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ON PLANAR FUZZY TERNARY RINGS

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Abstract. In this paper, we extend the process of coordinatization of projective planes to the fuzzy projective planes and introduce notion of fuzzy ternary ring which determines its associated fuzzy projective plane. Later, we give some propositions concerned with linearity of the defined fuzzy ternary operation.

1. Introduction

Geometric properties of fuzzy sets have been of interest since fuzzy sets were first introduced in the mid-1960; in fact, the first paper on fuzzy sets [13] deal with their convexity. Since the late 1970's there has been increasing interest in geometric properties of fuzzy sets, including adjacency and connectedness; distance; relative position; convexity; area, perimeter, and diameter. The concept of fuzzy projective plane has been introduced by K.C. Gupta- S. Ray in [6], and three models of the fuzzy projective planes have been used to investigate some important properties of these geometric structures. Also, it is well known that although they are clumsy and strange systems the ternary rings give the best algebraic representation of non-Desarguesian projective planes. And then the classification of fuzzy vector planes and 3-dimensional vector spaces of fuzzy $4-$dimensional vector space and fuzzy projective lines and planes of fuzzy $3-$dimensional projective space from fuzzy $4-$dimensional vector space are given in [2], [3]. It is well known that every projective plane also has an algebraic structure obtained by coordinatization. Conversely, certain algebraic structures can be used to construct projective planes. Certain algebraic structures in some projective planes are used by many authors. For instance, Çifçi-Kaya [5], Akça-Günaltılı-Güney [1], etc. In the present paper, we introduce notion of fuzzy ternary ring which determines its associated fuzzy projective plane and show that linearity of defined ternary operation implies the validity of a special case of the fuzzy small Desargues' proposition given in [6]. Finally, we give a proposition which determines a simple configuration that gives linearity of the fuzzy ternary operation.

2. Preliminaries

We now get down to definitions. The following three definitions concerning the basic concepts of the subject has been taken from [6].

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Let S be any non-empty set. A mapping from S to the unit closed interval $[\mathbf{0},\mathbf{1}]$ is a fuzzy subset of S, [13]. For $x \in S$ and $\mathbf{0} < \lambda \leq \mathbf{1}$, the fuzzy point x_λ of S is the fuzzy subset of S that maps x to λ and other points of S to 0. Instead of x_λ we shall use the notation (x, λ), which is convenient for geometrical discussion.

Definition 2.1 Let S be a non-empty set. A collection Π of fuzzy points of S is called *a complete set of fuzzy points* if, given $x \in S$, there exists $\mathbf{0} < \lambda \leq \mathbf{1}$ such that $(x, \lambda) \in \Pi$. For the same x in S there may exist $\mathbf{0} < \alpha \neq \beta \leq \mathbf{1}$ such that (x, α), $(x, \beta) \in \Pi$. Such points are called *(fuzzy) vertical points* in Π. Two points (x, α) and (y, β) of Π are said to be *fuzzy distinct* if $x \neq y$. Two distinct fuzzy points in Π are either fuzzy distinct or fuzzy vertical. A non-zero fuzzy subset l of S is called *a fuzzy line through* Π if, for all $x \in S$,

$$l(x) > \mathbf{0} \ \Rightarrow (x, l(x)) \in \Pi.$$

We say that a fuzzy line l contains (or passes through) the fuzzy point (x, λ) of Π if $l(x) = \lambda$. In that case we say that the fuzzy point (x, λ) lies on the fuzzy line l. In fact we have here a symmetric *incidence relation* $|$ such that $l \ | \ (x, \lambda)$ or $(x, \lambda) \ | \ l$ means that l contains the fuzzy point (x, λ).

Definition 2.2 *A fuzzy plane projective geometry* $(FPPG)$ is an axiomatic theory with the triple $(\Pi, \Lambda, |)$ as its fundamental notions and F1, F2, F3 as its axioms. Π is a complete set of fuzzy points of a non-empty set S and Λ is a collection of fuzzy lines through Π.

F1a. Given two fuzzy distinct points in Π, there is at least one fuzzy line in Λ incident with both points.

F1b. Given two fuzzy distinct points in Π, there is at most one fuzzy line in Λ incident with both points.

F2. Given two distinct fuzzy lines in Λ, there exists at least one fuzzy point in Π incident with both lines.

