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Feedforward Friction Compensator Design for Shake Table System

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Abstract

This study deals with the design of feedforward compensator for the shake table system via Higher Order Sinusoidal Input Describing Functions (HOSIDFs) in order to reduce the performance degrading effect of the friction existing in the system. In this study, HOSIDFs are used to analyze the effect of Coulomb type friction on the output of the system. The study consists of implementation and design of feedforward compensator whose coefficients are calculated via HOSIDF based cost function given in the study. The study also involves harmonic plots and time-domain result of the system output which is the position of the shake table in order to illustrate the effect of the proposed feedforward compensator which improves the reference tracking performance via the reduction of the friction effect in the shake table system.

Keywords: Describing functions, Shake table, Coulomb friction model

Titreşim Masası Sisteminde Sürtünme Etkisini Azaltan İleri Beslemeli Kompanzatör Tasarımı

Öz

Bu çalışmada, yüksek mertebeli sinüzoidal tanımlama fonksiyonları kullanılarak titreşim masası sisteminde etkin olan mekanik sürtünmenin sistemin referans takip performansını azaltan etkisini düşürmek üzere ileri beslemeli bir kompanzatör tasarımı ele alınmıştır. Çalışmada, yüksek mertebeli sinüzoidal tanımlama fonksiyonları sistemde bulunan ve Coulomb tipi olarak ifade edilen mekanik sürtünmenin sistem çıkışına olan etkisini analiz etmek amacıyla kullanılmıştır. Bu çalışma, ilerleyen bölümlerde detaylı olarak verilen bir maliyet fonksiyonu üzerinden hesaplanan ileri beslemeli kompanzatörün tasarımı ve sisteme uygulanmasını içermektedir. Ayrıca çalışmada tasarlanan kompanzatörün sistemdeki sürtünme etkisini azaltarak sistemin referans takip performansını iyileştirdiğini göstermek üzere titreşim masasının hareket eden üst tablasının pozisyonu olan sistem çıkışının harmonik grafiklerine ve zaman cevaplarına da yer verilmiştir.

Anahtar Kelimeler: Tanımlama fonksiyonları, Titreşim masası, Coulomb sürtünme modeli

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1. INTRODUCTION

Friction compensation has become an important topic in numerous engineering applications in the last decades. The main reason of this fact is directly linked with the performance degrading effect of friction for many different types of systems. It is also an important fact that friction is a nonlinear structure that exists in many different applications from different engineering fields.

There exist many different approaches in the control literature in order to deal with friction effect in the physical systems. Mostly, the main idea of these studies is to design a compensator in order to reduce the performance degrading effect of friction as much as possible. There exist studies that may be considered as major examples of friction compensator design [1,2]. These studies cover the application of model-based adaptive friction compensation and compensation algorithm consists of a PID component and an adaptive component for estimating friction and force ripple.

Moreover, there exist research papers in literature dealing with friction compensation where different friction-compensation strategies such as friction-model-based feedforward and inverse model based disturbance observer are taken into consideration for a linear motor-based XY feed drive of a high-speed milling machine in addition to compensator design via dynamic LuGre friction model [3,4].

The proposed compensator is designed as a feedforward structure where the optimal value of the static compensator gain is calculated via frequency dependent cost function. The feedforward compensator and controller design is an important topic that attracts researchers' attention recently. There exist numerous studies in control literature that deal with feedforward design [5,6]. Some of these studies focus on disturbance compensation [7] whereas there are researches dealing with trajectory planning in literature [8]. In addition, some studies dealing with feedforward controller design also involve application and experimental results [9].

This study utilizes HOSIDF theory in order to design the feedforward compensator. The research paper where the frequency based approach called as HOSIDF theory is introduced, is one of the major references for this approach in literature [10]. Here, the main idea is to define the nonlinear effects in the system via harmonic analysis of the system output where a sinusoidal signal is applied as the input to the system. There exist recent studies that extend this approach to Lure type systems [11]. Moreover, there are studies where HOSIDF theory is used for friction compensation for different kind of systems and applications [12,13]. In addition to these studies, there are researches involving application results where the analysis of the nonlinearities in the system are carried out via HOSIDF theory [14]. A recent survey paper about frequency domain system identification techniques, compares HOSIDF theory with other major approaches from different points of view [15].

