Dissatisfaction levels of earliness and tardiness durations by relaxing common due date on single machine scheduling problems

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Abstract —*This paper investigates single machine earliness/tardiness problem considering the decision maker's tolerances for earliness and tardiness durations in case of a restrictive common due date. In many classical or basic earliness/tardiness problems, due dates are accepted as deterministic or rigid numbers. In this paper, common due date in a single machine scheduling problem is relaxed with lower and upper bounds and these bounds are used for illustrating the decision maker's tolerances or satisfaction levels by using fuzzy sets. As a complementary set of satisfaction levels, dissatisfaction levels can be encoded with fuzzy sets. Then, this paper uses dissatisfaction levels in order to introduce a new objective criterion that minimizes the products of earliness and tardiness durations with dissatisfaction levels.*

Keywords: Earliness, tardiness, single machine, scheduling, fuzzy sets, dissatisfaction levels, common due date
Mathematics Subject Classification: 65K05, 90C70.

1 Introduction

Earliness/tardiness (E/T) problems are significant for the companies having the Just-in-Time philosophy. Determining earliness and tardiness weights or penalties may not always be an easy task. The decision maker (DM) uses E/T weights in order to show his/her biased importance factors. In some cases, DM may use real penalty costs in currencies as important factors for scheduling problems. In this paper, dynamic weights for E/T durations are introduced as decision variables in a single machine E/T problem with a common due date by using fuzzy membership functions of relaxed common due date with upper and lower bounds. Arık and Toksarı [1] considered a multi-objective fuzzy parallel machine scheduling problem under effects of fuzzy learning and deterioration where the objectives are to minimize earliness cost, to minimize tardiness cost and to minimize the cost of setting due dates. In their study, due dates are in form of fuzzy numbers as decision variables. They proposed a Local Search algorithm.

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Jayanthi et al. [2] investigated a single machine scheduling problem with trapezoidal processing times and triangular due dates. In order to solve their problem where the objective is to minimize total weighted earliness and tardiness costs, they proposed a Quantum Particle Swarm Optimization algorithm. Niroomand et al. [3] considered a single machine scheduling problem with a fuzzy common due date by proposing hybrid greedy algorithms in order to minimize fuzzy earliness/tardiness costs. Geng et al. [4] investigated flow shop scheduling problems for earliness/tardiness minimization with uncertain processing times and distinct due windows. They proposed a Scatter Search Based Particle Swarm Optimization. Kır and Yazgan [5] used Fuzzy Axiomatic Design to determine earliness and tardiness penalty costs in a single machine scheduling problem where dairy products are considered. They proposed a two-stage solution method that firstly creates an initial solution with Tabu Search and then improves that initial solution with Genetic Algorithm. Li and Zhang [6] considered single machine due date assignment problems where the objective is to minimize the possibilistic mean of total E/T cost with fuzzy processing times and precedence constraints. Behnamian and Fatemi Ghomi [7] considered a bi-objective hybrid flow shop scheduling problems with fuzzy tasks' operation times, due dates and sequence-dependent setup times. The objectives in their problem are to minimize makespan and the total sum of E/T cost simultaneously. In the study of Engin et al. [8], fuzzy sets were used to encode uncertainties in processing times and due dates in a fuzzy job shop scheduling problem with availability constraints. They proposed a Scatter Search (SS) method to solve these problems. Yan et al. [9] investigated flow shop scheduling problems with fuzzy processing times and due windows in order to minimize total weighted E/T cost by proposing a hybrid algorithm consist of quantum genetic algorithm and particle swarm optimization. Xu and Gu [10] considered a zero-wait multiproduct scheduling with due dates under uncertainty, where the total weighted earliness/tardiness penalty is to be minimized. Li et al. [11] investigated single machine scheduling problems where the objective is to minimize total weighted possibilistic mean of E/T cost with fuzzy processing time and they investigated how to predict due dates of jobs. Lu et al. [12] studied a multi-objective scheduling problem for a single batch-processing machine with non-identical job sizes with fuzzy processing times and fuzzy due dates. The objectives in their study are to minimize cost combination of makespan, earliness/tardiness penalties and processing cost. Wang and Shi [13] considered a multi-objective job shop scheduling problem with fuzzy processing times and due windows for E/T performance criterion and they proposed a genetic algorithm for their problem.