F3. Π contains at least four (fuzzy) distinct points such that no three of them are incident with one and the same fuzzy line in Λ.

Definition 2.3 Consider the relation $\sim$ in a $FPPG$ $(\Pi, \Lambda, |)$ defined in the set Π as follows:

$$(x, \alpha) \sim (y, \beta) \text{ iff } \ x = y.$$

This equivalence relation induces a partition of Π in *vertical classes.* The vertical class containing the fuzzy point (x, α) is denoted by $V(x)$. A vertical class $V(x)$ is called *a vertical barrier* if every fuzzy line in Λ intersects $V(x)$ nontrivial. A vertical class is called *trivial* if it consists of exactly one element.

Not all vertical classes are vertical barriers: for instance, Kuijken and Van Maldeghem [10], a trivial vertical class is never a vertical barrier as otherwise every line would contain the unique element (x, α) of that class; now (F3) guarantees the existence of three fuzzy distinct points a, b, c not contained in a common line. The unique line joining a, b, joining b, c and c, a, respectively, all contain (x, α). Axiom (F2) now implies that $a = b = c = (x, \alpha)$, a contraduction.

Theorem 2.1 ***(see [10])*** Let $(\Pi, \Lambda, |)$ be any finite $FPPG$. Identifying each fuzzy line and vertical class with the set of its nonzero-valued fuzzy points, we obtain exactly one of the following structures:

(1) a projective plane of order q where one line L is considered as a vertical class and all other lines as fuzzy lines: there exists one nontrivial vertical class, and it is a vertical barrier.

(2) a projective plane of order q where a point p is removed and where all but exactly one line L through p are considered as vertical classes and all other lines as fuzzy lines: there exist q nontrivial vertical classes, and none of them is a vertical barrier.

(3) a projective plane of order q where a point p is removed and where the $q+1$ lines through that point are considered as vertical classes and all other lines as fuzzy lines: there exist $q+1$ nonvertical classes, and they all are vertical barriers.

(4) a projective plane of order q possessing only trivial vertical classes.

Conversely, every structure mentioned in (1) up to (4) above gives rise to a $FPPG$ by first arbitrarily assigning nonzero membership degrees to the elements of Π in such a way that the points of a vertical class all have different membership degree, and then identifying the "base points" of the same vertical class.

For the description of the algebraic systems which can be obtained from projective planes, we need the idea of a ternary function, which is a simple generalization of the binary functions.

Definition 2.4 Let M, N, R be any sets. If T is a rule that associates with each ordered triple of cartesian product set $M \times N \times R$, (x_1, x_2, x_3), a unique element of R then T is called a ternary operation:

$$T(x_1, x_2, x_3) = x \in R.$$

3. Coordinatization a Fuzzy Projective Plane

We now introduce the notion of coordinatization of a fuzzy projective plane.

Let f be a fuzzy set (subset) of a set S which is a one-to-one function $f : S \to (\mathbf{0}, \mathbf{1}]$. Now consider the $FPPG$ of order n (that is, every fuzzy line has exactly $n+1$ nonzero-valued elements), associated with f , and choose four fuzzy distinct points X, Y, O, U no three on a fuzzy line. Let cardinality of S be n, and assume that S contains the symbols "0" and "1" but not the symbol ∞. On the fuzzy line OU give coordinates $(0, f(0))$ to O, $(1, f(1))$ to U and the single coordinate $(1i)$ to the point C which is the intersection of OU and XY the fuzzy line of infinity. For other fuzzy points of OU assign coordinates $(x, f(x))$, taking different symbols x for different fuzzy points. Considering the correspondence

$$\{0, 1, x, y, ...\} \to \{(0, f(0)),\ (1, f(1)),\ (x, f(x)), (y, f(y)), ...\}$$

one can easily say that there exists an one-to-one mapping between the set S and the set of fuzzy points on the fuzzy line OU except the fuzzy point $(1i)$. For a fuzzy point P not on XY, let XP vertical barrier intersect OU in $(x, f(x))$ and YP intersect OU in $(y, f(y))$. Then assign coordinates $(x, f(y))$ to P.

Let the fuzzy line joining $(0, f(0))$ and $(1, f(m))$ intersect XY in a fuzzy point M. Assign to M the single coordinate (mi), $m \in S\backslash\{0\}$. We have assigned coordinates to every fuzzy point except Y, and to this we arbitrarily assign the single coordinate (∞), $\infty \notin S$.