Different from the existing literature, this paper deals with the implementation of HOSIDF theory for Coulomb type friction compensation in shake table system. The reference tracking performance degrading effect of Coulomb type modeled friction is reduced via the proposed feedforward compensator structure implemented into the shake table system. Here, the proposed structure is designed as a secondary compensator in addition to the existing controller in the system which guarantees the stability. The contribution to the existing literature can be summarized as the implementation of HOSIDF-based feedforward compensator into a shake table system with Coulomb type friction model where the proposed compensator improves the reference tracking performance of the shake table system.

The rest of the paper is organized as follows: Preliminaries and mathematical background of HOSIDF technique is given in Section 2. Section 3 explains the implementation of HOSIDF technique for friction compensation. The shake table system involving Coulomb type friction is introduced in Section 4. The simulation studies of the feedforward compensator designed for friction compensation in shake table system including frequency-domain results are given in Section 5. Finally, conclusion and future studies are presented in Section 6.

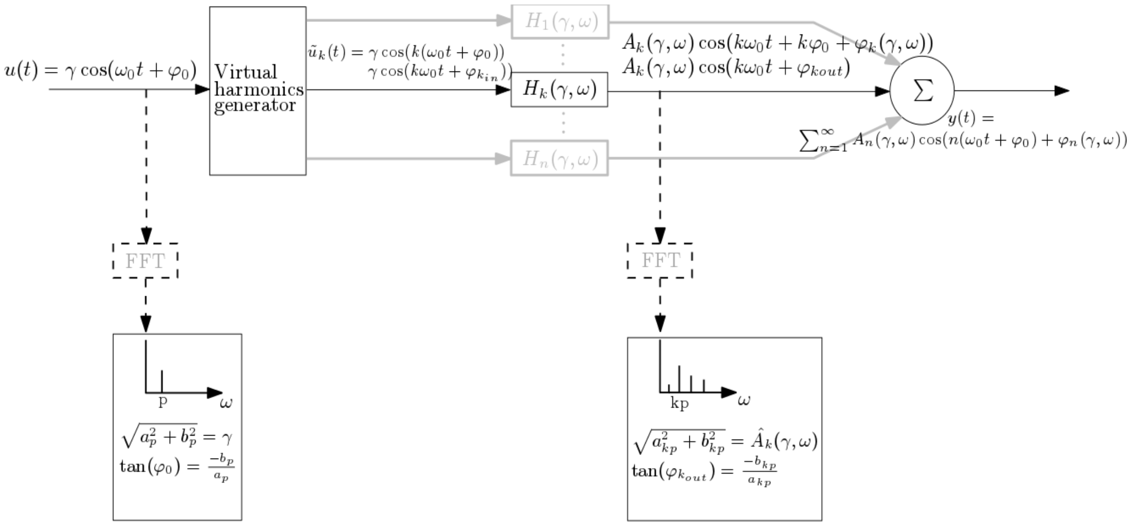

Figure 1. Calculation of HOSIDFs via FFT method [10]

2. PRELIMINARIES

HOSIDF theory which can be considered as a subclass of describing functions is based on the idea to define a non-linear structure in the system with a quasi-linear descriptor. Here, the gain is defined as a function of the amplitude of sinusoidal input signal.

If a sinusoidal excitation signal defined as Equation 1

$$u(t)=\gamma\cos(\omega_0 t+\varphi_0) \qquad (1)$$

is applied to a time-invariant, stable SISO nonlinear system which has a harmonic response due to the harmonics of the input frequency when it is subject to a sinusoidal excitation signal, the output can be structured as Equation 2

$$y(t)=\sum_{k=0}^{K}|H_k(\omega_0,\gamma)|\gamma^k\cos\left(k\left(\omega_0 t+\varphi_0\right)\right)+\angle H_k(\omega_0,\gamma) \qquad (2)$$

where k stands for the harmonics and $H_k(\omega_0,\gamma)$ is the *k-th* order HOSIDF. Here, $U(\omega), Y(\omega)$ are defined as single-sided spectra that are obtained from system input and output, respectively. The single-sided spectrum has the following characteristic properties such as $U_{single}(\omega)=2U_{double}(\omega)$, $\omega\in R>0$ and in addition to this equality, $U_{single}(0)=U_{double}(0)$ where $U_{single}(\omega)=0, \forall\omega<0$. Hence, every HOSIDF can be determined via aforementioned properties of given spectrum as $H_k(\omega_0,\gamma)=\frac{Y(k\omega_0)}{U^k(\omega_0)}$ where $U^k(\omega_0)=\prod_{l=1}^{k} U(\omega_0)$.