Wang et al. [14] proposed different genetic algorithms including different crossover operator a for single machine E/T problem with fuzzy processing times. Wang et al. [14] investigated a multi-objective job shop scheduling problem with fuzzy processing times and flexible due dates by proposing a genetic simulated annealing algorithm. Wu [15] considered fuzzy earliness and fuzzy tardiness in scheduling problems by using extension principle of fuzzy set theory for triangular fuzzy processing times and trapezoidal fuzzy due dates. Li et al. [16] proposed a due date assignment problem with fuzzy processing times and precedence constraints. They showed that their problem can be polynomially solvable without precedence constraints and the problem with precedence constraints is NP-hard. Lai and Wu [17] investigated fuzzy earliness and tardiness by using the concept of possibility and necessity measures in fuzzy set theory with fuzzy processing times and fuzzy due dates. They considered lots of E/T combinations in view of possibility and necessity measures and proposed a genetic algorithm approach for these different E/T combinations.

Dong [18] considered a fuzzy single machine scheduling problem with fuzzy processing times in order to minimize weighted E/T and resource costs. Dong [19] proposed a two-stage solution approach for the problem. Lam and Cai [20] considered a single machine weighted E/T problem with a fuzzy triangular common due date and they introduced job dependent weights for their objective function. Furthermore, they stated an optimal job sequence must be V-shaped in terms of weighted processing time when the problem is agreeably weighted. In another study of Lam and Cai [21], they used genetic algorithm and fuzzy distance function for solving a single machine E/T problem with fuzzy due dates. Murata et al. [22] examined the characteristic features of multi-objective scheduling problems formulated with the concept of fuzzy due-date. Ishibuchi et al. [23] investigated fuzzy scheduling problems and conventional scheduling problems with earliness and tardiness penalties. They showed the relations between fuzzy scheduling problems and conventional scheduling problems by solving them with a proposed genetic algorithm. Some of other recent papers about fuzziness in scheduling are conducted by Toksarı and Arık [24], Arık and Toksarı [25], Jia et al. [26], Golneshini and Fazlollahtabar [27], Arık [28], Saraçoğlu and Süer [29], Liao and Su [30], Liu et al. [31] and Arık and Toksarı [32].

2 Problem formulation

The earliness penalties or costs of early jobs in scheduling problems are considered as deterministic in scheduling problems. With classical set theory; if a job completed before its due date, then this job belongs to the set of early jobs, else this job is not a member of the set of early jobs. Belonging to the set of early jobs is not a desired situation and this does not satisfy DM. Equation (1) shows the classical membership function $\mu_{E_i}(C_i)\colon \mathbb{R}^+ \to [0,1]$ of DM's satisfaction level for an early job with respect to completion time of that job considering a common due date for all jobs.

$$\mu_{E_i}(C_i) = \begin{cases} 1, & if\ C_i \geq d, \\ 0, & if\ C_i < d, \end{cases} \tag{1}$$

where C_i is the completion time of job i and d is common due date for all jobs in the scheduling environment. Figure 1 illustrates classical membership function in Equation (1).

The early job's satisfaction level in Equation (1) is a rigid number. Like the most cases of the real life, this rigid approach for earliness may be tolerated in view of DM's tolerance degree or satisfaction degree to an unacceptable situation. In order to evaluate DM's satisfaction degree, common due date d may be relaxed with a lower bound $\underline{d_i}$ of common due date. Thus, if job i is completed on the interval between $\underline{d_i}$ and d, DM may not be fully satisfied because of this earliness amount but he/she may tolerate this earliness amount.

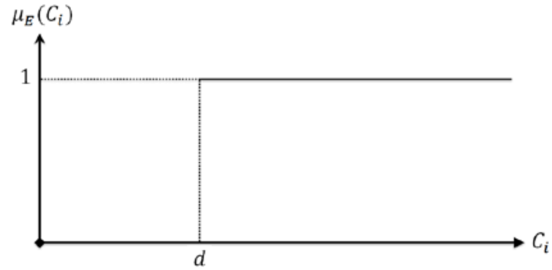

Figure 1. Classical earliness membership function

The degree of DM's satisfaction with respect to completion time of that job considering a relaxed common due date with a lower bound $\underline{d_i}$ for job i can be encoded with fuzzy sets as illustrated in Figure 2 and Equation (2).

