Commun.Fac.Sci.Univ.Ank.Series A1
Volume 56, Number 2, Pages 1–9 (2007)
ISSN 1303–5991

SLIGHTLY δ-PRECONTINUOUS FUNCTIONS

AYSE NAZLI URESIN, AYNUR KESKIN, TAKASHI NOIRI*

ABSTRACT. In this paper, a new weak form of slight precontinuity, called slight δ-precontinuity, is given and studied. Also, it is shown that slight δ-precontinuity is weaker than both almost δ-precontinuity and (δ-pre, s)-continuity.

1. Introduction

Recently, C.W.Baker [1] has introduced the notion of slight precontinuity and has shown that slight precontinuity is weaker than slight continuity and precontinuity.Quite recently, Erdal Ekici [4] has introduced the notion of almost δ-precontinuity and (δ-pre, s)-continuity by using δ-preopen sets. The aim of this paper is to introduce a new weak form of slight precontinuity which we shall call slight δ-precontinuity. Also, basic properties and preservation theorems of slightly δ-precontinuous functions are investigated.

2. Preliminaries

In this paper, (X,τ) and (Y,σ) (or X and Y) denote topological spaces. Let A be a subset of X. We denote the interior and the closure of a set A by $Int\,(A)$ and $Cl\,(A)$, respectively. A subset A is called δ-closed [14] if $A=\delta Cl\,(A)$, where $\delta Cl\,(A)=\{x\in X : A\cap Int(Cl(U))\neq\emptyset, U\in\tau,\ x\in U\}$. The complement of a δ-closed set is called δ-open [14]. A subset A is said to be preopen [7] (resp. δ-preopen [10], regular open [13], regular closed) if $A\subset Int(Cl\,(A))$ (resp. $A\subset Int(\delta Cl\,(A))$, $A=Int(Cl\,(A))$, $A=Cl\,(Int\,(A))$). The complement of a preopen set is said to be preclosed [7].

The complement of a δ-preopen set is said to be δ-preclosed [10]. The intersection of all δ-preclosed sets of X containing A is called the δ-preclosure of A and is denoted by $\delta pCl\,(A)$[10]. The union of all δ-preopen sets of X contained in A is called δ-preinterior [10]of A and is denoted by $\delta pInt\,(A)$.

Received by the editors May 4, 2007; Accepted: Sept. 12, 2007.
2000 *Mathematics Subject Classification.* 54C05, 54C08, 54C10.
Key words and phrases. preopen sets, δ-preopen sets, slight precontinuity, (δ-pre, s)-continuity.

©2007 Ankara University

The family of all preopen (resp. δ-preopen, regular open) sets of X is denoted by $PO(X)$ (resp. $\delta PO(X)$, $RO(X)$).The family of all preopen (resp. δ-preopen, regular open) sets of X containing $x \in X$ is denoted by $PO(X,x)$ (resp. $\delta PO(X,x)$, $RO(X,x)$).

A subset A of X is said to be clopen if it is both open and closed. The family of all clopen sets of X is denoted by $CO(X)$. The family of all clopen sets of X containing $x \in X$ is denoted by $CO(X,x)$. The clopen (resp. regular open) subsets of (X,τ) may be used as a base for a topology on X. The topology is called the ultra-regularization [9] (resp. semiregularization [13]) of τ and is denoted by τ_U (resp. τ_s).In case when $\tau = \tau_s$, the space (X,τ) is called semi-regular.

Definition 2.1. A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is said to be slightly precontinuous [1] if $f^{-1}(V)$ is preopen in X for every clopen set V of Y.

Definition 2.2. A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is said to be almost δ-precontinuous [4] if $f^{-1}(V)$ is δ-preopen in X for every regular open set V of Y.

Definition 2.3. A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is called (δ-pre, s)-continuous [4] if $f^{-1}(V)$ is δ-preopen in X for every regular closed set V of Y.

