Commun.Fac.Sci.Univ.Ank.Series A1
Volume 62, Number 1, Pages 121–129 (2013)
ISSN 1303–5991

EXISTENCE AND UNIQUENESS OF SOLUTION FOR A SECOND ORDER BOUNDARY VALUE PROBLEM*

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Abstract. This paper deals with a second order boundary value problem with only integrals conditions. Our aim is to give new conditions on the nonlinear term, then, using Banach contraction principle and Leray Schauder nonlinear alternative, we establish the existence of nontrivial solution of the considered problem. As an application, some examples to illustrate our results are given.

1. Introduction

We study the existence of solutions for the following second-order boundary value problem (BVP)(P_1):

$$u''(t) + f(t, u(t)) = 0, \ 0 < t < 1 \tag{1.1}$$

$$u(0) = \int_0^1 u(t)\,dt, \ u(1) = \int_0^1 tu(t)\,dt, \tag{1.2}$$

where $f : [0,1] \times \mathbb{R} \to \mathbb{R}$ is a given function. We mainly use the Banach contraction principle and Leray Schauder nonlinear alternative to prove the existence and uniqueness results. For this, we formulated the boundary value problem (P_1) as fixed point problem. We also study the compactness of solutions set.

The second order equations (1.1) are used to model various phenomena in physics, chemistry and epidemiology. In general nonlinearities that refer to source terms represent specific physical laws, in chemistry, for example, if $f(t,u) = ug(u)e^{\frac{u-1}{\varepsilon}}$, then it represents Arheninus law for chemistry reactions, where the positive parameter ε represents the activation energy for the reaction and the continuous function g represents the concentration of the chemical product, see [1].

Received by the editors Nov. 11, 2012; Accepted: June 27, 2013.
2010 *Mathematics Subject Classification.* 34B10, 34B15, 34B18, 34G20.
Key words and phrases. Fixed point theorem, two-point boundary value problem, Banach contraction principle, Leray Schauder nonlinear alternative, second-order equation.

The main results of this paper were presented in part at the conference Algerian-Turkish International Days on Mathematics 2012 (ATIM' 2012) to be held October 9–11, 2012 in Annaba, Algeria at the Badji Mokhtar Annaba University.

Non local conditions come up when values of the function on the boundary are connected to values inside the domain. The integral conditions arise in quasi-stationary thermoelasticity theory, in modeling the technology of integral circuits,.... Some times it is better to impose integral conditions because they lead to more precise measures than those proposed by a local conditions.

Very recently there have been several papers on second and third order boundary value problems, we can cite the paper Graef and Yang [6], Guo et al [10], Hopkins and Kosmatov [11], and Shunhong et al [15] . Excellent surveys of theoretical results can be found in Agarwal [1] and Ma [14]. More results can be found in [2, 3, 4, 5, 7, 8, 9, 10, 12, 13] . Most of the results dealing with these problems used the nonlinear alternative of Leray-Schauder, or more generally the theory of fixed point on the cone.

This paper is organized as follows. In section 2 we list some preliminaries materials to be used later. Then in Section 3, we give our main results which consist in uniqueness and existence theorems. We end our work with some illustrating examples.

2. Preliminaries

Let $E = C([0,1], \mathbb{R})$ be the Banach space of all continuous functions from $[0,1]$ into $\mathbb{R}$ with the norm $||y|| = \max_{t\in[0,1]} |y(t)|$. We denote by $L^1([0,1], \mathbb{R})$ the Banach space of Lebesgue integrable functions from $[0,1]$ into $\mathbb{R}$ with the norm $||y||_{L^1} = \int_0^1 |y(t)|\, dt$.

Definition 2.1. A function $f : [0,1] \times \mathbb{R} \to \mathbb{R}$ is called L^1-Carathéodory if

(i) The map $t \mapsto f(t,u)$ is measurable for all $u \in \mathbb{R}$.
(ii) The map $u \mapsto f(t,u)$ is continuous for almost each $t \in [0,1]$.
(iii) For each $r > 0$, there exists an $\psi_r \in L^1[0,1]$ such that for almost all $t \in [0,1]$ and $|u| \le r$ we have $|f(t,u)| \le \psi_r(t)$.