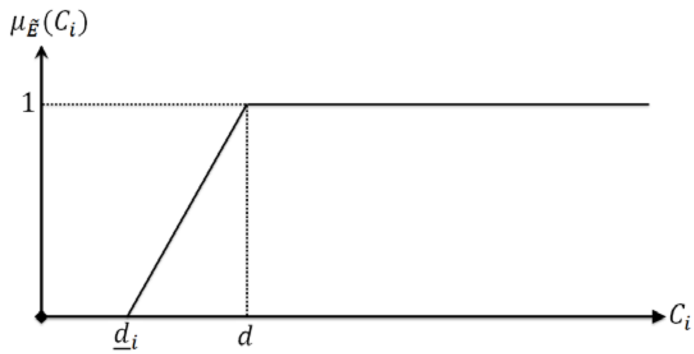

Figure 2: Fuzzy earliness membership function

$$\mu_{\tilde{E}_i}(C_i) = \begin{cases} 1, & if\ C_i \geq d, \\ \frac{C_i - \underline{d_i}}{d - \underline{d_i}}, & if\ \underline{d_i} \leq C_i < d, \\ 0, & if\ C_i < \underline{d_i}, \end{cases} \tag{2}$$

where $\mu_{\tilde{E}_i}(C_i): \mathbb{R}^+ \rightarrow [0,1]$ is the membership function of DM's satisfaction level for earliness with respect to completion time of that job considering a relaxed common due date with a lower bound $\underline{d_i}$ for job i. With classical scheduling triple notation, $1|d_i = d|\sum \alpha_i E_i$ denotes a single machine scheduling problem where the objective is to minimize total weighted earliness costs for all jobs by considering jobs' weight coefficients α_i. The weight coefficients for earliness or tardiness (E/T) are mostly assumed as deterministic values. This paper proposes dynamic weight coefficients for scheduling problems, especially for E/T problems. Furthermore, dissatisfaction levels of jobs for earliness of tardiness are proposed as dynamic penalty weights in this paper. The dissatisfaction level $\tilde{\alpha}_i$ of DM for any early job i is a complementary fuzzy set of fuzzy satisfaction level $\tilde{E}_i$ such as $\mu_{\tilde{\alpha}_i}(C_i) = 1 - \mu_{\tilde{E}_i}(C_i)$. Satisfaction or dissatisfaction level is on the closed interval between 0 and 1. The complementary part of satisfaction level can be called as dissatisfaction level. Figure (3) and Equation (3) show the membership function of DM's dissatisfaction level $\mu_{\tilde{\alpha}_i}(C_i): \mathbb{R}^+ \rightarrow [0,1]$ for earliness with respect to completion time of that job considering a relaxed common due date with a lower bound $\underline{d}_i$ for job i.

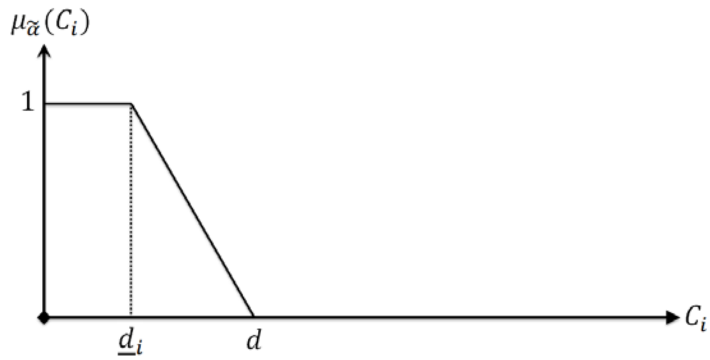

Figure 3: Fuzzy earliness weight membership function

$$\mu_{\tilde{\alpha}_i}(C_i) = \begin{cases} 0, & if\ C_i \geq d, \\ \frac{d-C_i}{d-\underline{d}_i}, & if\ \underline{d}_i \leq C_i < d, \\ 1, & if\ C_i < \underline{d}_i\ . \end{cases} \quad (3)$$

By following the same approach, the classic tardiness classical membership function $\mu_{T_i}(C_i): \mathbb{R}^+ \rightarrow [0,1]$ of DM's satisfaction level for an tardy job with respect to completion time of that job considering a common due date for all jobs can be illustrated as in Figure 4 and Equation (4).

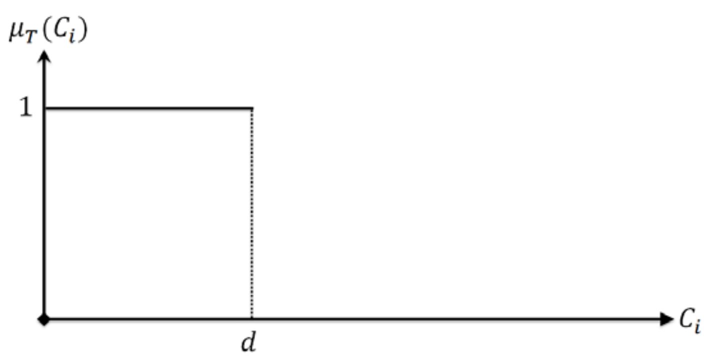

Figure 4: Classical tardiness membership function

$$\mu_{T_i}(C_i) = \begin{cases} 0, & if\ C_i > d, \\ 1, & if\ C_i \leq d, \end{cases} \quad (4)$$

equation (4) can be relaxed with an upper bound $\overline{d}_i$ of common due date for any tardy job i as seen in Figure 5 and Equation (5).

