau$. Then $A \subset int^*(cl^*(A))$ and $B = int(B) \subset int^*(B)$. Thus, we obtain

$$\begin{aligned}
A \cap B &\subset int^*(cl^*(A)) \cap int^*(B) \\
&= int^*(cl^*(A) \cap B) \\
&= int^*((A^* \cup A) \cap B) \\
&= int^*((A^* \cap B) \cup (A \cap B)) \\
&\subset int^*((A \cap B)^* \cup (A \cap B)) \\
&= int^*(cl^*(A \cap B))
\end{aligned}$$

□

Remark 2.16. In general, a finite intersection of the $S.P^* - I - open$ sets need not be $S.P^* - I - open$, as shown by the following example:

Example 2.17. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $I = \{\phi, \{a\}, \{d\}, \{a, d\}\}$. We can easily conclude that $A = \{c, d\}$ and $B = \{b, d\}$ are $S.P^* - I - open$ sets, but $A \cap B = \{d\}$ is not $S.P^* - I - open$.

Theorem 2.18. Let (X, τ, I) be an ideal topological space, where I is codense then the following hold:

1. Every $S.P^* - I - open$ set is a $strong\ \beta - I - open$ set.
2. Every $S.P^* - I - open$ set is a $\beta - open$ set.
3. Every $S.P^* - I - open$ set is a $weakly\ semi - I - open$ set.
4. Every $S.P^* - I - open$ set is a $pre - open$ set.

PROOF. It is obvious. □

Remark 2.19. The reverse of the above theorem is not true in general as shown in the following examples:

Example 2.20. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\phi\}$. Then we obtain:

1. $A = \{a, c\}$ is $strong\ \beta - I - open$ set, but it is not $S.P^* - I - open$.
2. $A = \{b, c\}$ is $\beta - open$ set, but it is not $S.P^* - I - open$.
3. $A = \{a, d\}$ is $weakly\ semi - I - open$ set, but it is not $S.P^* - I - open$

Example 2.21. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a, b\}\}$ and $I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$. If we take $A = \{b\}$, then we get A is a $pre - open$ set, but it is not $S.P^* - I - open$.

Theorem 2.22. Let (X, τ, I) be an ideal topological space, such that every $open$ set is $* - closed$, then every $strong\ \beta - I - open$ set is $S.P^* - I - open$.

Proof. Let A is a $strong\ \beta - I - open$, then $A \subset cl^*(int(cl^*(A)))$. Since $int(cl^*(A))$ is $open$, by hypothesis $int(cl^*(A)) = cl^*(int(cl^*(A)))$. So $A \subset cl^*(int(cl^*(A))) = int(cl^*(A) \subset int^*(cl^*(A))$. Which shows that A is $S.P^* - I - open$. □

Theorem 2.23. Let (X, τ, I) be an ideal topological space. If A is $* - perfect$, then the following hold:

1. Every $S.P^* - I - open$ set is $almost\ strong - I - open$.
2. A is a $S.P^* - I - open$ set if and only if it is $I - open\ set$.

Proof. (1) Let A is an $S.P^* - I - open$, then $A \subset int^*(cl^*(A)) = int(cl^*(A)) \subset cl^*(int(cl^*(A))) = cl^*(int(A^*))$. This implies A is $almost\ strong - I - open$.
(2) Let A is a $S.P^* - I - open$, then $A \subset int^*(cl^*(A)) \subset int^*(cl(A)) = int(A^*)$. Hence A is $I - open$. Conversely, if A is $I - open$, then $A \subset int(A^*) \subset int^(*)(A^*) = int^*(cl^*(A))$. Hence A is $S.P^* - I - open$. □

Corollary 2.24. Let (X, τ, I) be an ideal topological space, If A is $* - perfect$, then every $pre^* - I - open$ set is $S.P^* - I - open$.

Proof. Let A is $pre^* - I - open$ set, since it is $* - perfect$, then $A \subset int^*(cl(A)) = int^*(cl^*(A))$. Hence A is $S.P^* - I - open$. □

Corollary 2.25. Every $I - open$ set is $S.P^* - I - open$.