In a similar way, we will label the fuzzy lines of the $FPPG$. If a fuzzy line not through Y intersects XY at a fuzzy point labelled (mi) and also intersects the vertical barrier $V(0)$ at a fuzzy point $(0, f(k))$ then we will label $[m, f(k)]$, $m \in S\backslash\{0\}$, $k \in S$. If a fuzzy line intersects XY at Y and also intersects the vertical barrier $V(0)$ at a fuzzy point $(0, f(k))$ then we will label $[f(k)]$, $k \in S$. Label XY as $[\infty]$, such that $\infty \mid [\infty]$ and $(mi) \mid [\infty]$, $m \in S\backslash\{0\}$. Yet, we do not know when a fuzzy point $(x, f(y))$ is on a fuzzy line $[m, f(k)]$. For this, we need:

Proposition 3.1 Let a $FPPG$ be coordinatized as above. If T is a fuzzy subset such that

$$T: \ S\backslash\{0\} \times S \times f(S) \to f(S), \quad f(S) \subset (\mathbf{0}, \mathbf{1}]$$

$$T(m, x, f(k)) = f(y) \quad \Leftrightarrow \quad (x, f(y)) \mid [m, f(k)]$$

then, T is a ternary operation.

Proof To show that T is a ternary operation, one must show that it is well defined. Consider $[f(y)] \cap [m, f(k)]$. Since $(\Pi, \Lambda, |)$ is a fuzzy projective plane, the intersection must be a unique fuzzy point. And since $[f(y)] \cap [\infty] = (\infty)$ while $[m, f(k)] \cap [\infty] = (mi)$, this point can not be on $[\infty]$. Additionally, recall that all of the fuzzy points of $[f(y)]$ have the same second coordinate $f(y)$ and consequently $[f(y)] \cap [m, f(k)] = (x, f(y))$ for some unique $f(y) \in f(S)$. Therefore, if $T(m, x, f(k)) = f(y_1)$ and $T(m, x, f(k)) = f(y_2)$ then $f(y_1) = f(y_2)$.

Proposition 3.2 *Let S be a coordinatizing set of a fuzzy projective plane. The ternary operation T defined as*

$$T: \ S\backslash\{0\} \times S \times f(S) \to f(S) \qquad \ni \qquad T(m, x, f(k)) = f(y)$$

satisfies the following properties:

T1) *$T(m, 0, f(k)) = f(k)$, for all $k \in S$, $m \in S\backslash\{0\}$.*

T2) *$T(1, x, f(0)) = f(x)$ and $T(m, 1, f(0)) = f(m)$, for all $x \in S$, $m \in S\backslash\{0\}$.*

T3) *Given $x, y \in S$, $m \in S\backslash\{0\}$, there exists exactly one $f(k) \in f(S)$ such that $T(m, x, f(k)) = f(y)$.*

T4) *If $m_1 \neq m_2$, $m_1, m_2 \in S\backslash\{0\}, k_1, k_2 \in S$ are given, then there exists a unique $x \in S$ such that $T(m_1, x, f(k_1)) = T(m_2, x, f(k_2))$.*

T5) *If $x_1 \neq x_2$, $y_1, y_2 \in S$ are given, then there exists a unique pair $[m, f(k)]$ such that $T(m, x_1, f(k)) = f(y_1)$ and $T(m, x_2, f(k)) = f(y_2)$.*

Proof Properties T1 and T2 are immediate consequence of the definition of the ternary operation T. From the definition of T one get $[m, f(k)] = (0, f(k)) \cup (mi)$, and $(x, f(y)) \mid [m, f(k)]$. So, the fuzzy points $(0, f(k)), (x, f(y))$, and (mi) are fuzzy concurrent. By property F1, there exists a unique $f(k) \in f(S)$ such that