Definition 2.4. A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is called almost precontinuous [8] if $f^{-1}(V)$ is preopen in X for every regular open set V of Y.

Definition 2.5. A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is called slightly continuous [6] if $f^{-1}(V)$ is open in X for every clopen set V of Y.

Definition 2.6. A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is called almost contra-precontinuous [3] if $f^{-1}(V)$ is preclosed in X for every regular open set V of Y.

3. Slightly δ-precontinuous functions

Definition 3.1. A function $f : (X,\tau) \longrightarrow (Y,\sigma)$ is said to be slightly δ-precontinuous if for each point $x \in X$ and each $V \in CO(Y)$ containing $f(x)$, there exists a δ-*preopen* set U in X containing x such that $f(U) \subset V$.

Theorem 3.2. *The following statements are equivalent for a function $f : (X,\tau) \longrightarrow (Y,\sigma)$:*

(a) f is slightly δ-precontinuous,
(b) For every clopen set $V \subset Y$, $f^{-1}(V)$ is δ-preopen,
(c) For every clopen set $V \subset Y$, $f^{-1}(V)$ is δ-preclosed,
(d) For every clopen set $V \subset Y$, $f^{-1}(V)$ is δ-preclopen,
(e) For every $A \subset X$, $f(\delta pCl(A)) \subset Cl_{\sigma_U}(f(A))$.

The following diagram holds:

$$\begin{array}{ccc} \text{almost } \delta\text{-precontinuous} & \Longleftarrow & \text{almost precontinuous} \\ \Downarrow & & \Downarrow \\ \text{slightly } \delta\text{-precontinuous} & \Longleftarrow & \text{slight precontinuous} \\ \Uparrow & & \Uparrow \\ (\delta\text{-}pre,\ s)\text{-continuous} & \Longleftarrow & \text{almost contra-precontinuous} \end{array}$$

Remark 3.3. None of these implicaitons is reversible.

Example 3.4. Let X=Y={a,b,c}, τ={X,$\emptyset$,{c},{b,c}} and σ ={X,$\emptyset$,{a,b},{c}}. Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be the identity mapping. The set {a,b} is clopen in (X,σ) and f^{-1}({a,b}) ={a,b}. Since {a,b} is not preopen in (X,τ), f is not slightly precontinuous, but f is almost δ-precontinuous.

Example 3.5. Let τ_c be the cofinite topology and σ be the usual topology on $\mathbb{R}$. Let $f : (\mathbb{R},\tau_c) \longrightarrow (\mathbb{R},\sigma)$ be the identity mapping. Since the only clopen subsets of $(\mathbb{R},\sigma)$ are $\mathbb{R}$ and $\emptyset$, f is slightly precontinuous. However, the open interval $]a,b[$ is regular open in $(\mathbb{R},\sigma)$, and $f^{-1}(]a,b[) =]a,b[$ is not δ-*preopen* in $(\mathbb{R},\tau_c)$. Therefore, f is not almost δ-*pre*continuous.

Example 3.6. Let X ={a,b,c,d}, σ ={X,$\emptyset$,{a},{b,c},{a,b,c}} and τ ={X,$\emptyset$,{a},{c},{a,c}}. Then the identity $f : (X,\tau) \longrightarrow (X,\sigma)$ is slightly precontinuous, but not (δ-*pre*, s)-continuous.

Remark 3.7. The function in Example 5 of [4] is (δ-*pre*, s)-continuous, but it is not almost contra-precontinuous.

Recall that a space X is said to be:
(a) *extremally disconnected* if the closure of each open set of X is open in X,
(b) *0-dimensional* if its topology has a base consisting of clopen sets.