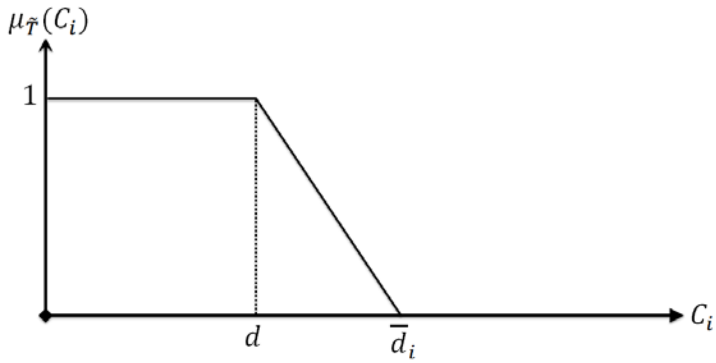

Figure 5: Fuzzy tardiness membership function

$$\mu_{\tilde{T}}(C_i) = \begin{cases} 1, & if\ d \leq C_i, \\ \frac{C_i - d}{\overline{d}_i - d}, & if\ \overline{d}_i \geq C_i > d, \\ 0, & if\ \overline{d}_i < C_i, \end{cases} \tag{5}$$

with classical scheduling triple notation, $1|d_i = d|\sum \beta_i T_i$ denotes a single machine scheduling problem where the objective is to minimize total weighted tardiness costs for all jobs by considering jobs' weight coefficients β_i. The complementary set of the satisfaction level $\tilde{T}_i$ for tardiness is dissatisfaction level $\tilde{\beta}_i$ with a membership function such as $\mu_{\tilde{\beta}_i}(C_i) = 1 - \mu_{\tilde{T}_i}(C_i)$ as shown in Figure 6 and Equation (6).

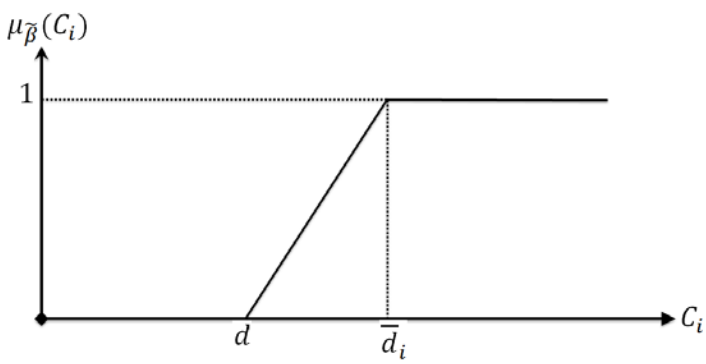

Figure 6: Fuzzy tardiness weight membership function

$$\mu_{\tilde{\beta}_i}(C_i) = \begin{cases} 0, & if\ d \leq C_i, \\ \frac{\overline{d}_i - C_i}{\overline{d}_i - d}, & if\ \overline{d}_i \geq C_i > d, \\ 1, & if\ \overline{d}_i < C_i. \end{cases} \tag{6}$$

The weighted single machine E/T problem $1|d_i = d|\sum \alpha_i E_i + \sum \beta_i T_i$ can be expressed as $1|\underline{d_i} < d < \overline{d_i}|\sum \mu_{\tilde{\alpha}_i}(C_i)E_i + \sum \mu_{\tilde{\beta}_i}(C_i)T_i$ for minimizing earliness and tardiness amounts and dissatisfaction of DM, simultaneously. This new performance criterion aims to minimize the sum of the products of earliness/tardiness durations and dissatisfaction levels of them in view of DM.

3 Mixed integer non-linear mathematical model

In this section of the paper, a mixed integer non-linear mathematical programming (MINLP) model is proposed. $\mu_{\tilde{\alpha}_i}(C_i)$ and $\sum \mu_{\tilde{\beta}_i}(C_i)$ are piecewise linear functions, $T_i = \max(0,\ C_i - d)$ and $E_i = \max(0,\ d - C_i)$. $\sum \mu_{\tilde{\alpha}}(C_i)E_i + \sum \mu_{\tilde{\beta}_i}(C_i)T_i$ is a non-linear objective function. Each of $\mu_{\tilde{\alpha}_i}(C_i)$ and $\sum \mu_{\tilde{\beta}_i}(C_i)$ functions has three intervals on the real axis as shown in Figures 7 and 8. In order to simplify mathematical model, these intervals are used in the proposed MINLP. The completion time C_i can be placed on any of these intervals in Figure 7 and Figure 8.