Lemma 2.2. [4] *Let F be a Banach space and Ω a bounded open subset of F, $0 \in \Omega$. Let $T : \overline{\Omega} \to F$ be a completely continuous operator. Then, either there exists $x \in \partial\Omega$, $\lambda > 1$ such that $T(x) = \lambda x$, or there exists a fixed point $x^* \in \overline{\Omega}$ of T.*

Lemma 2.3. *Let $y \in L^1([0,1], \mathbb{R})$. Then the solution of the following boundary value problem*

$$\begin{aligned} &u''(t) + y(t) = 0,\ 0 < t < 1 \\ &u(0) = \int_0^1 u(t)\, dt,\ u(1) = \int_0^1 tu(t)\, dt, \end{aligned} \tag{2.1}$$

is

$$u(t) = \frac{1}{3}\int_0^1 G(t,s)y(s)ds,$$

where

$$G(t,s)=\begin{cases} 23s+3s^2t-6st-4s^2-6s^3-10 & , \quad 0\le s\le t\le 1 \\ (1-s)\left(10s+3t-3st+6s^2-10\right) & , \quad 0\le t\le s\le 1. \end{cases}$$

Proof. Rewriting the differential equation (2.1) as $u''(t)=-y(t)$, then integrating two times, we obtain

$$u(t)=-\int_0^t (t-s)\,y(s)ds+At+B. \tag{2.2}$$

Using the first integral condition we get $B=\int_0^1 u\,(s)\,ds$. Substituting B in (2.2) and using the second integral condition we get

$$A=\int_0^1 (1-s)\,y(s)ds+\int_0^1 su\,(s)\,ds-\int_0^1 u\,(s)\,ds.$$

Substituting A in (2.2) we obtain

$$\begin{aligned} u(t) \quad = \quad & -\int_0^t (t-s)\,y(s)ds+t\int_0^1 (1-s)\,y(s)ds \\ & +t\int_0^1 su\,(s)\,ds+(1-t)\int_0^1 u\,(s)\,ds. \end{aligned} \tag{2.3}$$

Integrating (2.3) over $[0,1]$, it yields

$$\int_0^1 u\,(s)\,ds=-\int_0^1 (1-s)^2\,y(s)ds+\int_0^1 (1-s)\,y(s)ds+\int_0^1 su\,(s)\,ds. \tag{2.4}$$

Substituting (2.4) in (2.3) then integrating the resultant equality over $[0,1]$ we get

$$\begin{aligned} u(t) \quad = \quad & -\int_0^t (t-s)\,y(s)ds-(1-t)\int_0^1 (1-s)^2\,y(s)ds \\ & +\int_0^1 (1-s)\,y(s)ds+\int_0^1 su\,(s)\,ds. \end{aligned} \tag{2.5}$$

Multiplying (2.5) by t then integrating the resultant equality over $[0,1]$ we obtain

$$\begin{aligned} \int_0^1 su\,(s)\,ds \quad = \quad & -2\int_0^1 (1-s)^2\,(s+2)\,y(s)ds \\ & -\frac{1}{3}\int_0^1 (1-s)^2\,y(s)ds+\int_0^1 (1-s)\,y(s)ds. \end{aligned} \tag{2.6}$$

Substituting (2.6) in (2.5) it yields

$$\begin{aligned} u(t) &= -\int_0^t (t-s)\, y(s)ds + \frac{1}{3}\int_0^1 (1-s)^2 (3t-6s-16)\, y(s)ds \qquad (2.7)\\ &\quad +2\int_0^1 (1-s)\, y(s)ds \\ &= \frac{1}{3}\int_0^t \left(23s+3s^2t-6st-4s^2-6s^3-10\right) y(s)ds \\ &\quad +\frac{1}{3}\int_t^1 (1-s)\left(10s+3t-3st+6s^2-10\right) y(s)ds \\ &= \frac{1}{3}\int_0^1 G(t,s)y(s)ds. \end{aligned}$$

□

3. Existence and Uniqueness Results

Theorem 3.1. *Assume that the following hypotheses hold.*

(A1) *f is an L^1-Carathéodory function.*

(A2) *There exists a nonnegative function $g \in L^1([0,1], \mathbb{R}_+)$ such that*

$$|f(t,x)-f(t,y)| \leq g(t)\,|x-y|\,, \forall x,y \in \mathbb{R},\ t\in[0,1]\,, \qquad (3.1)$$

$$\int_0^1 g(s)ds < \frac{3}{10}, \qquad (3.2)$$

then the BVP (P_1) has a unique solution u in E.