Theorem 3.8. *If $f : X \longrightarrow Y$ is slightly δ-precontinuous and Y is extremally disconnected, then f is (δ-pre, s)-continuous and almost δ-precontinuous.*

Proof. Let V be a regular closed (resp. regular open) in Y. Since Y is extremally disconnected, V is open (resp. closed) in Y. Hence, V is clopen in Y. Then $f^{-1}(V)$ is δ-preopen in X. Therefore, f is (δ-*pre*, s)-continuous (resp. almost δ-precontinuous). □

Theorem 3.9. *If $f : X \longrightarrow Y$ is slightly δ-precontinuous and Y is 0-dimensional, then f is almost δ-precontinuous.*

Proof. Let $x \in X$ and $V \in RO(Y,\ f(x))$. Since Y is 0-dimensional, there exists $G \in CO(Y,\ f(x))$ such that $f(x) \in G \subset V$. Since f is slightly δ- precontinuous, there exists a δ-preopen subset U in X containing x such that $f(U) \subset G \subset V$. Therefore, f is almost δ-precontinuous. □

Theorem 3.10. *If $f : X \longrightarrow Y$ is slightly δ-precontinuous and X is semi-regular, then f is slightly precontinuous.*

Proof. Since X is semi-regular, δ-closure and closure of a set coincide. Therefore, f is slightly precontinuous. □

The composition of two slightly δ-precontinuous functions need not be slightly δ-precontinuous.

Example 3.11. Let X ={a,b,c}, τ ={X,∅,{c},{b,c}}, σ ={X,∅,{a,b},{c}}, τ_t ={X,∅}, and let $f : (X, \tau) \longrightarrow (X, \tau_t)$, $g : (X, \tau_t) \longrightarrow (X, \sigma)$ be the identity function. Then f and g are slightly δ-precontinuous , but $g \circ f$ is not slightly δ-precontinuous.

Definition 3.12. A function $f : X \longrightarrow Y$ is called

(a) δ^*- almost continuous [11] if for every δ-*preopen* subset V in Y, $f^{-1}(V)$ is δ-*preopen* in X;

(b) δ-*preopen* [4] if for every δ-*preopen* subset W in X, $f\ (W)$ is δ-*preopen* in Y.

Theorem 3.13. *Let $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be functions, then the following properties hold:*

(a) If f is slightly δ-precontinuous and g is slightly continuous, then $g \circ f : X \longrightarrow Z$ is slightly δ-precontinuous.

(b) If f is δ^*- almost continuous and g is slightly δ-precontinuous, then $g \circ f : X \longrightarrow Z$ is slightly δ-precontinuous.

(c) If f is δ^*- almost continuous and g is slightly continuous, then $g \circ f : X \longrightarrow Z$ is slightly δ-precontinuous.

Proof. (a) Let $W \in CO(Z)$. Since g is slightly continuous, $g^{-1}(W)$ is clopen in Y. Since f is slightly δ-precontinuous, $f^{-1}\left(g^{-1}(W)\right) = (g \circ f)^{-1}(W)$ is δ-preopen in X. Therefore, $g \circ f$ is slightly δ-precontinuous.

(b) Let $W \in CO(Z)$. Since g is slightly δ-precontinuous, $g^{-1}(W)$ is δ-preopen in Y. Since f is δ^*-almost continuous, $f^{-1}\left(g^{-1}(W)\right) = (g \circ f)^{-1}(W)$ is δ-preopen in X.

(c) can be obtained similarly. □

Theorem 3.14. *Let $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be functions.*

(a) If f is a δ-preopen surjection and $g \circ f : X \longrightarrow Z$ is slightly δ-precontinuous, then g is slightly δ-precontinuous.

(b) Let f be a δ-preopen and δ^*-almost continuous surjection. Then g is slightly δ-precontinuous if and only if $g \circ f : X \longrightarrow Z$ is slightly δ-precontinuous.

Proof. (a) Let $G \in CO(Z)$. Since $g \circ f$ is slightly δ-precontinuous, $f^{-1}\left(g^{-1}(G)\right)$ is δ-preopen in X. Since f is δ-preopen and surjective, $f\left(f^{-1}\left(g^{-1}(Z)\right)\right) = g^{-1}(Z)$ is δ-preopen in Y. Therefore, g is slightly δ-precontinuous.

