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# A new filled function method with two parameters in a directional search

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**Abstract** — *In this paper, we investigate the solution of the problem of finding the global minimizer for the unconstrained objective function, for that, a new algorithm developed, this algorithm based on two steps. First, we transform the problem into a one-dimensional according to the number of directions. Second, we construct a new filled function at each direction in order to minimize the one-dimensional problem and then to find the global minimizer of the multi-dimensional function. We present the results of numerical experiments using test problems taken from literature studies. The experiment results indicate the effectiveness and accuracy of the purposed filled function methods.*

**Keywords:** global optimization, dimensional search, filled function method.
**Mathematics Subject Classification:** 80C50, 30A40, 90C26.

## 1 Introduction

Global optimizations are important tools for examining complicated function spaces such as these located in modern high-fidelity engineering models. Those models present increasingly accurate insights within system behaviours but are usually costly to estimate and difficult to search. While methods exist for determining global optimization problems there is yet room for improving faster, more reliable, and easier to implement algorithms [1]. The filled function method that firstly introduced via Ge(1987) [2,3], and then reviewed in various searches, is an efficient method for determining the global optimization approaches. It modifies the objective function as a filled function and then obtains a best local minimizer frequently by optimizing the filled function formed on the minimizer found previously [4]. The main purpose of this paper is to introduce and formalize a new filled function in two parameters. Firstly, we provide formal definitions and assumptions. Next, we offer and investigate the theoretical prosperities of the filled function and

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propose the solution algorithm. Finally, we report experimental results by applying the algorithm on several test problems to confirm the effectiveness of the new method.

## 2 Basic Concepts

Suppose the unconstrained problem:

$$min f(x) \quad s.t \quad x \in S, \quad S \subset \mathbb{R}^n \tag{1}$$

where $f : S \longrightarrow R$ is a continuously differentiable function. Now, we introduce the following definitions.

**Definition 2.1.** [5] A point $x^* \in S$ is said to be a global minimizer of the function $f$ on $S$ if:

$$f(x^*) \leq f(x)\, \forall x \in S,$$

and it is called a strict global minimizer point of $f$ on $S$ if:

$$f(x^*) < f(x) \qquad \forall x \in S, \quad x \neq x^*.$$

**Definition 2.2.** [5] A point $x_k^* \in S$ is said to be a local minimizer of $f$ on $S$ if there exists a neighborhood $B(x_k^*; \epsilon)$, with $\epsilon > 0$ such that

$$f(x_k^*) \leq f(x) \quad \forall x \in S \cap B(x_k^*; \epsilon)$$

and it is called a strict local minimizer of $f$ on $S$ if there exists a neighborhood $B(x_k^*; \epsilon)$, with $\epsilon > 0$ such that

$$f(x_k^*) < f(x) \qquad \forall x \in S \cap B(x_k^*; \epsilon), \quad x \neq x_k^*.$$

**Definition 2.3.** [5] A basin of $f(x)$ at an isolated minimizer $x_k^*$ is a connected domain $B(x_k^*)$ which contains $x_k^*$ and in which starting from any point the steepest descent trajectory of $f(x)$ converges to $x_k^*$. but outside which the steepest descent trajectory of $f(x)$ does not converge to $x_k^*$. A hill of $f(x)$ at $x_k^*$ is the basin of $-f(x)$ at its minimizer $x_k^*$, if $x_k^*$ is a maximizer of $f(x)$.

**Definition 2.4.** [2] Let $x_k^*$ is a current minimizer of $f$. Let $B(x_k^*)$ is the basin of $f$ at $x_k^*$ over $S$. A function $F : S \to \mathbb{R}$ is said to be a filled function of $f$ at $x_k^*$ if it satisfies the following properties:

- $x_k^*$ is a maximizer of $F$ and whole basin $B(x_k^*)$ of $f$ at $x_k^*$ over $S$ becomes a part of a hill of $F$;
- $F$ has no stationary points in any basin of $f$ higher than $B(x_k^*)$;
- If $f$ has a basin $B(x_{k+1}^*)$ at $x_{k+1}^*$ lower than $B(x_k^*)$, then there exists a point $x' \in B(x_{k+1}^*)$ is a minimizer of $F$.