Proof. If A is $I - open$, then $A \subset int(A^*) \subset int(A^* \cup A) \subset int^*(cl^*(A))$. Hence A is $S.P^* - I - open$. □

Theorem 2.26. Let (X, τ, I) be an ideal topological space, Where I is codense, then the following are equivalent:

1. A is $pre^* - I - open$.
2. A is $S.P^* - I - open$.

Proof. It is obvious. □

Theorem 2.27. Let (X, τ, I) be an ideal topological space and $A \subset X$ be a $pre - open$ and $semi - closed$. Then A is $S.P^* - I - open$.

Proof. Let A is $pre - open$, then $A \subset int(cl(A))$. Since A is $semi - closed$ then $int(cl(A)) = int(A)$, now $A \subset int(A) \subset int^*(cl^*(A))$. Which shows A is $S.P^* - I - open$. □

Theorem 2.28. Let (X, τ, I) be an ideal topological space and $A \subset X$ be an $S.P^* - I - open$ and $* - closed$. Then A is $S.S^* - I - open$.

Proof. Let A is $S.P^* - I - open$, then $A \subset int^*(cl^*(A))$. Since A is $* - closed$ then $int^*(cl^*(A)) = int^*(A)$. Now $A \subset int^*(A) \subset cl^*(int^*(A))$. Which shows A is $S.S^* - I - open$. □

Theorem 2.29. Let (X, τ, I) be an ideal topological space, and $A \subset X$, then the followings hold:

1. A is $S.P^* - I - open$ set, if it is both $weakly\ semi - I - open$ and $strong\ S - I - set$.
2. A is $S.P^* - I - open$ set, if it is both $semi - I - open$ set and $S - I - set$.

PROOF. (1) Let A is $weakly\ semi-I-open$ set, then $A \subset cl^*(int(cl(A)))$. Since A is $strong\ S-I-set$ then, $int(A) = cl^*(int(cl(A)))$. Now $A \subset int(A) \subset int^*(cl^*(A))$. Hence A is $S.P^* - I - open$.
(2) Let A is $semi-I-open$ set, then $A \subset cl^*(int(A))$. Since A is $S-I-set$ then, $int(A) = cl^*(int(A))$. Now $A \subset int(A) \subset int^*(cl^*(A))$. Hence A is $S.P^* - I - open$. □

Theorem 2.30. Let (X, τ, I) be an ideal topological space. A is an $S.P^* - I - open$ set if it is both $Pre^* - I - open$ and $closed$.

PROOF. Let A is $pre^* - I - open$ set, then $A \subset int^*(cl(A))$. Since A is closed set, then $A \subset int^*(cl(A)) = int^*(A) \subset int^*(cl^*(A))$. Hence A is $S.P^* - I - open$. □

Theorem 2.31. Let (X, τ, I) be an $I - extremally\ disconnected$ space and $A \subset X$. Then every $semi - I - open$ set is an $S.P^* - I - open$ set.

PROOF. Let A is $semi - I - open$, then $Acl^*(int(A))$. By Lemma 1.11, we obtain $A \subset int(cl^*(A)) \subset int^*(cl^*(A))$. Which shows A is $S.P^* - I - open$. □

Lemma 2.32. An ideal topological space (X, τ, I) is $I - extremally\ disconnected$ if and only if $cl^*(int^*(A)) \subset int^*(cl^*(A))$, for every subset A of X.

PROOF. From Definition 1.9., we obtain $cl^*(A)$ is $open$. Thus $cl^*(int^*(A)) \subset cl^*(A) = int(cl^*(A)) \subset int^*(cl^*(A))$. Hence $cl^*(int^*(A)) \subset int^*(cl^*(A))$. Conversely, since $cl^*(int(A)) \subset cl^*(int^*(A)) \subset int^*(cl^*(A)) \subset int^*(cl(A))$. Then X is $I - extremally\ disconnected$. □

Corollary 2.33. Let (X, τ, I) be an $I - extremally\ disconnected$ space and $A \subset X$. Then every $strong\ semi^* - I - open$ set is $S.P^* - I - open$.

PROOF. It is obvious by Lemma 2.32. □

Theorem 2.34. Let (X, τ, I) be an ideal topological space, A and B are subsets of X. If A is an $S.P^* - I - open$ set and B is a $pre - open$ set, then $A \cup B$ is $pre^* - I - open$.