(b) ($\Rightarrow$): Let g be slightly δ-precontinuous, then by Theorem 5 (b) $g \circ f$ is slightly δ-precontinuous.

($\Leftarrow$): Let $g \circ f$ be slightly δ-precontinuous. Then by (a) g is slightly δ-precontinuous. □

Theorem 3.15. *Let $f : X \longrightarrow Y$ be a function and $g : X \longrightarrow X \times Y$ be the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is slightly δ-precontinuous, then f is slightly δ-precontinuous.*

Proof. Let $V \in CO(Y)$, then $X \times V \in CO(X \times Y)$. Since g is slightly δ-precontinuous, then $f^{-1}(V) = g^{-1}(X \times V) \in \delta PO(X)$. Thus, f is slightly δ-precontinuous. □

Lemma 3.16. *Let A and X_0 be subsets of a space (X, τ). If $A \in \delta PO(X)$ and $X_0 \in \delta O(X)$, then $A \cap X_0 \in \delta PO(X_0)$* [10].

Theorem 3.17. *If $f : X \longrightarrow Y$ is a slightly δ-precontinuous function and $A \in \delta O(X)$, then the restriction $f|_A : A \longrightarrow Y$ is slightly δ-precontinuous.*

Proof. Let $V \in CO(Y)$.Then $f|_A^{-1}(V) = f^{-1}(V) \cap A$. Since $f^{-1}(V)$ is δ-preopen in X and $A \in \delta O(X)$, it follows from Lemma 3.16 that $f|_A^{-1}(V)$ is δ-preopen in the subspace A. □

Lemma 3.18. *Let (X, τ) be a topological space and $A \subset X_0 \subset X$. If $X_0 \in \delta O(X)$ and $A \in \delta PO(X_0)$, then $A \in \delta PO(X)$* [10].

Theorem 3.19. *Let $\{U_\alpha : \alpha \in I\}$ be a δ-open cover of a topological space X. If the restriction $f|_{U_\alpha} : U_\alpha \longrightarrow Y$ is slightly δ-precontinuous for each $\alpha \in I$, then $f : X \longrightarrow Y$ is a slightly δ-precontinuous function.*

Proof. Let $V \in CO(Y)$. Since $f|_{U_\alpha}$ is slightly δ-precontinuous for each $\alpha \in I$, $f|_{U_\alpha}^{-1}(V) \in \delta PO(U_\alpha)$. Since $U_\alpha \in \delta O(X)$, $f|_{U_\alpha}^{-1}(V) \in \delta PO(X)$ for each $\alpha \in I$. Then $f^{-1}(V) = \bigcup_{\alpha \in I} \left[f|_{U_\alpha}^{-1}(V) \right] \in \delta PO(X)$. Hence, f is slightly δ-precontinuous. □

Definition 3.20. A space X is said to be

(a) δ-*pre*-Hausdorff if for each pair of distinct points x and y in X, there exist $U \in \delta PO(X, x)$ and $V \in \delta PO(X, y)$ such that $U \cap V = \emptyset$ [4].

(b) δ-*pre*-T_1 if for each pair of distinct points x and y in X, there exist δ-preopen sets U and V containing x and y, respectively, such that $y \notin U$ and $x \notin V$ [4].

Definition 3.21. A space X is said to be

(a) *clopen* T_1 if for each pair of distinct points x and y in X, there exist $U \in CO(X, x)$ and $V \in CO(X, y)$ such that $y \notin U$ and $x \notin V$ [5].

(b) *ultra*-Hausdorff if for each pair of distinct points x and y in X, there exist $U \in CO(X, x)$ and $V \in CO(X, y)$ such that $U \cap V = \emptyset$ [12].