The evolution of the filled functions supports the subsequent periods. The typical models of the filled functions as a first creation are the function (2) and (3) [6] which offered as following

$$F(x, a, \beta) = exp(-\frac{\|x - x_k^*\|}{\beta^2})\frac{1}{(a + f(x))} \tag{2}$$

$$D(x,a,\beta) = -[\beta^2 \ln(a + f(x)) + \|x - x_k^*\|^p] \tag{3}$$

These functions have a common feature, there are two flexible parameters $a$ and $\beta$. However, the task of modifying these parameters is extremely challenging. Due to this restriction, the next creation filled functions were introduced which have only a single parameter. For instance, the function introduced in (4)[6] is performed by

$$E(x,r) = -(f(x) - f(x_k^*))exp(r\|x - x_k^*\|^2). \tag{4}$$

The function given in (4) is much simpler than the those presented in the prior generation. Furthermore, as the parameter $r$ grows larger and larger, the swiftly growing of the exponential function value could lead to an influx of the computation [7]. To beat this lack, another filled function suggested as follows:

$$Q(x,r) = \frac{1}{ln(1 + f(x) - f(x_k^*))} - r\|x - x_k^*\|^2, \tag{5}$$

this filled function still holds the feature of function (5) with one parameter, in addition to that, it has no exponential terms. It can be considered as the third generation filled functions(for more samples see [9-11]).

Throughout the rest of this paper, we assume that the following assumptions are satisfied:

*Assumption 1.* The function $f(x)$ is differential in $R^n$ and the number of minimizers can be infinite, but the number of the different value of minimizers is finite.

*Assumption 2.* $f(x) : R^n \to R$ is coercive, i.e., $f(x) \to +\infty$ as $\|x\| \to +\infty$.

## 3 Transforming the problem into one-dimensional

Directional search method is based on the directions $d_k$, $k = 1, ..., m$. If we have an objective function $f(x)$ with n-dimensions, we can use the line $l_\alpha = x_0 + \alpha d_k, \alpha \in R$ to construct a one-dimensional problem $L(\alpha)$. Moreover, we might want to choose $\alpha_k^*$ as the answer of

$$min_\alpha L(\alpha) = f(x_0 + \alpha d_k), \tag{6}$$

that means $\alpha_k^*$ at the direction $d_k$ can be a result of a one-dimensional minimization problem (for more information see [8]). We obtain a local minimizer $\alpha_k^1$ of $L(\alpha)$ then we construct the filled function on $L(\alpha)$, next, we take an initial point as a starting to find the second minimizer $\alpha_k^2$ of $L(\alpha)$. By repeat the above process we will obtain the global minimizer $\alpha_k^*$ at the direction $d_k$ as a solution of one-dimensional problem $L(\alpha)$, and by using $\hat{x}_k = x_0 + \alpha_k^* d_k$ we can minimize $f(x)$ when we use $\hat{x}_k$ as a starting point. Consequently, by comparing all minimizer points $\hat{x}_k$, $k = 1, ..., m$ with each other we will obtain the global minimizer of the problem $f$.

In the next section, we introduce a new filled function to minimize the one-dimensional problem $L(\alpha)$.

## 4 A new filled function

We suppose that the point $\alpha_k^1$ is a local minimizer of the function $L(\alpha)$ that can be determined by any efficient method.

Reducing the objective function from a multi-dimensional as a one-dimensional function making the minimization process easier and more efficient. For this purpose, we offer a new filled function as follows:

$$I_\alpha(\alpha, \alpha_k^1) = G(L(\alpha) - L(\alpha_k^1))U(|\alpha - \alpha_k^1|^2), \tag{7}$$

and

$$G(u) = \begin{cases} cos(\beta u), & u < 0; \\ 1, & \text{othewise,} \end{cases}$$

where $u = L(\alpha) - L(\alpha_k^1)$, $\beta > 1$, and $U(\kappa)$ is an escape function. The private form of $U(\kappa)$ presented in literature into several forms, for instance,$\kappa^\rho$ ,$exp(\kappa^\rho)$ and $arctan(\kappa^\rho)$, where $\rho$ is a positive integer. In the proposed paper the function $U$ will be selected as $-\rho|\alpha - \alpha_k^1|^2$,$\rho > 0$, that is the final form of the filled function will be as following:

$$I_\alpha(\alpha, \alpha_k^1) = -\rho|\alpha - \alpha_k^1|^2 G(L(\alpha) - L(\alpha_k^1)), \tag{8}$$

where the parameters $\beta$ and $\rho$ require to be adjusted appropriately.
The proposed filled function is continuously differentiable with two parameters. the new idea and advantages of the proposed algorithm are: First, this algorithm converts the objective function from multi-dimensional as a one-dimensional function this allows us to obtain the global minimizer easier. Second, the trigonometric function $cos(\beta u)$ allows to add many stationary points in the lower basin, this idea has many advantages, for example, it helps to reduce the time and the function evaluations which are very important in cases like this as we see can clearly in the experimental results. Now, let $\alpha_k^1$ be the current local minimizer of $L(\alpha)$, then we can define:

$$L_{S_1} = \{\alpha | L(\alpha) \geq L(\alpha_k^1), \alpha \in R, \alpha \neq \alpha_k^1\}, \ and \ L_{S_2} = \{\alpha | L(\alpha) < L(\alpha_k^1), \alpha \in R\}.$$

The next theorems show that the function $I_\alpha(\alpha, \alpha_k^1)$ achive Definition 2.4.