Proof. We transform the boundary value problem (1.1)-(1.2) to a fixed point problem. Define the integral operator $T : E \to E$ by

$$Tu(t) = \frac{1}{3}\int_0^1 G(t,s) f(s,u(s))\, ds, \forall t \in [0,1]\,.$$

From Lemma 2.3, the BVP (1.1)-(1.2) has a solution if and only if the operator T has a fixed point in E. Using elementary computations we prove that $|G(t,s)| \leq 10$. Let $u, v \in E$, applying (3.1) we get

$$\begin{aligned} |Tu(t)-Tv(t)| &\leq \frac{1}{3}\int_0^1 |G(t,s)|\,|f(s,u(s))-f(s,v(s))|\, ds \\ &\leq \frac{10}{3}\int_0^1 g(s)\,|u(s)-v(s)|\, ds. \end{aligned}$$

Due to (3.2), we obtain $\|Tu-Tv\| < \|u-v\|$. Consequently T is a contraction, hence it has a unique fixed point which is the unique solution of the BVP (1.1)-(1.2). □

Now we give some existence results for the BVP (1.1)-(1.2).

Theorem 3.2. *Assume that the following hypotheses hold*

(B1) f *is an* L^1*-Carathéodory function, the map* $t \to f(t,0)$ *is continuous and* $f(t,0) \neq 0$, *for any* $t \in [0,1]$.

(B2) *There exist nonnegative functions* $h, k \in L^1([0,1], \mathbb{R}_+)$ *and* $0 < \alpha < 1$, *such that*

$$|f(t,x)| \leq k(t)|x|^{\alpha} + h(t), \quad (t,x) \in [0,1] \times \mathbb{R}. \tag{3.3}$$

Then the BVP (1.1)-(1.2) has at least one nontrivial solution $u^* \in E$ *and the set of its solutions is compact.*

Proof. To prove this Theorem, we apply Leray Schauder nonlinear alternative. First we prove that T is completely continuous.

(i) T is continuous. Let (u_n) be a sequence that converges to u in E. Using the fact that $|G(t,s)| \leq 10$, we obtain

$$|Tu_n(t) - Tu(t)| \leq \frac{10}{3} \int_0^1 |f(s, u_n(s)) - f(s, u(s))| \, ds.$$

Moreover

$$\|Tu_n - Tu\| \leq \frac{10}{3} \|f(., u_n(.)) - f(, u(.))\|.$$

(ii) T maps bounded sets into relatively compact sets in E. Let $B_r = \{u \in E; \|u\| \leq r\}$ be a bounded subset.

(a) For any $u \in B_r$ and $t \in [0,1]$

$$\begin{aligned} |Tu(t)| &\leq \frac{10}{3} \int_0^1 (k(s)|u(s)|^{\alpha} + h(s)) \, ds \\ &\leq \frac{10}{3} r^{\alpha} \int_0^1 k(s) ds + \frac{10}{3} \int_0^1 h(s) ds, \end{aligned}$$

then $T(B_r)$ is uniformly bounded.

(b) $T(B_r)$ is equicontinuous. Indeed for all $t_1, t_2 \in [0,1], u \in B_r$, we have from (3.3) that

$$\begin{aligned} &|Tu(t_1) - Tu(t_2)| \\ &\leq \frac{10}{3} \int_0^1 |G(t_1,s) - G(t_2,s)| (k(s)|u(s)|^{\alpha} + h(s)) \, ds \\ &\leq \frac{10r^{\alpha}}{3} \int_0^1 |G(t_1,s) - G(t_2,s)| k(s) ds + \frac{10}{3} \int_0^1 |G(t_1,s) - G(t_2,s)| h(s) ds, \end{aligned}$$

when $t_1 \to t_2$, then $|Tu(t_1) - Tu(t_2)|$ tends to 0. Consequently $T(B_r)$ is equicontinuous. Then T is completely continuous operator.

Now we apply Leray Schauder nonlinear alternative.

Let $m = \left(\frac{10}{3}\int_0^1 k(s)ds + \frac{10}{3}\int_0^1 h(s)ds\right)^{\frac{1}{1-\alpha}}$, $M = \max(1, m)$, $0 < \lambda < 1$, $\Omega = \{u \in E : \|u\| < M + 1\}$, $u \in \partial\Omega$, such that $u = \lambda Tu$. Using (3.3) we get

$$\begin{aligned} |u(t)| &= \lambda |Tu(t)| \leq \frac{10}{3}\int_0^1 \left(k(s)|u(s)|^\alpha + h(s)\right) ds \\ &\leq \frac{10}{3}\|u\|^\alpha \int_0^1 k(s)ds + \frac{10}{3}\int_0^1 h(s)ds \end{aligned}$$

so,

$$\|u\| \leq \frac{10}{3}\|u\|^\alpha \int_0^1 k(s)ds + \frac{10}{3}\int_0^1 h(s)ds.$$

