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# Some connections between various classes of analytic functions associated with the power series distribution

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**Abstract**

The primary motivation of the paper is to investigate the power series distribution (Pascal model) for the analytic function classes $T\mathcal{G}(\alpha)$, $T\mathcal{G}\mathcal{S}^*(\alpha,\rho)$ and $T\mathcal{G}\mathcal{C}(\alpha,\rho)$. Furthermore, we give necessary and sufficient conditions for the Pascal distribution series belonging to these classes.

**Keywords:** Analytic functions, power series distribution

## 1. INTRODUCTION

The power series distribution is very useful in multivariate data research fields. This family of distributions, particularly is used in survival and reliability studies. However, nowadays, the elementary distributions such as the Poisson, the Pascal, the Logarithmic, the Binomial, the Burr-Weibull have been partially studied in the Geometric Function Theory from a theoretical point of view (see [1], [2], [3], [4]). In this paper, we focus on the Pascal power series distribution.

Let us consider a non-negative discrete random variable $\mathcal{X}$ with a Pascal probability generating function

$$P(\mathcal{X}=j)=\binom{j+\mathrm{t}-1}{\mathrm{t}-1}p^j(1-p)^{\mathrm{t}},\quad j\in\{0,1,2,3,\ldots\},$$

where $p$ $(0\le p\le 1)$, t are called the parameters.

Let $\mathcal{A}$ represent the class of functions $f$ of the form

$$f(z)=z+\sum_{j=2}^{\infty}a_j z^j, \tag{1}$$

which are analytic in the open unit disk $\mathfrak{U}=\{z: z\in\mathbb{C} \text{ and } |z|<1\}$. Let $\mathcal{S}$ be the subclass of $\mathcal{A}$ consisting of functions which are univalent in $\mathfrak{U}$ and $T$ be the subclass of $\mathcal{S}$ consisting of functions whose coefficients, from the second on, are non-negative given by (see [5])

$$f(z)=z-\sum_{j=2}^{\infty}|a_j|z^j. \tag{2}$$

Furthermore, by $\mathcal{G}(\alpha)$, $\mathcal{G}\mathcal{S}^*(\alpha,\rho)$ we shall denote the class of all functions $f\in\mathcal{A}$ which satisfy the following conditions

$$\mathrm{Re}\left\{\left(1+e^{i\theta}\right)\frac{zf'(z)}{f(z)}-e^{i\theta}\right\}\ge\alpha\left(0\le\alpha<1,\theta\in\mathbb{R}\right), \tag{3}$$

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$$\mathrm{Re}\left\{\left(1+\rho e^{i\theta}\right)\frac{zf'(z)}{f(z)}-\rho e^{i\theta}\right\}\geq\alpha \qquad (4)$$
$$\left(0\leq\alpha<1,\rho\geq0,\theta\in\mathbb{R}\right),$$

respectively.

Let also $\mathcal{GC}(\alpha,\rho)$ denote the class of all functions $f\in\mathcal{A}$ which satisfy the following condition

$$\mathrm{Re}\left\{\left(1+\rho e^{i\theta}\right)\left(1+\frac{zf''(z)}{f'(z)}\right)-\rho e^{i\theta}\right\}\geq\alpha \qquad (5)$$
$$\left(0\leq\alpha<1,\rho\geq0,\theta\in\mathbb{R}\right).$$

We next must write

$$T\mathcal{G}(\alpha)=\mathcal{G}(\alpha)\cap T,$$

$$T\mathcal{GS}^*(\alpha,\rho)=\mathcal{GS}^*(\alpha,\rho)\cap T$$

and

$$T\mathcal{GC}(\alpha,\rho)=\mathcal{GC}(\alpha,\rho)\cap T.$$

These classes introduced and studied by Ronning [6].

The primary motivation of the paper is to investigate the Pascal power series distribution for the analytic function classes $T\mathcal{G}(\alpha)$, $T\mathcal{GS}^*(\alpha,\rho)$ and $T\mathcal{GC}(\alpha,\rho)$.

## 2. PASCAL POWER SERIES DISTRIBUTION

We start by stating the Pascal power series and the basis lemmas for our further investigations.

Based upon the Pascal distribution, consider the following power series:

$$P(\mathrm{t},p,z)=z+\sum_{j=2}^{\infty}\binom{j+\mathrm{t}-2}{\mathrm{t}-1}p^{j-1}(1-p)^{\mathrm{t}}z^j \qquad (6)$$
$$\left(\mathrm{t}\geq0,0\leq p\leq1,z\in\mathfrak{U}\right).$$

Note that, by using ratio test we conclude that the radius of convergence of the above power series is infinity.

**Lemma 1.** A function $f\in\mathcal{A}$ given by (1) is in the class $\mathcal{G}(\alpha)$ if it satisfies the following condition

$$\sum_{j=1}^{\infty}(2j-1-\alpha)\left|a_j\right|\leq2(1-\alpha). \qquad (7)$$

**Lemma 2.** A function $f\in\mathcal{A}$ given by (1) is in the class $\mathcal{GS}^*(\alpha,\rho)$ if it satisfies the following condition

$$\sum_{j=1}^{\infty}\left[\rho(j-1)+j-\alpha\right]\left|a_j\right|\leq2(1-\alpha). \qquad (8)$$

**Lemma 3.** A function $f\in\mathcal{A}$ given by (1) is in the class $\mathcal{GC}(\alpha,\rho)$ if it satisfies the following condition

$$\sum_{j=1}^{\infty}j\left[\rho(j-1)+j-\alpha\right]\left|a_j\right|\leq2(1-\alpha). \qquad (9)$$

**Lemma 4.** A function $f\in T$ given by (2) is in the class $T\mathcal{G}(\alpha)$ if and only if it satisfies the following condition (7).