Figure 1 illustrates HOSIDF theory block diagram representation in addition to the calculation of k-th order HOSIDFs using FFT method. It is shown in Figure 1 that the sinusoidal excitation signal is applied to the system and frequency components of the virtual harmonic generator output can be determined via the amplitude γ and frequency ω_0 of the sinusoidal input signal. Here, FFT method is utilized for the system input and system output in order to calculate the magnitude and phase of every HOSIDF corresponding to the harmonics of system output. The necessary equations used in the calculation are also given in detail in Figure 1.

In order to make the proposed compensator design suitable for noisy environment where the noise signal may affect the measured data received from the sensors, some statistical properties are used in the proposed HOSIDF model. By the help of these statistical properties, it is possible to select the "meaningful" data that are taken into consideration for the optimization process via HOSIDF based cost function. Here, the "meaningful" data term represents the harmonic values related with the nonlinearity of the system.

The expected value of the system output is defined as Equation 3

$$\overline{Y}(k\omega_0)=\frac{1}{N}\sum_{n=1}^{N} Y_n(k\omega_0) \tag{3}$$

for N periods of excitation signal. The variance of the average is represented as Equation 4

$$\sigma_{\overline{Y}}^2(k\omega_0)=\frac{1}{N^2-N}\sum_{n=1}^{N}\left(\left(Y_n(k\omega_0)-\overline{Y}(k\omega_0)\right)\right)^2 \tag{4}$$

and the average of the variance is given via Equation 5

$$\overline{\sigma}_{\overline{Y}}^2=\frac{1}{K-1}\sum_{k=1}^{K}\overline{\sigma}_{\overline{Y}}^2(k\omega_0) \tag{5}$$

The variance of the variance can be calculated as Equation 6

$$\delta_{\overline{\sigma}}^2=\frac{1}{K-1}\sum_{k=1}^{K}\left(\sigma_{\overline{Y}}(k\omega_0)-\overline{\sigma}_{\overline{Y}}\right)^2 \tag{6}$$

where $k=1,2,\cdots,K$ stands for the "effective" harmonics that are analyzed in the system output [12].

3. FRICTION COMPENSATION VIA HOSIDF THEORY

The proposed compensator design is mainly based on the idea that the relevant nonlinearity that can be identified from the system output spectrum in the shake table system is due to the nonlinear structure of mechanical friction existing in the system. The reason of this fact is that the mathematical model of the shake table is a linear time-invariant system. Hence, when the visible indicators of nonlinearity such as the harmonics existing at the system output are reduced, it is guaranteed that the degrading impact of friction is also decreased as much as possible. It is emphasized that as the sinusoidal excitation signal whose spectrum consists of the frequency line at $f_0 \in F_{in}$ is applied to the nonlinear system, the spectrum of the steady-state output $Y(\omega)$ will contain the harmonics due to the friction existing in the system in addition to the original spectral line of the input signal.

Hence, a cost function based on HOSIDFs which characterizes the amplitude of output harmonics, is constructed. Here, the main idea is to calculate the optimal feedforward value which minimizes the cost function. The compensator FF is selected such that Equation 7

$$FF=\arg\min_{FF\in R\geq 0}\frac{1}{N_K}\frac{\sum_{k\in K}|E\{Y(k\omega_0)\}|}{|E\{Y(\omega_0)\}|} \tag{7}$$

where $K=\{k\in N\geq 2|E\{Y(k\omega_0)\}\neq 0$. Here, $|E\{Y(k\omega_0)\}|$ stands for the expected value of $Y(k\omega_0)$ as it is given in Section 2.

Since the real measurement data received from the sensors in experimental applications may be very noisy, the expected value and variance definitions are used to select "meaningful" spectral lines to be included in the cost function. If $|E\{\overline{Y}(k\omega_0)\}|\neq 0$ is η confidence level, which is equivalent to Equation 8

$$\frac{|\overline{Y}(k\omega_0)|^2}{\sigma_{\overline{Y}}^2(k\omega_0)}>F_{2,2(N-1)}^{\eta} \tag{8}$$

where the cumulative $F_{2,2(N-1)}$ distribution satisfies $cdf\left(F_{F_{2,2(N-1)}}^{\eta}\right)=\eta$ condition, then the sample mean is called η-significant [12].