**Theorem 4.1.** *Let $\alpha_k^1$ be a local minimizer of $I_\alpha(\alpha, \alpha_k^1)$, then $\alpha_k^1$ is a strictly local maximizer of $I_\alpha(\alpha, \alpha_k^1)$.*

*Proof.* Since $\alpha_k^1$ is a local minimizer of $L(\alpha)$, there exists a neighborhood $N(\alpha_k^1, \epsilon^*)$ of $\alpha_k^1$, $\epsilon^* > 0$ such that $L(\alpha) \geq L(\alpha_k^1)$ for all $\alpha \in N(\alpha_k^1, \epsilon^*)$. Then, for all $\alpha \in N(\alpha_k^1, \epsilon^*)$ , $\alpha \neq \alpha_k^1$, we have:
$I_\alpha(\alpha, \alpha_k^1) = -\rho|\alpha - \alpha_k^1|^2 < 0 = I_\alpha(\alpha_k^1, \alpha_k^1)$.
Thus, $\alpha_k^1$ is a strict local maximizer of $I_\alpha(\alpha, \alpha_k^1)$. □

**Theorem 4.2.** *Assume that $\alpha_k^1$ is a local minimizer of $L(\alpha)$ and $\alpha$ is any point in $L_{S_1}$ then $I_\alpha(\alpha, \alpha_k^1)$ has no a stationary point on $L_{S_1}$ .*

*Proof.* Since $L(\alpha) \geq L(\alpha_k^1)$ and $\alpha \neq \alpha_k^1$, we have:
$I_\alpha(\alpha, \alpha_k^1) = -\rho|\alpha - \alpha_k^1|^2$,$\nabla I_\alpha(\alpha, \alpha_k^1) = -2\rho(\alpha - \alpha_k^1)$.
This means that $\nabla I_\alpha(\alpha, \alpha_k^1) \neq 0$, i.e. $\alpha$ is not a stationary point of $I_\alpha(\alpha, \alpha_k^1)$. □

**Theorem 4.3.** *Suppose $\alpha_k^1$ is a local minimizer of $L(\alpha)$ but not a global minimizer, and $L_{S_2} = \{\alpha | L(\alpha) < L(\alpha_k^1), \alpha \in R\}$ is not empty, then there exists a point $\alpha' \in L_{S_2}$ is a local minimizer of $I_\alpha(\alpha, \alpha_k^1)$.*

*Proof.* Let $L_{S_3} = \{\alpha | L(\alpha) \leq L(\alpha_k^1), \alpha \in R\}$ and $\partial L_{S_2} = \{\alpha | L(\alpha) = L(\alpha_k^1), \alpha \in R\}$, then $L_{S_3} = L_{S_2} \bigcup \partial L_{S_2}$, that means $\partial L_{S_2}$ is the boundary of the sets $L_{S_2}$ and $L_{S_3}$. Since $L(\alpha)$ is continuous, then $\partial L_{S_2}$ and $L_{S_3}$ are bounded and closed sets.

Now for any $\alpha \in \partial L_{S_2}$ we have

$$I_\alpha(\alpha, \alpha_k^1) = -\rho|\alpha - \alpha_k^1|^2,$$

also, for any $\alpha \in L_{S_2}$ we have

$$I_\alpha(\alpha, \alpha_k^1) = -\rho|\alpha - \alpha_k^1|^2 \cos(\beta(L(\alpha) - L(\alpha_k^1))).$$

Since $I_\alpha(\alpha, \alpha_k^1)$ is continuously differentiable and has the term $\cos(\beta(L(\alpha) - L(\alpha_k^1)))$, $\beta > 1$ then there is at least one point exists $\alpha' \in L_{S_2}$ is a minimizer of the function $I_\alpha(\alpha, \alpha_k^1)$. □

## Algorithm

According to the investigation and hypotheses in the earlier section, a new algorithm to obtaining the global minimizer of the function $f(x)$ will be proposed, and the experimental results will be provided as follows.