**Lemma 5.** A function $f\in T$ given by (2) is in the class $T\mathcal{GS}^*(\alpha,\rho)$ if and only if it satisfies the following condition (8).

**Lemma 6.** A function $f\in T$ given by (2) is in the class $T\mathcal{GC}(\alpha,\rho)$ if and only if it satisfies the following condition (9).

## 3. APPLICATION

By considering above definitions and lemmas, we have the following necessary and sufficient conditions for the function $P$.

**Theorem 1.** For $p\neq1$, the function $P$ given by (6) is in the class $\mathcal{G}T\mathcal{S}^*(\alpha,\rho)$ if and only if

$$\frac{(\rho+1)\mathrm{t}p}{(1-p)^{\mathrm{t}+1}}\leq1-\alpha. \qquad (10)$$

**Proof.** According to Lemma 2, we must show that

$$\sum_{j=2}^{\infty}\binom{j+t-2}{t-1}[\rho(j-1)+j-\alpha]p^{j-1}(1-p)^{t} \leq 1-\alpha.$$

Therefore, by combining the relation (6) and implication (10), we have the equality

$$\sum_{j=2}^{\infty}\binom{j+t-2}{t-1}[\rho(j-1)+j-\alpha]p^{j-1}(1-p)^{t}$$

$$=(\rho+1)tp(1-p)^{t}\sum_{j=2}^{\infty}\binom{j+t-2}{t}p^{j-2}$$

$$+(1-\alpha)(1-p)^{t}\sum_{j=2}^{\infty}\binom{j+t-2}{t-1}p^{j-1}$$

$$=(\rho+1)tp(1-p)^{t}\sum_{j=0}^{\infty}\binom{j+t}{t}p^{j\,t}$$

$$+(1-\alpha)(1-p)^{t}\sum_{j=2}^{\infty}\binom{j+t-1}{t-1}p^{j}-(1-\alpha)(1-p)$$

$$=\frac{(\rho+1)tp}{1-p}+(1-\alpha)-(1-\alpha)(1-p)^{t} \leq 1-\alpha.$$

Thus the proof of Theorem 1 is now completed.

**Corollary 1.** For $p \neq 1$, the function $P$ given by (6) is in the class $T\mathcal{G}(\alpha)$ if and only if

$$\frac{2tp}{(1-p)^{t+1}} \leq 1-\alpha.$$

In what follows, we shall give the results for the class $T\mathcal{GC}(\alpha,\rho)$.

**Theorem 2.** For $p \neq 1$, the function $P$ given by (6) is in the class $T\mathcal{GC}(\alpha,\rho)$ if and only if

$$\frac{(\rho+1)t(t+1)p^{2}}{(1-p)^{t+2}}+\frac{(3+2\rho-\alpha)tp(1-p)^{t}}{(1-p)^{t+1}} \leq 1-\alpha. \quad (11)$$

**Proof.** According to Lemma 3, we must show that

$$\sum_{j=2}^{\infty}\binom{j+t-2}{t-1}j[1-\alpha+(\rho+1)(j-1)]p^{j-1}(1-p)^{t} \leq 1-\alpha.$$

Therefore, by combining the relation (6) and implication (12), we have the equality

$$\sum_{j=2}^{\infty}\binom{j+t-2}{t-1}j[1-\alpha+(\rho+1)(j-1)]p^{j-1}(1-p)^{t}$$

$$=(\rho+1)t(t+1)p^{2}(1-p)^{t}\sum_{j=3}^{\infty}\binom{j+t-2}{t+1}p^{j-3}$$

$$+(3+2\rho-\alpha)tp(1-p)^{t}\sum_{j=2}^{\infty}\binom{j+t-2}{t}p^{j-2}$$

$$+(1-\alpha)(1-p)^{t}\sum_{j=2}^{\infty}\binom{j+t-2}{t-1}p^{j-1}$$

$$=(\rho+1)t(t+1)p^{2}(1-p)^{t}\sum_{j=0}^{\infty}\binom{j+t+1}{t+1}p^{j}$$

$$+(3+2\rho-\alpha)tp(1-p)^{t}\sum_{j=0}^{\infty}\binom{j+t}{t}p^{j}$$

$$+(1-\alpha)(1-p)^{t}\sum_{j=0}^{\infty}\binom{j+t-1}{t}p^{j}-(1-\alpha)(1-p)^{t}$$

$$=\frac{(\rho+1)t(t+1)p^{2}}{(1-p)^{2}}+\frac{(3+2\rho-\alpha)tp(1-p)^{t}}{1-p}$$

$$+(1-\alpha)-(1-\alpha)(1-p)^{t} \leq 1-\alpha.$$

Thus, according to Lemma 3, we conclude that $f \in T\mathcal{GC}(\alpha,\rho)$.

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