If $\|u\| \geq 1$, then

$$\|u\| \leq \left(\frac{10}{3}\int_0^1 k(s)ds + \frac{10}{3}\int_0^1 h(s)ds\right)^{\frac{1}{1-\alpha}} = m. \tag{3.4}$$

Consequently $\|u\| \leq \max(1, m) = M$, then (3.4) contradicts the fact that $u \in \partial\Omega$. By Lemma 2.2 we conclude that the operator T has a fixed point $u^* \in \overline{\Omega}$ and then the BVP $(1.1) - (1.2)$ has a nontrivial solution $u^* \in E$.

Let Σ be the set of solutions, we shall prove that Σ is compact, for this, we apply Arzela-Ascoli Theorem. Let $\{u_n\}_{n\geq 1}$ be a sequence in Σ, using the same reasoning as above, we prove that the sequence $\{u_n\}_{n\geq 1}$ is bounded and equicontinuous, consequently there exists a uniformly convergent subsequence $\{u_{n'}\}_{n'\geq 1}$ of $\{u_n\}_{n\geq 1}$, such $u_{n'} \to u$.

Now we prove that Σ is closed. From the condition (B2) we have

$$|f(t, u_{n'})| \leq k(t)|u_{n'}|^\alpha + h(t) \leq k(t)m^\alpha + h(t), \quad (t, x) \in [0, 1] \times \mathbb{R}.$$

By Lebesgue Dominated Convergence Theorem and the assumption f is an L^1-Carathéodory function one can guaranty that

$u(t) = \lim u_n(t) = -\int_0^1 G(t, s) f(s, u(s))\, ds, \forall t \in [0, 1]$, hence $u \in \Sigma$ and consequently Σ is compact. $\square$

Theorem 3.3. *Assume that the following hypotheses hold:*

(C1) *f is an L^1-Carathéodory function, the map $t \mapsto f(t, 0)$ is continuous and $f(t, 0) \neq 0$, for any $t \in [0, 1]$.*

(C2) *There exist nonnegative functions $h, k \in L^1([0, 1], \mathbb{R}_+)$ such that*

$$\begin{aligned} &|f(t, x)| \leq k(t)|x| + h(t), \quad (t, x) \in [0, 1] \times \mathbb{R}, \\ &\int_0^1 k(s)ds < \frac{3}{10}. \end{aligned}$$

Then the BVP (1.1)-(1.2) has at least one nontrivial solution $u^* \in E$ and the set of its solutions is compact.

Proof. From the proof of Theorem 3.2, we know that T is completely continuous. Let $M_1 = \dfrac{10\int_0^1 h(s)ds}{3-10\int_0^1 k(s)ds}$, $\Omega = \{u \in E : \|u\| < M_1 + 1\}$, $u \in \partial\Omega$, $0 < \lambda < 1$, such that $u(t) = \lambda Tu(t)$. From hypotheses (C1) and (C2), we have

$$\|u\| \leq \frac{10}{3}\|u\| \int_0^1 k(s)ds + \frac{10}{3}\int_0^1 h(s)ds,$$

consequently $\|u\| \leq M_1$, this contradicts the fact that $u \in \partial\Omega$. By Lemma 2.2 we conclude that the operator T has a fixed point $u^* \in \overline{\Omega}$ and then the BVP (1.1)-(1.2) has a nontrivial solution $u^* \in E$.

The proof of the compacity of the set of solutions is similar to the case $\alpha \in [0,1[$. □

Theorem 3.4. *Assume that the following hypotheses hold:*

(E1) *f is an L^1-Carathéodory function, the map $t \to f(t,0)$ is continuous and $f(t,0) \neq 0$, for any $t \in [0,1]$.*

(E2) *There exist nonnegative functions $h, k \in L^1([0,1], \mathbb{R}_+)$ and $\alpha > 1$ such that*

$$|f(t,x)| \leq k(t)|x|^\alpha + h(t), \quad (t,x) \in [0,1] \times \mathbb{R},$$
$$M = \frac{10}{3}\int_0^1 k(s)ds < \frac{1}{2}, N = \frac{10}{3}\int_0^1 h(s)ds < \frac{1}{2}.$$

Then the BVP (1.1)-(1.2) has at least one nontrivial solution $u^* \in E$ and the set of its solutions is compact.