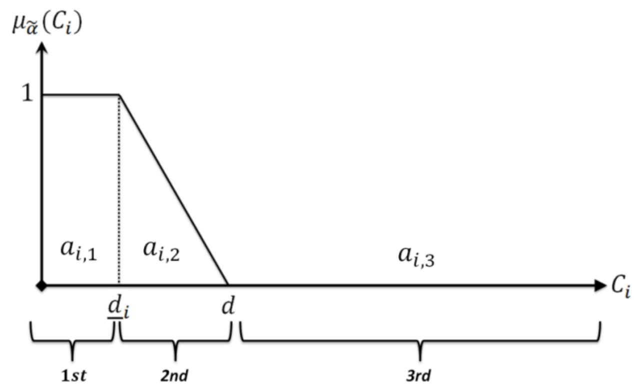

Figure 7: Intervals of $\mu_{\tilde{\alpha}}(C_i)$ functions on the real axis

In case of earliness, C_i can be represented with $A_{i,k}$, where k =1,2,3 and k is index for intervals in Figure 7. In order to determine the interval that C_i is on, assignment variables $a_{i,k}$ can be used as follows:

$$C_i = \sum_{k=1}^{3} a_{i,k} A_{i,k} \ \ \forall i \tag{7}$$

$$\sum_{k=1}^{k=3} a_{i,k} = 1 \ \forall i \tag{8}$$

$$A_{i,1} \leq \underline{d}_i a_{i,1} \ \forall i \tag{9}$$

$$\underline{d}_i a_{i,2} \leq A_{i,2} \leq d a_{i,2} \ \forall i \tag{10}$$

$$A_{i,3} \geq d a_{i,3} \ \forall i \tag{11}$$

where $a_{i,k} \in \{0,1\}$ and $A_{i,k} \geq 0 \ \forall\ i, k$. $\mu_{\tilde{\alpha}_i}(C_i)$ value is simply obtained by using $a_{i,k}$ decision variables as follows:

$$\mu_{\tilde{\alpha}_i}(C_i) = a_{i,1} * 1 + a_{i,2} * \frac{d-C_i}{d-\underline{d}_i} + a_{i,3} * 0 \; \forall i. \tag{12}$$

Equation (12) can be written as follows:

$$\mu_{\tilde{\alpha}_i}(C_i) = a_{i,1} + a_{i,2} * \frac{d-C_i}{d-\underline{d}_i} \; \forall i. \tag{13}$$

Equation (13) can be simply regulated as $\mu_{\tilde{\alpha}_i}(C_i) = \min(1, E_i/P_i h_i)$.

In case of tardiness, C_i can be represented with $B_{i,t}$, where t =1,2,3 and t is index for intervals in Figure 8. In order to determine the interval that C_i is on, assignment variables $b_{i,t}$ can be used as follows:

$$C_i = \sum_{t=1}^{3} b_{i,t} B_{i,t} \; \forall i \tag{14}$$

$$\sum_{t=1}^{t=3} b_{i,t} = 1 \; \forall i \tag{15}$$

$$B_{i,1} \leq d b_{i,1} \; \forall i \tag{16}$$

$$d b_{i,2} \leq B_{i,2} \leq \overline{d_i} b_{i,2} \; \forall i \tag{17}$$

$$B_{i,3} \geq \overline{d_i} b_{i,3} \; \forall i \tag{18}$$

where $b_{i,t} \in \{0,1\}$ and $B_{i,t} \geq 0 \; \forall \; i, t$. $\mu_{\tilde{\beta}_i}(C_i)$ value is simply obtained by using $b_{i,t}$ decision variables as follows:

$$\mu_{\tilde{\beta}_i}(C_i) = b_{i,1} * 0 + b_{i,2} * \frac{\overline{d}_i - C_i}{\overline{d}_i - d} + b_{i,3} * 1 \; \forall i. \tag{19}$$

Equation (19) can be written as follows:

$$\mu_{\tilde{\beta}_i}(C_i) = b_{i,2} * \frac{\overline{d}_i - C_i}{\overline{d}_i - d} + b_{i,3} \; \forall i. \tag{20}$$

Equation (20) can be simply regulated as $\mu_{\tilde{\beta}_i}(C_i) = \min(1, T_i/P_i h_i)$.