$$(0, f(k)) \cup (mi) = (x, f(y)) \cup (mi) = [m, f(k)]$$

which gives T3. Now, by the definition of T, $T(m_1, x, f(k_1)) = f(y)$ if and only if $(x, f(y))|[m_1, f(k_1)]$ and $T(m_2, x, f(k_2)) = f(y)$ if and only if $(x, f(y))|[m_2, f(k_2)]$. Then, $[m_1, f(k_1)]$ and $[m_2, f(k_2)]$ intersect $[\infty]$ in distinct fuzzy points. This means that their unique fuzzy point of intersection is $(x, f(y))$. So, there exists a unique $x \in S$, and T4 is valid. Similarly, $T(m, x_1, f(k)) = f(y_1)$ if and only if $(x_1, f(y_1))|[m, f(k)]$ and $T(m, x_2, f(k)) = f(y_2)$ if and only if $(x_2, f(y_2))|[m, f(k)]$. Thus, there exists a unique fuzzy line $[m, f(k)]$ by F1, and T5 is valid.

Definition 3.1 Let $FPPG$ be a fuzzy projective plane and S be its coordinatizing set. If T is ternary operation which satisfies the properties T1, T2, T3, T4 and T5 in proposition 3.2, then (S,T) is called a *fuzzy ternary ring.*

The following proposition gives that every fuzzy ternary ring determines its associated $FPPG$, that is, (S,T) is planar.

Proposition 3.3 *Every (S,T) fuzzy ternary ring is a planar fuzzy ternary ring of a $FPPG$, denoted by $FPPG_{(S,T)}$.*

Proof Let (S,T) be a fuzzy ternary ring and $\infty \notin S$. Let $(\Pi, \Lambda, |)$ be a geometric structure of fuzzy points and fuzzy lines, with

$$\Pi = \{(x, f(y)) : x, y \in S,\ f : S \to (\mathbf{0}, \mathbf{1}]\} \cup \{(mi) : m \in S \backslash \{0\}\} \cup \{(\infty)\}$$

$$\Lambda = \{[m, f(k)] : k, m \in S, m \neq 0, f : S \to (0,1]\} \cup \{[f(k)] : k \in S, f : S \to (0,1]\} \cup \{[\infty]\}$$

and using the function $f : S \to (\mathbf{0}, \mathbf{1}]$, $x, y, k \in S, m \in S\backslash\{0\}$ define an incidence relation $|$ such that

$$\begin{array}{l} (x, f(y)) \mid [m, f(k)] \Leftrightarrow T(m, x, f(k)) = f(y) \\ (x, f(y)) \mid [f(k)] \Leftrightarrow f(y) = f(k) \\ (xi) \mid [m, f(k)] \Leftrightarrow x = m \\ (mi) \mid [\infty] \\ (\infty) \mid [\infty] \\ (\infty) \mid [f(k)]. \end{array}$$

Clearly, $(x, f(y)) \nmid [\infty]$, $(mi) \nmid [f(k)]$, $(\infty) \nmid [m, f(k)]$, where $\nmid$ stands for negation of $|$.

Now, it is an easy exercise to show F1, F2 and F3 using the definition of T and its properties T1-T5.

Example Consider a planar ternary ring of a $FPPG$ where $S = \mathbb{R}$ and $f(x) = \frac{1}{\pi}\cot^{-1}(x)$ then $FPPG_{(S,T)}$ is the straight line model given in [6].

4. **Linear Fuzzy Ternary Rings**

Linearity of a ternary ring plays a central role to represent the algebraic structure of the corresponding projective plane [12]. G. Pickert has given an example of a non-Desarguesion plane coordinatized by a field with the non-linear ternary operation [11]. Linearity of ternary rings can be given in terms of some special cases of Desargues or dual Pappus axioms [8], [9], [11].

Definition 4.1 Let T be a ternary operation of a planar fuzzy ternary ring (S,T). Then

$$T(1, x, f(k)) = f(x + k) \qquad \text{and} \qquad T(m, x, f(0) = f(mx)$$

are called *fuzzy addition* and *fuzzy multiplication*, respectively.

Proposition 4.1 *If $a, b, c, d \in S$, then*
1) *There exists a unique $x \in S$ such that $f(a \circ x) = f(b)$,*

2) *There exists a unique* $y \in S$ *such that* $f(y \circ c) = f(d)$,
3) *There exists unique* $e \in S$ *such that* $f(e \circ x) = f(x \circ e) = f(x)$,
where $\circ$ *stands for each of the fuzzy addition or multiplication.*

Proof If $\circ$ is the fuzzy addition, let $e = 0$. For $a, b \in S$, there exists a unique $f(x)$, such that $f(x) \in f(S)$ and $T(1, a, f(x)) = f(b)$ by T3, and $T(1, a, f(x)) = f(a + x)$. Thus, there is a unique $x \in S$ such that $f(a + x) = f(b)$ since f is one-to-one.