Proof. Let $m = \left(\dfrac{N}{M}\right)^{1/n}$, where n is the entire part of α. Setting

$$\Omega = \{u \in E : \|u\| < m\},$$

$u \in \partial\Omega$, $\lambda > 1$ such that $Tu(t) = \lambda u(t)$ and using the same arguments as previous, we get

$$\lambda\|u\| \leq \frac{10}{3}\|u\|^\alpha \int_0^1 k(s)ds + \frac{10}{3}\int_0^1 h(s)ds = \|u\|^\alpha M + N$$

that implies $\lambda m \leq m^\alpha M + N$, then $\lambda \leq M^{((n+1)-\alpha)/n} N^{(\alpha-1)/n} + M^{1/n} N^{1-1/n}$. From hypotheses we know that $n \leq \alpha < n+1$, $M < 1/2$ and $N < 1/2$ so $M^{((n+1)-\alpha)/n} < (1/2)^{((n+1)-\alpha)/n}$, $N^{(\alpha-1)/n} < (1/2)^{(\alpha-1)/n}$, $M^{1/n} < (1/2)^{1/n}$ and $N^{1-1/n} < (1/2)^{1-1/n}$, consequently $\lambda < 1$, this contradicts the fact that $\lambda > 1$. By Lemma 2.2 we conclude that the operator T has a fixed point $u^* \in \overline{\Omega}$ then the BVP (P_1) has a nontrivial solution $u^* \in E$.

The proof of the compacity of the set of solutions is similar to the case $\alpha \in [0,1[$. □

Example 3.5. Consider the following boundary value problem

$$\begin{cases} u'' + u\frac{\sin^3 t}{4+2\cos t}e^t = 0 & , \quad 0 < t < 1, \\ u(0) = \int_0^1 u(t)\,dt & , \quad u(1) = \int_0^1 tu(t)\,dt. \end{cases} \tag{3.5}$$

One can check that $|f(t,x) - f(t,y)| \leq g(t)\,|x-y|\,, \forall x, y \in \mathbb{R},\ t \in [0,1]$, where $g(t) = \frac{\sin^3 t}{4+2\cos 1}$ and $\int_0^1 \frac{\sin^3 t}{4+2\cos 1}dt = 0.03522 < \frac{3}{10}$. From Theorem 3.1, the BVP (3.5) has a unique solution u in E.

Example 3.6. Consider the following boundary value problem

$$\begin{cases} u'' + \frac{1}{3}u^{\frac{1}{4}}\left(t^3 + \cos t\right) + \arcsin t = 0 & , \quad 0 < t < 1, \\ u(0) = \int_0^1 u(t)\,dt & , \quad u(1) = \int_0^1 tu(t)\,dt. \end{cases} \tag{3.6}$$

We have $f(t,u) = \frac{1}{3}\left(u^{\frac{1}{4}}\right)\left(t^3 + \cos t\right) + \arcsin t$, $f(t,0) \neq 0$, $0 < \alpha = \frac{1}{4} < 1$ and

$$|f(t,u)| \leq \frac{1}{3}\left(t^3 + \cos t\right)|u|^{\frac{1}{4}} + \arcsin t = k(t)\,|u|^{\frac{1}{4}} + h(t).$$

Using Theorem 3.2, we conclude that the BVP (3.6) has at least one nontrivial solution u^* in E.

Example 3.7. Consider the following boundary value problem

$$\begin{cases} u'' + \frac{3u^4}{10(1+u^2)}\sin t + e^{-2t}\cos(1+t) = 0 & , \quad 0 < t < 1 \\ u(0) = \int_0^1 u(t)\,dt & , \quad u(1) = \int_0^1 tu(t)\,dt. \end{cases} \tag{3.7}$$

We have $f(t,u) = \frac{3u^4}{10(1+u^3)}\sin t + e^{-2t}\cos(1+t)$, so $|f(t,u)| \leq k(t)\,|u|^2 + h(t)$, $\alpha = 2$, $k(t) = \frac{3\sin t}{10}$, $h(t) = e^{-2t}\cos(1+t)$. $M = \int_0^1 \sin s ds = 0.45970 < \frac{1}{2}$ and $N = \frac{10}{3}\int_0^1 e^{-2s}\cos(1+s)ds = 0.316\,55 < \frac{1}{2}$. Hence, from Theorem 3.4, we deduce that the BVP (3.7) has at least one nontrivial solution u^* in E.

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