Theorem 3.22. *Let $f : X \longrightarrow Y$ be a slightly δ-precontinuous injection. Then the following properties hold:*

(a) If Y is ultra-Hausdorff, then X is δ-pre-Hausdorff.

(b) If Y is clopen T_1, then X is δ-pre-T_1.

Proof. (a) Suppose that Y is ultra-Hausdorff. Then for any distinct points x and y in X, there exist $V \in CO\,(Y, f(x))$,$W \in CO\,(Y, f(y))$, such that $W \cap V = \emptyset$. Since f is slightly δ-precontinuous, $x \in f^{-1}(V) \in \delta PO\,(X, f(x))$ and $y \in f^{-1}\,(W) \in \delta PO\,(X, f(y))$ such that $f^{-1}\,(V) \cap f^{-1}\,(W) = \emptyset$. This shows that X is δ-pre-Hausdorff.

(b) Suppose that Y is clopen T_1.Then for any distinct points x and y in X, there exist clopen sets U and W containing $f(x)$ and $f(y)$, respectively, such that $f(y) \notin U$ and $f(x) \notin W$. Since f is slightly δ-precontinuous, $f^{-1}\,(U)$ and $f^{-1}\,(W)$ are δ-preopen subsets of X such that $x \in f^{-1}\,(U)$, $y \notin f^{-1}\,(U)$, $x \notin f^{-1}\,(W)$ and $y \in f^{-1}\,(W)$ .Therefore, X is δ-pre-T_1. □

Theorem 3.23. *If $f : X \longrightarrow Y$ is a slightly continuous function, $g : X \longrightarrow Y$ is a slightly δ-precontinuous function and Y is ultra-Hausdorff, then $E = \{x \in X : f(x) = g(x)\}$ is δ-preclosed in X.*

Proof. Let $x \in X - E$, then it follows that $f(x) \neq g(x)$. Since Y is ultra-Hausdorff, there exist $V \in CO(Y)$ and $W \in CO(Y)$ containing $f\,(x)$ and $g(x)$, respectively, such that $V \cap W = \emptyset$. Since f is slightly continuous, $f^{-1}(V)$ is clopen and hence regular open in X. Since g is slightly δ-precontinuous, $g^{-1}(W)$ is δ-preopen in X and $x \in g^{-1}(W)$. Set $O = f^{-1}(V) \cap g^{-1}(W)$. O is δ-preopen in X. Therefore, $f(O) \cap g(O) = \emptyset$ and it follows that $x \notin pCl_\delta\,(E)$. This shows that E is δ-preclosed in X. □

Definition 3.24. A graph $G\,(f)$ of a function $f : X \longrightarrow Y$ is said to be δ^*-*preclosed* if for each $(x, y) \in (X \times Y) - G\,(f)$, there exist a δ-*preclopen* subset U of X containing x and a clopen subset V of Y containing y such that $(U \times V) \cap G(f) = \emptyset$.

Lemma 3.25. *A graph $G\,(f)$ of a function $f : X \longrightarrow Y$ is δ^*-preclosed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - G\,(f)$ there exist a δ-preclopen subset U of X containing x and a clopen subset V of Y containing y such that $f\,(U) \cap V = \emptyset$.*

Theorem 3.26. *If $f : X \longrightarrow Y$ is slightly δ-precontinuous and Y is clopen T_1, then $G\,(f)$ is δ^*-preclosed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G\,(f)$, then $f(x) \neq y$ and there exist a clopen set T of Y such that $f(x) \in T$ and $y \notin T$. Since f is slightly δ-precontinuous, then $f^{-1}\,(T)$ is δ-preclopen subset of X containing x. Set $U = f^{-1}\,(T)$. We have $f(U) \subset T$. Therefore, we obtain $f\,(U) \cap (Y - T) = \emptyset$ and $Y - T \in CO\,(Y, y)$. This shows that $G(f)$ is δ^*-preclosed. □