Let $c, d \in S$, then

$$T(1, y, f(c)) = f(d) \quad \Leftrightarrow \quad (y, f(d)) \quad | \quad [1, f(c)] \tag{1}$$

and further

$$(y, f(d)) \quad | \quad [f(d)]. \tag{2}$$

Since there exists a unique fuzzy point $(y, f(d))$ such that $[1, f(c)] \cap [f(d)] = (y, f(d))$ by (1) and (2), there exists a unique $y \in S$ such that $f(y + c) = f(d)$.

$$\begin{aligned} f(x+0) &= T(1, x, f(0)) \quad \text{by the definition} \\ &= f(x) \quad \text{by T2} \end{aligned}$$

and

$$\begin{aligned} f(0+x) &= T(1, 0, f(x)) \quad \text{by the definition} \\ &= f(x) \quad \text{by T1.} \end{aligned}$$

Thus, $f(x + 0) = f(0 + x) = f(x)$.

A similar proof can be given for the case where $\circ$ is the fuzzy multiplication and $e = 1$.

Definition 4.2 Let (S, T) be a fuzzy ternary ring. If T has the property

$$T(m, x, f(k)) = f(mx + k), \text{ for all } x, k \in S, m \in S \backslash \{0\}$$

then (S, T) is called *linear fuzzy ternary ring.*

Now, we need the following definitions to give the geometric version of the linearity.

Definition 4.3 Two or more fuzzy points A_i $(i = 1, 2, ...)$ are *(fuzzy) collinear* if there is a fuzzy line with which each of them is incident. Clearly, no two of them are fuzzy vertical. The fuzzy lines l_i $(i = 1, 2, ...)$ are *(fuzzy) concurrent* if there is a fuzzy point with which each of them is incident. If $P \mid l$, we write $P \in l$.

Definition 4.4 *A fuzzy triangle* in a $FPP_{(S,T)}$ is a set of three fuzzy distinct points A_1, A_2, A_3 and a set of three fuzzy lines a_1, a_2, a_3 such that $A_i \in a_k$ for $i \neq k$, but $A_i \notin a_i$ $(i, k = 1, 2, 3)$. The points A_i are the *vertices*, the lines a_i are the *sides* of a triangle. The triangle is denoted by $A_1A_2A_3$.

FSDP (Fuzzy small Desargues' proposition): *Let two fuzzy triangles* $A_1A_2A_3$ *and* $B_1B_2B_3$ *be given such that the corresponding vertices are fuzzy distinct and corresponding sides are distinct. Let* $C_i = a_i \cap b_i$ $(i = 1, 2, 3)$. *The lines connecting corresponding vertices are incident with a fuzzy point* P. *There is an extra incidence* $A_1 \in b_1 = B_2B_3$. *Then* C_1, C_2, C_3 *are either fuzzy collinear or fuzzy vertical.*

Proposition 4.2 *If a fuzzy ternary ring* (S,T) *is linear then* $FSDP$ is valid in $FPPG_{(S,T)}$ provided that $P=(\infty)$, $C_1C_2=[\infty]$ *and* $B_1\in a_1=A_2A_3$.

Proof Let $A_1A_2A_3$ and $B_1B_2B_3$ be two fuzzy triangles such that the corresponding vertices are fuzzy distinct and the corresponding sides are distinct. Then, we take $A_1=(a,f(0))$, $A_2=(b,f(u))$, $A_3=(c,f(v))$, $B_1=(a',f(0))$, $B_2=(b',f(u))$ and $B_3=(c',f(v))$. Recall that $C_i=a_i\cap b_i$, $i=1,2,3$, and let $C_1=(pi)$ and $C_2=(qi)$. We have to show that $C_3\mid[\infty]$, where $C_3=a_3\cap b_3=A_2A_3\cap B_2B_3$. Clearly, the fuzzy lines $A_1B_1=[f(0)], A_2B_2=[f(u)], A_3B_3=[f(v)]$ which are concurrent at (∞). The sides of the triangles are as follows: $A_1A_2=[p,f(-(pa))]$, $A_1A_3=[q,f(-(qa))]$, $B_1B_2=[p,f(-(pa'))]$, $B_1B_3=[q,f(-(qa'))]$. Let $A_2A_3=[r,f(k)]$ and $B_2B_3=[r',f(k')]$, and suppose that $r\neq r'$. Since B_1,A_2,A_3 are fuzzy collinear $B_1\mid A_2A_3$. That is,