PROOF. Let A is $S.P^* - I - open$ then $A \subset int^*(cl^*(A))$, and B is a $pre - open$ then $B \subset int(cl(B))$. Now:

$$\begin{aligned} A \cup B &\subset int^*(cl^*(A)) \cup int(cl(B)) \\ &\subset int^*(cl(A)) \cup int^*(cl(B)) \\ &\subset int^*(cl(A \cup B)). \end{aligned}$$

Hence $A \cup B$ is a $pre^* - I - open$ set. □

Theorem 2.35. Let (X, τ, I) be an ideal topological space, A and B are subsets of X. If A is an $S.P^* - I - open$ set and B is a $weakly\ semi - I - open$ set, then $A \cup B$ is $\beta^* - I - open$.

PROOF. Let A is $S.P^* - I - open$, then $A \subset int^*(cl^*(A))$, B is $weakly\ semi - I - open$ then $B \subset cl^*(int(cl(B)))$ Now :

$$\begin{aligned} A \cup B &\subset int^*(cl^*(A)) \cup cl^*(int(cl(B))) \\ &\subset cl(int^*(cl(A))) \cup cl(int^*(cl(B))) \\ &= cl(int^*(cl(A)) \cup int^*(cl(B))) \\ &\subset cl(int^*(cl(A \cup B))). \end{aligned}$$

Hence $A \cup B$ is a $\beta^* - I - open$ set. □

Theorem 2.36. Let (X, τ, I) be an ideal topological space, where I is codense then A is $\alpha - I - open$ if and only if it is an $S.S^* - I - open$ and $S.P^* - I - open$.

Proof. Necessity, this is obvious.
Sufficiency, Let A is an $S.S^* - I - open$ and $S.P^* - I - open$, we have:

$$\begin{aligned} A &\subset int^*(cl^*(A)) \\ &\subset int^*(cl^*(cl^*(int^*(A)))) \\ &= int^*(cl^*(int^*(A))) \\ &= int(cl^*(int(A))). \end{aligned}$$

Hence A is $\alpha - I - open$. □

3. Strong Pre*-I-Closed Sets

Definition 3.1. A subset A of an ideal topological space (X, τ, I) is said to be *strong* $pre^* - I - closed$ (briefly $S.P^* - I - closed$) if its complement is $S.P^* - I - open$. We denote that all $S.P^* - I - closed$ by $S.P^* - I - C(X)$.

Lemma 3.2. Let (X, τ, I) be an ideal topological space, the followings hold, for any subset A of X:

1. Every $pre - I - closed$ set is a $S.P^* - I - closed$.
2. Every $S.P^* - I - closed$ set is a $pre^* - I - closed$.

The following diagram holds for any subset A of an ideal topological space (X, τ, I).

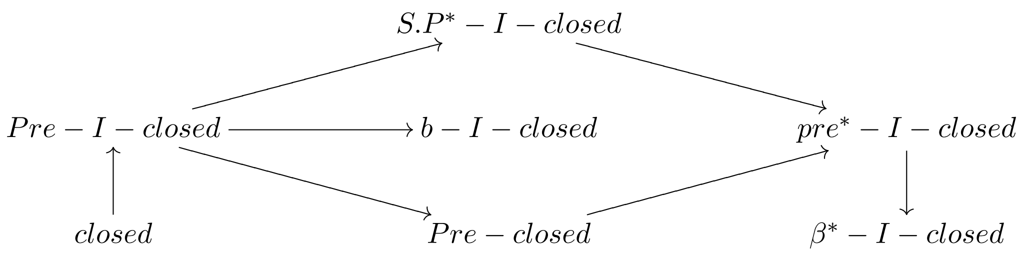

Figure 2. The implication between some generalizations of closed sets

Theorem 3.3. A subset A of a space (X, τ, I) is said to be an $S.P^* - I - closed$ if and only if$cl^*(int^*(A)) \subset A$.

Proof. Let A be an $S.P^* - I - closed$ of (X, τ, I), then $(X - A)$ is an $S.P^* - I - open$ and hence $(X - A) \subset int^*(cl^*(X - A)) = X - cl^*(int^*(A))$. Therefore, we obtain $cl^*(int^*(A)) \subset A$.
Conversely, let $cl^*(int^*(A)) \subset A$, then $(X-A) \subset int^*(cl^*(X - A))$ and hence $(X-A)$ is $S.P^*-I-open$. Therefore, A is an $S.P^* - I - closed$. □

Theorem 3.4. Let (X, τ, I) be an ideal topological space, if I is codense, then A is an $S.P^*-I-closed$ if and only if $cl^*(int(A)) \subset A$.