Corollary 1. *If $f : X \longrightarrow Y$ is slightly δ-precontinuous and Y is ultra-Hausdorff, then $G(f)$ is δ^*-preclosed in $X \times Y$.*

Theorem 3.27. *Let $f : X \longrightarrow Y$ have a δ^*-preclosed graph $G(f)$. If f is injective, then X is δ-pre-T_1.*

Proof. Let x and y be any two distinct points of X. Then we have $(x, f(y)) \in (X \times Y) - G(f)$. Since $G(f)$ is δ^*-preclosed, there exist a δ-preclopen subset U of X and $V \in CO(Y)$ such that $(x, f(y)) \in U \times V$ and $f(U) \cap V = \emptyset$. Hence, $U \cap f^{-1}(V) = \emptyset$ and $y \notin U$. This shows that X is δ-pre-T_1. □

Theorem 3.28. *Let $f : X \longrightarrow Y$ have a δ^*-preclosed graph $G(f)$. If f is a surjective δ-preopen function, then Y is δ-pre-Hausdorff.*

Proof. Let $y_1, y_2 \in Y$ and $y_1 \neq y_2$. Since f is surjective, there exists a $x \in X$ such that $f(x) = y_1$ and $(x, y_2) \in (X \times Y) - G(f)$. Since $G(f)$ is δ^*-preclosed, there exist a δ-preclopen subset U of X and $V \in CO(Y)$ such that $(x, y_2) \in U \times V$ and $(U \times V) \cap G(f) = \emptyset$. Then, we have $f(U) \cap V = \emptyset$. Since f is δ-preopen, then $f(U)$ is δ-preopen in Y such that $f(x) = y_1 \in f(U)$. This implies that Y is δ-pre-Hausdorff. □

4. Covering properties

Definition 4.1. A space X is said to be

(a) δ-pre-compact [4] if every δ-*preopen* cover of X has a finite subcover.

(b) countably δ-pre-compact [4] if every countable cover of X by δ-*preopen* sets has a finite subcover.

(c) δ-pre-Lindelof [4] if every δ-*preopen* cover of X has a countable subcover.

(d) mildly compact [12] if every clopen cover of X has a finite subcover.

(e) mildly countably compact [12] if every countable cover of X by clopen sets has a finite subcover.

(f) mildly Lindelof [12] if every clopen cover of X has a countable subcover.

Theorem 4.2. *Let $f : X \longrightarrow Y$ be a slightly δ-precontinuous surjection. Then the following statements hold:*

(a) if X is δ-pre-compact, then Y is mildly compact.

(b) if X is δ-pre-Lindelof, then Y is mildly Lindelof.

(c) if X is countably δ-pre-compact, then Y is mildly countably compact.

Proof. (a) Let $\{U_\alpha : \alpha \in I\}$ be a clopen cover of Y. Since f is slightly δ-precontinuous, $\{f^{-1}(U_\alpha) : \alpha \in I\}$ is a δ-preopen cover of X and there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(U_\alpha) : \alpha \in I_0\}$. Hence, $\{U_\alpha : \alpha \in I_0\}$ is a finite subcover of $\{U_\alpha : \alpha \in I\}$. Therefore, Y is mildly compact.

(b) and (c) can be obtained similarly. □

Definition 4.3. A space X is said to be

(a) δ-preclosed-compact [4] if every δ-*preclosed* cover of X has a finite subcover.

(b) countably δ-preclosed-compact [4] if every countable cover of X by δ-*preclosed* sets has a finite subcover.

(c) δ-preclosed-Lindelof [4] if every cover of X by δ-*preclosed* sets has a countable subcover.

Theorem 4.4. *Let $f : X \longrightarrow Y$ be a slightly δ-precontinuous surjection. Then the following statements hold:*

(a) if X is δ-preclosed-compact, then Y is mildly compact.