The aforementioned definition is important for the proposed approach since we need to implement a measurement quality condition to select the harmonics that are included in the cost function to avoid the measurement noise as much as possible.

The proposed feedforward compensator structure is used to suppress the degrading effect of Coulomb friction on the reference tracking performance of shake table system. The optimal coefficient for the proposed compensator is calculated via frequency-domain cost function given above.

In order to summarize the algorithm of the proposed feedforward compensator design process, the procedure is explained step by step. The first step of the optimization process is to apply a sinusoidal excitation signal as the input of the shake table.

In order to get rid of the effect of transient response, the first period is not taken into account and the input and output signal spectrums are determined via FFT method. As it is explained in the previous section, the input signal spectrum consists of spectral lines at the frequency of the sinusoidal excitation signal whereas the output signal spectrum consists of the input frequency harmonics due to existence of Coulomb modeled friction in the system. Starting from the initial condition which is the case that there is no feedforward compensator in the system, a searching algorithm is utilized in order to find out the optimal coefficient. For every step of this procedure, the output spectrums, obtained for all values of feedforward compensator, are analyzed via HOSIDF theory according to the given cost function. Finally, the optimum compensator coefficient is determined as the value which minimizes the cost function.

The flowchart for the algorithm in order to calculate the proposed feedforward controller is given in Figure 2.

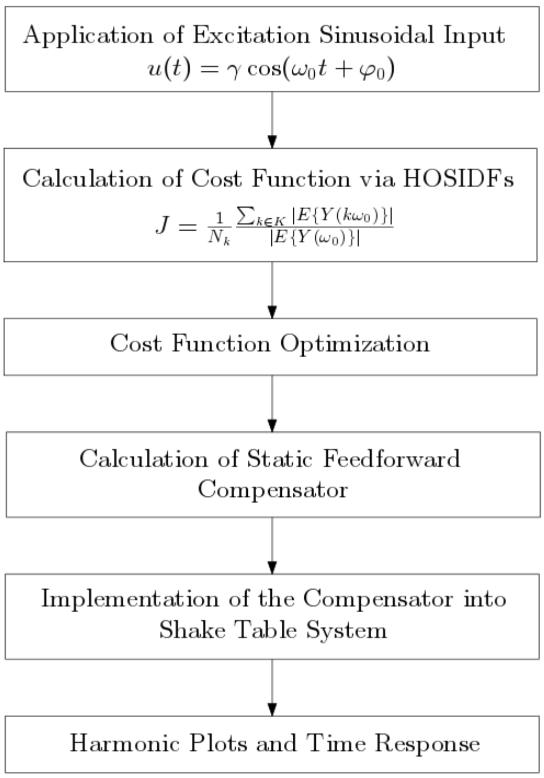

Figure 2. Algorithm flowchart for proposed controller design

4. SHAKE TABLE SYSTEM

This study deals with the feedforward compensator design for friction compensation of shake table system. Hence, detailed information about the mathematical model of shake table is also presented in this study. Shake table system can be considered as one of the major benchmark application examples in control theory. The main usage of shake table is defined as an earthquake simulator where the real earthquake data is applied as the reference signal to the system and the shake table simulates the behavior of the earthquake in x-coordinate. From the control theory point of view, the shake table system is a major example for reference tracking problem.

Here, a sinusoidal signal is applied as the reference input of the system and PD controller already existing in the system aims to force the actual output of the shake table which is the position in this case to track the sinusoidal reference signal with minimum tracking error.