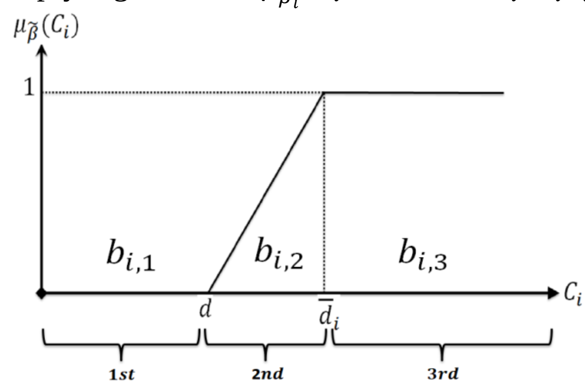

Figure 8: Intervals of $\mu_{\tilde{\beta}_i}(C_i)$ functions on the real axis

The mathematical model can be structured by using Equations (7-11) and Equations (14-18) as follows:

Indices:
i: index for jobs ($i = 1,2, \dots, n$)
r: index for position numbers ($i = 1,2, \dots, n$)
k: index for earliness time interval ($k = 1,2,3$)
t: index for tardiness time interval ($t = 1,2,3$)

Parameters:
P_i: *processing time of job i*
h_i: *relaxing factor for upper and lower bounds of common due date for job i*
h: *restrictive factor for common due date*

Decision Variables:
C_i: *completion time of job i*
E_i: *earliness time of job i*
T_i: *tardiness time of job i*
C_r: *completion time of the job assigned on position r*
P_r: *processing time of the job assigned on position r*
$A_{i,k}$: *completion time of job i on k^{th} earliness interval*
$B_{i,t}$: *completion time of job i on t^{th} tardiness interval*
d: *common due date for all jobs*
$\underline{d}_i$: *lower bound of common due date for job i*
$\overline{d}_i$: *upper bound of common due date for job i*
$X_{i,r}$: *if job i is assigned on position r on the machine, $X_{i,r} = 1$; otherwise, $X_{i,r} = 0$*
$a_{i,k}$: *if C_i is on k^{th} earliness interval, $a_{i,k} = 1$; otherwise, $a_{i,k} = 0$*
$b_{i,t}$: *if C_i is on t^{th} tardiness interval, $b_{i,k} = 1$; otherwise, $b_{i,k} = 0$*
$\mu_{\tilde{\beta}_i}(C_i)$: *dissatisfaction level of DM in case of tardiness for job i*
$\mu_{\tilde{\alpha}_i}(C_i)$: *dissatisfaction level of DM in case of earliness for job i*

Objective Function:

$$Min\ \sum_i^n \mu_{\tilde{\alpha}_i}(C_i)E_i + \sum_i^n \mu_{\tilde{\beta}_i}(C_i)T_i \tag{21}$$

Subject to:

$$d = h\sum_{i=1}^n P_i \tag{22}$$

$$\underline{d}_i = d - P_i h_i \tag{23}$$

$$\overline{d}_i = d + P_i h_i \tag{24}$$

$$\mu_{\tilde{\alpha}_i}(C_i) = a_{i,1} + a_{i,2}\frac{d - C_i}{d - \underline{d}_i} \quad \forall i \tag{25}$$

$$\mu_{\widetilde{\beta}_i}(C_i) = b_{i,2}\frac{\overline{d}_i - C_i}{\overline{d}_i - d} + b_{i,3}\ \forall i \tag{26}$$