**Step 1** (Initialization) Determine the parameters $\beta > 1$ and $\rho > 0$, choose a starting point $x_0 \in S$, generate direction $d_k$,$k = 1, 2, ..., m$, and set $\epsilon = 10^{-2}$;

**Step 2** Create $L(\alpha) = f(x_0 + \alpha d_k)$ as a one-dimensional function;

**Step 3**
1. Obtain the local minimizer $\alpha_k^i$ of $L(\alpha)$ starting from $\alpha_0$ and then choose $\varrho = -1$.
2. Construct the filled function $I_\alpha(\alpha, \alpha_k^i)$ at $\alpha_k^i$;
3. Start from $\alpha_0 = \alpha_k^i + \varrho\epsilon$ to find a minimizer $\upsilon_1$ of $I_\alpha(\alpha, \alpha_k^i)$;
4. If $\upsilon_1$ in $S$ go to (5) otherwise go to (7);
5. Minimize $L(\alpha)$ start from $\upsilon_1$ to obtain $\alpha_{k+1}^i$ and then, go to (6);
6. If the point $\alpha_{k+1}^i$ in $S$ let $\alpha_k^i = \alpha_{k+1}^i$, and go to (2).
7. If $\varrho = 1$ terminate the iteration and give $\alpha_k^* = \alpha_k^i$ otherwise; let $\varrho = 1$ go to(3).

**Step 4** Calculate $\hat{x}_k$ using $\hat{x}_k = x_0 + \alpha_k^* d_k$, and consequently, find $x_k^*$ of $f(x)$ by using $\hat{x}_k$ as the initial point.

**Step 5** If $k < m$, let $k = k + 1$ and produce $d_{k+1}$ as a new search direction and go to (Step 2) otherwise; go to (Step 6).

**Step 6** Pick out the global minimizer of $f(x)$ using :

$$x^* = \min\{f(x_1^*), f(x_2^*), ..., f(x_m^*)\}.$$

## 5 The Experimental Results

In order to achieve the merit of the proposed algorithm in this paper, we selected test functions taken from literature. The algorithm is examined on all proposed problems and the results are submitted in Tables 1. and a comparative with the algorithm in [8] submitted in Tables 2. The following symbols are used in this paper:

$x_0$ The starting point.

$f_{eval}$ total number of functions evaluations $f(x)$, $L(\alpha)$ and $I_\alpha(\alpha, \alpha_k^1)$.

$T$ the mean of sum running time.

$f_{mean}$ the mean of the best value in the 10 runs.

$f_{best}$ the best value in 10 runs.

$ratio$ the rate of successfully obtaining true optimal solution among 10 runs.

**Problem 1.** *(Two-dimensional function)*

$$minf(x) = (1 - 2x_2 + csin(4\pi x_2) - x_1)^2 + (x_2 + 0.5sin(2\pi x_1))^2$$

$$s.t \quad 0 \leq x_1 \leq 10, \quad -10 \leq x_2 \leq 0,$$

where $c = 0.2, 0.5, 0.05$. The global minimum function value $f(x^*) = 0$ for all $c$.

**Problem 2.** *(Three-hump back camel function)*

$$minf(x) = 2x_1^2 - 1.05x_1^4 + \frac{1}{6}x_1^6 - x_1x_2 + x_2^2$$

$$s.t \quad |x_i| \leq 3, \quad i = 1, 2.$$

The global minimizer is $x^* = (0, 0)^T$.

**Problem 3.** *(Six-hump back camel function)*

$$minf(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 - x_1x_2 - 4x_2^2 + 4x_2^4$$

$$s.t \quad |x_i| \leq 3, \quad i = 1, 2.$$

The global minimizer is $x^* = (-0.0898, -0.7127)^T$ or $x^* = (0.0898, 0.7127)^T$ .

**Problem 4.** *(Treccani function)*

$$minf(x) = x_1^4 + 4x_1^3 + 4x_1^2 + x_2^2$$

$$s.t \quad |x_i| \leq 3, \quad i = 1, 2.$$

The global minimizers are $x^* = (0, 0)^T$ and $x^* = (-2, 0)^T$.