$$\begin{aligned}(a',f(0))\mid[r,f(k)]\quad&\Leftrightarrow T(r,a',f(k))=f(0)\\&\Leftrightarrow f(ra'+k)=f(0)\text{ (since T is linear)}\\&\Leftrightarrow ra'+k=0\text{ (since f is one-to-one)}\\&\Leftrightarrow k=-(ra')\text{ (by proposition 3.1).}\end{aligned}$$

Thus, $A_2A_3=[r,f(-(ra'))]$. Again by $A_2\mid B_1A_3$

$$T(r,b,f(-(ra'))=f(u). \tag{3}$$

$A_1\mid B_2B_3$ since A_1,B_2,B_3 are fuzzy collinear, That is,

$$\begin{aligned}(a,f(0))\mid[r',f(k')]\quad&\Leftrightarrow T(r',a,f(k'))=f(0)\\&\Leftrightarrow f(r'a+k')=f(0)\\&\Leftrightarrow r'a+k'=0\\&\Leftrightarrow k'=-(r'a)\end{aligned}$$

which gives $B_2B_3=[r',f(-(r'a))]$. Similarly, $B_2\mid A_1B_3$ implies

$$T(r',b',f(-(r'a)))=f(u). \tag{4}$$

Now, using (3) and (4), one gets

$$T(r,b,f(-(ra')))=T(r',b',f(-(r'a)))=f(u) \tag{5}$$

it follows from equation (5) that $b=b'$ by T4. This a contradiction since A_2 and B_2 are fuzzy distinct. That is, $r=r'$. So, $A_2A_3=[r,f(-(ra'))], B_2B_3=[r,f(-(ra))]$ and $A_2A_3\cap B_2B_3=(ri)=C_3$. This completes the proof.

Proposition 4.3 *Consider the triples of fuzzy lines* $\alpha=[\infty]$, $\beta=[f(b)]$, $\gamma=[f(b+a)]$ *and* $\alpha^{'}=[\infty]$, $\beta^{'}=[c,f(a)]$, $\gamma^{'}=[c,f(0)]$, *which are concurrent at the fuzzy points* (∞) *and* (ci), *respectively. Define* $P=\beta\gamma^{'}$ *and* $Q=\beta^{'}\gamma$. *If* P and Q *are in a vertical barrier* $V(x)$ *then* T *is linear.*

Proof Clearly $\alpha\beta^{'}\cup\alpha^{'}\beta=\alpha\gamma^{'}\cup\alpha^{'}\gamma=[\infty]$ and $\beta\gamma^{'}=(x,f(b))$, $\beta^{'}\gamma=(x,f(b+a))$. We get $(x,f(b))\mid\gamma^{'}$,

$$\begin{aligned}(x,f(b))\ \mid\ [c,f(0)]\quad&\Leftrightarrow T(c,x,f(0))=f(b)\\&\Leftrightarrow f(cx)=f(b)\\&\Leftrightarrow cx=b\end{aligned}\tag{6}$$

and from $(x, f(b+a)) \mid \beta^{'}$,

$$(x, f(b+a)) \mid [c, f(a)] \Leftrightarrow T(c, x, f(a)) = f(b+a). \tag{7}$$

Thus, from (6) and (7),

$$T(c, x, f(a)) = f(cx+a)$$

is obtained. So, T is linear.

In a forthcoming work, we investigate properties of a linear fuzzy ternary ring and algebraic structures of the corresponding fuzzy projective planes.

ÖZET: Bu çalışmada, projektif düzlemlerin koordinatlama yöntemini fuzzy projektif düzlemlere genişletiyoruz ve fuzzy projektif düzlemin fuzzy üçlü halkası kavramını tanımlıyoruz. Daha sonrada, tanımlanan fuzzy üçlü işlemin lineerliği ile ilğili bazı özellikleri belirleyen önermeler veriyoruz.

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