$$C_i \geq \sum_{k=1}^{3} a_{i,k} A_{i,k}\ \ \forall i \tag{27}$$

$$\sum_{k=1}^{k=3} a_{i,k} = 1\ \forall i \tag{28}$$

$$A_{i,1} \leq \underline{d_i} a_{i,1}\ \forall i \tag{29}$$

$$\underline{d_i} a_{i,2} \leq A_{i,2}\ \ \forall i \tag{30}$$

$$A_{i,2} \leq d a_{i,2}\ \forall i \tag{31}$$

$$A_{i,3} \geq d a_{i,3}\ \forall i \tag{32}$$

$$C_i \geq \sum_{t=1}^{3} b_{i,t} B_{i,t}\ \ \forall i \tag{33}$$

$$\sum_{t=1}^{t=3} b_{i,t} = 1\ \forall i \tag{34}$$

$$B_{i,1} \leq d b_{i,1}\ \forall i \tag{35}$$

$$d b_{i,2} \leq B_{i,2}\ \ \forall i \tag{36}$$

$$B_{i,2} \leq \overline{d_i} b_{i,2}\ \forall i \tag{37}$$

$$B_{i,3} \geq \overline{d_i} b_{i,3}\ \forall i \tag{38}$$

$$\sum_{r=1}^{n} X_{i,r} = 1\ \forall i \tag{39}$$

$$\sum_{i=1}^{n} X_{i,r} = 1\ \forall r \tag{40}$$

$$C_i + E_i - T_i = d\ \forall i \tag{41}$$

$$C_i = \sum_{r=1}^{n} X_{i,r}\, C_r\ \forall i \tag{42}$$

$$C_r = C_{r-1} + P_r\ \forall r \tag{43}$$

$$P_r = \sum_{i=1}^{n} X_{i,r}\, P_i\ \forall r \tag{44}$$

$$C_{r=0} = 0 \tag{45}$$

$$\mu_{\widetilde{\alpha}_i}(C_i) \leq 1 \tag{46}$$

$$\mu_{\tilde{\beta}_i}(C_i) \leq 1 \tag{47}$$

$$C_r, P_r \geq 0 \ \forall r \tag{48}$$

$$C_i \geq 0 \ \forall i \tag{49}$$

$$A_{i,k} \geq 0 \ \forall \ i, k \tag{50}$$

$$B_{i,t} \geq 0 \ \forall \ i, t \tag{51}$$

$$a_{i,k} \ \in \{0,1\} \ \forall \ i, k \tag{52}$$

$$b_{i,t} \ \in \{0,1\} \ \forall \ i, t \tag{53}$$

$$X_{i,r} \ \in \{0,1\} \ \forall \ i, r \tag{54}$$

Objective function (21) is to minimize the products of earliness/tardiness with dissatisfaction levels of DM simultaneously. Constraint (22) shows that common due date d is equal to the product of the sum of processing times with a restrictive factor that is predetermined by DM. Constraints (23-24) shows that upper and lower bounds of common due date for job i are relaxed with the same amount that is equal to the product of processing time of job i with a predetermined relaxing factor h_i. Constraints (25-26) are dissatisfaction levels of DM for earliness and tardiness, respectively. These constraints are introduced in Equations (13-20) previously. Constraints (27-32) are to determine the earliness interval where C_i is placed on and these constraints are introduced in Equations (7-11). Constraints (33-38) are to determine the tardiness interval where C_i is placed on and these constraints are introduced in Equations (14-18). Constraint (39) assures that only one job can be assigned to any position of the machine. Constraint (40) guarantees that only one position number can be used to assign a job. Constraint (41) shows that completion time, earliness duration and tardiness duration must be balanced with the common due date. Constraint (42) is decision variable transformation between completion times that are dependent on job index and position index, respectively. Constraint (43) shows that completion time of the job on position r is equal to sum of previous position's completion time and processing time of the job on position r. Constraint (44) is to determine which job is assigned to position r. Constraint (45) assures that the machine is ready to process jobs at the beginning and all jobs have same release date. Constraints (46-47) assure that dissatisfaction levels are not more than 1. Constraints (48-54) define domains of decision variables.

4 Numerical example

In this section, a numerical example for the proposed problem is given for the readers. The numerical example in this section has 10 jobs that are ready to be processed on a single machine. Processing times of jobs are in Table 1. Preemption is not allowed and ready times of all jobs are equal to zero. Each job has same relaxing factor h_i=0.5. The restrictive factor h for common due date are predetermined by DM. For different restrictive factors between 0.1 and 1.5 by incrementing h with 0.1, the problem is solved and solutions of the problem for these restrictive factors are given in Table 2 and Figure 9. Solutions were obtained via Dicopt Solver in Gams 21.6 software.

Table 1: Processing times of the numerical example

i	1	2	3	4	5	6	7	8	9	10
P_i	5	6	16	8	9	16	10	12	23	11

Table 2: Solutions of the numerical example for different h levels

Restrictive factor h	**Common due date d**	**The optimal sequence**	**Objective Function**
0.1	11.6	1,2,4,5,7,10,8,3,6,9	394.920
0.2	23.2	8,5,2,4,7,1,10,6,3,9	330.676
0.3	34.8	3,6,1,5,2,10,7,4,8,9	302.116
0.4	46.4	3,1,5,2,4,7,10,8,6,9	257.040
0.5	58.0	6,1,3,8,10,2,7,4,9,5	268.727
0.6	69.6	9,3,5,6,4,2,1,8,7,10	233.160
0.7	81.2	9,2,7,10,8,3,6,5,4,1	303.480
0.8	92.8	9,3,6,7,5,10,8,2,4,1	270.340
0.9	104.4	9,3,6,8,10,7,5,2,1,4	299.653
1.0	116.0	9,3,6,8,10,7,5,4,2,1	381.000
1.1	127.6	9,3,6,8,10,7,5,4,2,1	497.000
1.2	139.2	9,3,6,8,10,7,5,4,2,1	613.000
1.3	150.8	9,3,6,8,10,7,5,4,2,1	719.000
1.4	162.4	9,3,6,8,10,7,5,4,2,1	845.000
1.5	174.0	9,3,6,8,10,7,5,4,2,1	961.000