Proof. Let A be a $S.P^* - I - closed$ set of X, then $A \supset cl^*(int^*(A)) = cl^*(int(A))$.
Conversely, let A be any subset of X, such that $A \supset cl^*(int(A))$. This implies that $A \supset cl^*(int^*(A))$, i.e., A is an $S.P^* - I - closed$ □

Theorem 3.5. Let (X, τ, I) be an ideal topological space, and $A \subset X$, then the followings hold:

1. If A is an $S.P^* - I - open$ set, then $SIcl(A) = int^*(cl(A))$.
2. If A is an $S.P^* - I - closed$ set, then $SIint(A) = cl^*(int(A))$.

PROOF. (1) Let A be an $S.P^* - I - open$ set in X. Then we have $A \subset int^*(cl^*(A)) \subset int^*(cl(A))$. Thus we have $SIcl(A) = int^*(cl(A))$.
(2) Let A be an $S.P^* - I - closed$ set in X ,then we have $A \supset cl^*(int^*(A)) \supset cl^*(int(A))$. Hence $SIint(A) = cl^*(int(A))$. □

Remark 3.6. 1The reverse of the above theorem is not true in general as shown in the following examples:

Example 3.7. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}\}$, $I = \{\phi, \{a\}, \{d\}, \{a, d\}\}$, and $A = \{c\}$. Then we obtain:

1. $SIcl(A) = int^*(cl(A))$, but A is not $S.P^* - I - open$.
2. $SIint(A) = cl^*(int(A))$, but it is not $S.P^* - I - closed$

Theorem 3.8. A subset A of a space (X, τ, I) is said to be $S.P^* - I - closed$ if and only if there exists an $S.P^* - I - closed$ set B such that $int^*(A) \subset B \subset A$.

PROOF. Let A be an $S.P^* - I - closed$ set of a space (X, τ, I), then $cl^*(int^*(A)) \subset A$. We put $B = cl^*(int^*(A))$ be a $* - closed\ set$. i.e, B is $S.P^* - I - closed$. And $int^*(A) \subset cl^*(int^*(A)) = B \subset A$. Conversely, if B is an $S.P^* - I - closed$ set such that $int^*(A) \subset B \subset A$, then $int^*(A) = int^*(B)$. On the other hand, $cl^*(int^*(B) \subset B$ and hence $A \supset B \supset cl^*(int^*(B)) = cl^*(int^*(A)$. Thus $A \supset cl^*(int^*(A))$. Hence A is $S.P^* - I - closed$. □

Corollary 3.9. a subset A of a space (X, τ, I) is an $S.P^* - I - closed$ set if and only if there exists a $* - closed$ set B such that $int^*(A) \subset B \subset A$.

Remark 3.10. The union of strong $pre^* - I - closed$ sets need not be an $S.P^* - I - closed$ set. This can be shown by the following example:

Example 3.11. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, X\}$ and $I = \{\phi, \{a\}, \{d\}, \{a, d\}\}$, then $A = \{b\}$ and $B = \{c\}$ are $S.P^* - I - closed$ sets but $A \cup B = \{b, c\}$ is not $S.P^* - I - closed$.

Theorem 3.12. Let (X, τ, I) be an ideal topological space, A and B are subsets of X. Then $A \cap B$ is a $pre^* - I - closed$ set, if A is $S.P^* - I - closed$ and B is $pre - closed$ set.

PROOF. It is proved similarly by Theorem 2.34. □

Theorem 3.13. Let (X, τ, I) be an ideal topological space, A and B are subsets of X. Then $A \cap B$ is a $B^* - I - closed$ set, if A is $S.P^* - I - closed$ and B is $weakly\ semi - I - closed$.

PROOF. It is proved similarly by Theorem 2.35. □

Theorem 3.14. Let (X, τ, I) be an ideal topological space, then each $pre - I - regular$ set in X is $S.P^* - I - open$ and $S.P^* - I - closed$ set.

PROOF. It follows from the fact that every $pre - I - regular$ set is $pre - I - open$ and $pre - I - closed$. This implies that it is $S.P^* - I - open$ and $S.P^* - I - closed$. □

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