(b) if X is δ-preclosed-Lindelof, then Y is mildly Lindelof.

(c) if X is countably δ-preclosed-compact, then Y is mildly countably compact.

Proof. (a) Let $\{A_\alpha : \alpha \in \Delta\}$ be any clopen cover of Y. Since f is slightly δ-precontinuous surjection, then $\{f^{-1}(A_\alpha) : \alpha \in \Delta\}$ is a δ-preclosed cover of X. Since X is δ-preclosed-compact, there exists a finite subset Δ_0 of Δ such that $X = \cup\{f^{-1}(A_\alpha) : \alpha \in \Delta_0\}$. Hence, $\{A_\alpha : \alpha \in \Delta_0\}$ covers Y. This shows that Y is mildly compact.

(b) and (c) can be obtained similarly. □

SLIGHTLY $\delta-$PRE SÜREKLİ FONKSİYONLAR

Özet: Bu makalede, slight δ-pre süreklilik olarak adlandırılan, slight pre sürekliliğin yeni bir zayıf çeşidi takdim edilmiş ve çalışılmıştır. Ayrıca, slight $\delta-$pre sürekliliğin hem almost $\delta-$pre süreklilikten hem de (δ-*pre,* s)-süreklilikten daha zayıf olduğu gösterilmiştir.

References

[1] C. W. Baker, Slightly precontinuous functions, *Acta Math. Hungar.*, **94** (1-2) (2002), 45-52.
[2] N. Bourbaki, General Topology, Part 1, Addison Wesley, Reading, Mass 1996.
[3] E. Ekici, Almost contra-precontinuous functions, *Bull. Malays. Math. Sci. Soc.* (2), **27** (2004), 53-65.
[4] E. Ekici, (δ-*pre*, s)-continuous functions, *Bull. Malays. Math. Sci. Soc.* (2), **27** (2004), 237-251.
[5] E. Ekici, Slightly γ-continuous functions, *Bol. Soc. Paran. Mat.,* (3s.) **v. 22** 2 (2004): 63-74.
[6] R. C. Jain, *The role of regularly open sets in general topology,* Ph.D. Thesis, Meerut University, Institute of Advanced Studies (Meerut, India, 1980).
[7] A. S. Mashour, M. E. Abd El-Monsef and S. N. El-Deep, On precontinuous and weak precontinuous mappings, *Proc. Math. Phys. Soc. Egypt.,* **53** (1982), 47-53.
[8] A. A. Nasef and T. Noiri, Some weak forms of almost continuity, *Acta Math. Hungar.,* **74** (1997), 211-219.
[9] T. Noiri, Slightly β-continuous functions, *Int. J. Math. Math. Sci.,* **28:8** (2001) 469-478.
[10] S. Raychaudhuri and N. Mukherjee, On δ-almost continuity and δ-preopen sets, *Bull. Inst. Math. Acad. Sinica,* **21** (1993), 357-366.
[11] S. Raychaudhuri, Concerning δ^*-almost continuity and δ-preregularity, *Bull. Calcutta Math. Soc.,* **85** (1993), 385-392.
[12] R. Staum, The algebra of bounded continuous functions into a nonarchimedean field, *Pasific J. Math.,* **50** (1974) 169-185.
[13] M. H. Stone, Applications of the theory of Boolean rings to general topology, *TAMS* **41** (1937), 375-381.

[14] N. V. Velicko, *H*-closed topological spaces, *Amer. Math. Soc. Transl.,* **78** (1968), 103-118.

Current address: Selcuk University Faculty of Science and Arts Department of Mathematics Campus 42075 Konya Turkey, 2949-1 Shiokita-cho, Hinagu, Yatsushiro-shi, Kumamoto-ken 869-5142, JAPAN

E-mail address: ayseuresin@mynet.com, akeskin@selcuk.edu.tr, *t.noiri@nifty.com

URL: http://math.science.ankara.edu.tr