Table 1: The results obtained by our algorithm

| $No$ | $n$ | $T$ | $f_{eval}$ | $x^*$ | $f_{mean}$ | $f_{best}$ | $ratio$ |
|---|---|---|---|---|---|---|---|
| 1. | 2(c=0.2) | 0.1658 | 128 | (0.4091; 0.2703) | 1.0004e-15 | 4.6810e-16 | 100 |
| | 2(c=0.5) | 0.2742 | 56 | (1.0000; 0.0000) | 8.8150e-13 | 7.3282e-31 | 100 |
| | 2(c=0.05) | 0.2464 | 56 | (1.0000; 0.0000) | 2.9313e-14 | 2.4652e-31 | 100 |
| 2. | 2 | 0.2699 | 28 | (0.0000; 0.0000) | 3.9199e-15 | 1.0793e-32 | 100 |
| 3. | 2 | 0.2786 | 364 | (0.0898; 0.7127) | -1.0316 | -1.0316 | 100 |
| 4. | 2 | 0.2382 | 280 | (0.0000; 0.000) | 3.0507e-16 | 1.5866e-32 | 100 |
| 5. | 2 | 0.2942 | 224 | (0.0000; -1.0000) | 3.0000 | 3.0000 | 100 |
| 6. | 2 | 0.7117 | 414 | (-1.4251; -0.8003) | -186.7309 | -186.7309 | 100 |
| 7. | 2 | 0.2927 | 336 | (1.0000; 1.0000) | 1.1647e-14 | 1.0980e-15 | 100 |
| | 3 | 0.4143 | 216 | (1.000; 1.000; 1.000) | 3.7328e-08 | 7.0755e-16 | 100 |

Table 2: The results obtained by algorithm [8] and our algorithm on the problems 1-7

| No | n | The algorithm in [8] | | | The proposed algorithm | | |
|---|---|---|---|---|---|---|---|
| | | T | $f_{eval}$ | $f_{best}$ | T | $f_{eval}$ | $f_{best}$ |
| 1(c=0.2) | 2 | 0.648842 | 518 | 1.0707e-30 | 0.1658 | 128 | 4.6810e-16 |
| 1(c=0.5) | 2 | 0.721799 | 522 | 1.0707e-30 | 0.2742 | 56 | 7.3282e-31 |
| 1(c=0.05) | 2 | 0.644013 | 306 | 1.4252e-18 | 0.2464 | 56 | 2.4652e-31 |
| 2 | 2 | 0.762039 | 360 | 2.1294e-16 | 0.2699 | 28 | 1.0793e-32 |
| 3 | 2 | 0.900348 | 384 | -1.0316 | 0.2786 | 364 | -1.0316 |
| 4 | 2 | 0.920637 | 364 | 2.7399e-17 | 0.2382 | 280 | 1.5866e-32 |
| 5 | 2 | 0.996568 | 400 | 3.0000 | 0.2942 | 224 | 3.0000 |
| 6 | 2 | 2.003763 | 480 | -186.7309 | 0.7117 | 414 | -186.7309 |
| 7 | 2 | 0.856628 | 244 | 2.3558e-31 | 0.2927 | 336 | 1.0980e-15 |
| 7 | 3 | 1.31539 | 244 | 1.5705e-31 | 0.4143 | 216 | 7.0755e-16 |

**Problem 5.** *(Goldstein and Price function)*

$$min f(x) = g(x)h(x)$$

$$s.t \quad |x_i| \leq 3, \quad i = 1, 2.$$

where

$$g(x) = 1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)$$

and

$$h(x) = 30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 - 48x_2 - 36x_1x_2 + 27x_2^2)$$

$x^* = (0, -1)^T$.

**Problem 6.** *(Two-dimensional Shubert function)*

$$minf(x) = (\sum_{i=1}^{5} icos[(i+1)x_1] + i)(\sum_{i=1}^{5} icos[(i+1)x_2] + i)$$

$$s.t \quad 0 \le x_i \le 10, \quad i = 1, 2, 3, 4.$$

This problem has 760 local minimizers in total. The global minimum value is$f(x^*) = -186.7309$.

**Problem 7.** *( n-dimensional function)*

$$minf(x) = \frac{\pi}{n}[10sin^2(\pi x_1) + g(x) + (x_n - 1)^2]$$

$$s.t \quad |x_i| \le 10, \quad i = 1, \cdots, 10$$

where

$$g(x) = \sum_{i=1}^{n-1}[(x_i - 1)^2(1 + 10sin^2(\pi x_{i+1}))].$$

The global minimizer of this problem is $x^* = (1, \cdots, 1)$ for all $n$.

It is seen from Tables 1 and 2 that the introduced algorithm has many advantages, for instance, the global minimizers of all test problems listed above can be found, this implies the effectiveness of the introduced algorithm. Moreover, from column ratio in Table 1, the ratio of the successful runs are 100%, which confirms that the introduced algorithm is stable. In addition, the difference between $f_{mean}$ and $f_{best}$ is small this implies that the introduced algorithm is stable and robust to the initial points and parameter variation.

## 6 Conclusion

In this paper, a new filled function introduced for global optimization. The main approach was to transform the objective function into one-dimensional function depending on the directional search and minimize it in each direction. The computational results confirm that this algorithm is actually effective and reliable and the comparison with an actual algorithm confirmed that the introduced method was more efficient and relevant.

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