As seen in Table 2, while restrictive factor h is increasing and the problem is still restricted ($d < \sum P_i$), the sequence is changing and objective function values fluctuate because the common due date is increasing with restrictive factor. Increasing the common due date leads the schedule is changed because there is a similar v-shaped property for the problem. The v-shaped property presents a sequence where jobs are ordered in decreasing order of their weighted processing times until the common due date and then the remaining jobs are ordered in increasing order of their weighted processing times. This property is common for classical single machine weighted earliness/tardiness scheduling problems and as seen from Table 2, this property can be seen for $1|\underline{d_i} < d < \overline{d_i}|\sum \mu_{\tilde{\alpha}_i}(C_i)E_i + \sum \mu_{\tilde{\beta}_i}(C_i)T_i$ problem. While problem is a non-restricted ($d \geq \sum P_i$) and h is increasing, the sequence stays same and objective function values are increasing because of earliness.

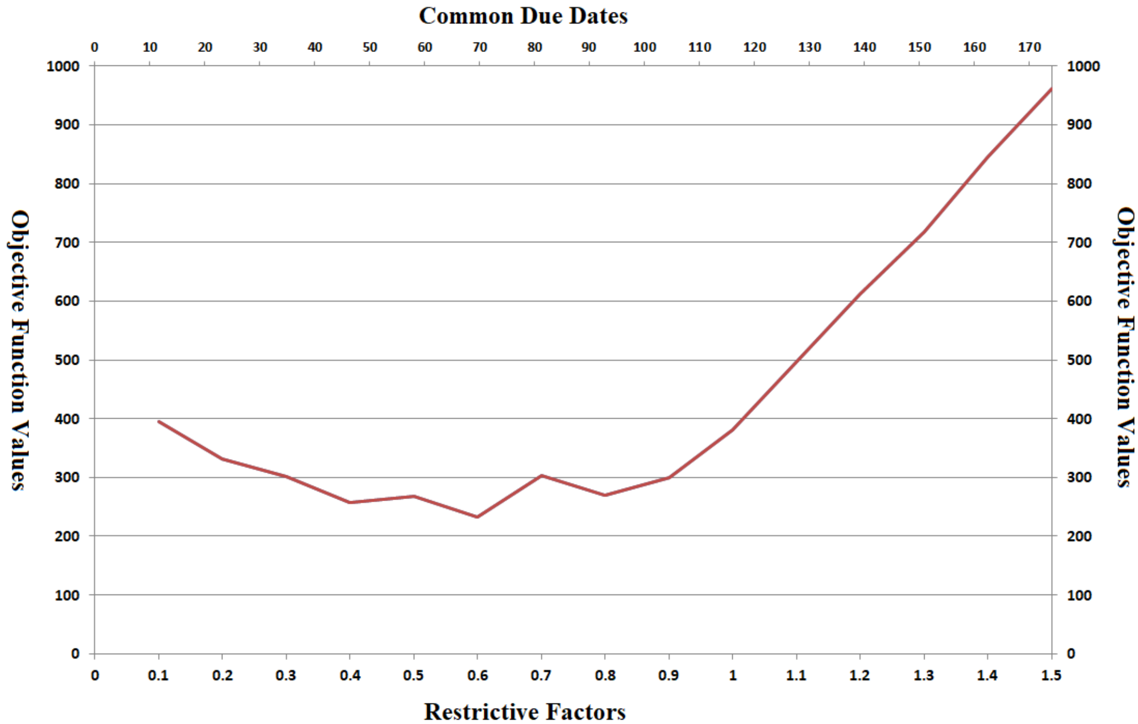

Figure 9: Solutions of the numerical example for different h levels and common due dates

5 Conclusion

In this paper, a new performance criterion $1|\underline{d_i} < d < \overline{d_i}|\sum \mu_{\tilde{\alpha}_i}(C_i)E_i + \sum \mu_{\tilde{\beta}_i}(C_i)T_i$ that minimizes the sum of the products of earliness/tardiness durations and dissatisfaction levels of them in view of DM is introduced. Dissatisfaction levels denote tolerances for earliness and tardiness durations considering a common due date. This approach may be used for different due dates of jobs. A numerical example for different restrictive levels is given in this paper. Single machine scheduling problems are basic of scheduling problems. Therefore, this approach can be used in more complex production systems that are mainly considered as a part of the companies having Just-in-time philosophy. The extending of this performance criterion for more complex scheduling environments and fuzzification of other parameters such as processing times can be considered in future researches.

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