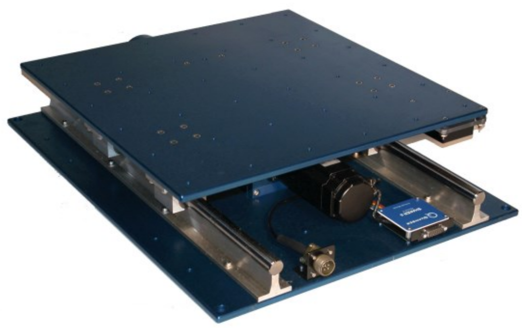

Figure 3. Shake table system

The shake table system which is illustrated in Figure 3 involves a top stage that is driven by a brushless DC motor. There exist two metal shafts where stage rides via linear bearings which allow for smooth linear motions. An encoder is placed in the motor in order to measure the position of the stage in addition to an analog accelerometer mounted on the platform to measure the acceleration of the stage directly. In order to design the PD controller, the transfer function is used to represent the mathematical relationship in s-domain between the current applied to the motor, I_m and position of the top stage of the shake table, X given as Equation 9

$$X(s)=\frac{I_m(s)}{K_f s^2} \tag{9}$$

where K_f is defined as the open-loop gain of the transfer function. The open-loop gain is defined as Equation 10

$$K_f=\frac{M_t P_b}{K_t} \tag{10}$$

where M_t stands for the total mass. Here, P_b and K_t are the coefficients of the motor in the system which are given in the next section. Here, the current is transformed to acceleration by the help of K_f coefficient and the position of the stage is obtained from the acceleration via the given transfer function given in (9). Since the effect of friction is neglected during the design of the classical PD controller, some deficiencies at the reference tracking performance of the PD controller occur during the process. Hence, the proposed feedforward compensator is utilized in order to improve the tracking performance of the overall system.

5. SIMULATION STUDIES

The proposed compensator is utilized for the shake table system which is highly used in different experimental studies especially in mechanical and civil engineering problems. It is mostly used in the studies about structural dynamics and vibration isolation.

The shake table system taken into account for the simulation studies, is used with an additional load of 7.26 kg which makes the total load including the stage up to 15 kg that is driven by the electrical motor in the system. The motor current torque constant K_t is taken as 0.360 Nm/A and the ball screw pitch P_b which is the relationship of the distance taken by the stage in every revision of the rod connected to the electrical motor is given as 0.0127 m/rev. Moreover, in order to set the performance of the PD controller the damping ratio ζ and the natural frequency of the system ω_n are chosen as 0.75 and 15 Hz, respectively. Here, according to given specifications and system parameters, the coefficients of PD controller, K_P=4700.4 and K_D=74.8, are calculated via second order approximation of shake table system and Ziegler-Nichols method.

The block diagram of the overall system including the transfer function of the shake table, Coulomb friction model and the proposed feedforward compensator in addition to PD controller is given in Figure 4 where du/dt and sgn blocks stand for the derivative action and sign function, respectively.

Here, the static Coulomb friction coefficient is chosen as 2 which is an applicable value for the shake table system. Coulomb friction coefficient is defined as the static gain between the velocity and the friction force. In order to reduce the degrading effect of the friction existing in the system, a feedforward compensator as given in Figure 4 is implemented into the system. The static coefficient of the feedforward compensator is calculated via the minimization of HOSIDF based cost function which is explained in detail in the previous sections. The optimal value for the feedforward gain is calculated as 21. When the calculated optimal value is applied to the system, it is illustrated in Figure 5 and 6 that there exists an improvement in the reference tracking performance of the system. The harmonic plot given in Figure 4 represents that the harmonics except the first harmonic are suppressed via the proposed feedforward compensator design. In Figure 5, the square markers represent the magnitudes of harmonics for uncompensated case and the circle markers illustrate the harmonic magnitudes for the compensated case. As Figure 5 is analyzed in detail, the magnitude values for harmonics are significantly reduced when the proposed compensator is implemented into the system. Here, the fact that the harmonics different than the first harmonic are present due to the nonlinearities in the system when a sinusoidal input is applied to the system, approve that suppressing harmonics directly relate with the reduction of nonlinear friction effect on the system output. Hence, it is illustrated that the reference tracking performance of the system is improved by reducing the effect of the nonlinear structure (friction) in the shake table system.

It is also represented in Figure 6 that the success of the proposed compensator can be analyzed in terms of time-domain result. During the simulation studies, the time-domain plot is obtained for all the periods of the sinusoidal position reference signal with a magnitude of 10 cm and then a closer look by zooming into one period is given in Figure 6 in order to illustrate the improvement in detail. Here, as the uncompensated (dashed dotted line), compensated (dashed line) and reference signal (solid line) are compared with each other, there exists approximately 1% improvement at the reference tracking performance of the closed loop system involving the proposed feedforward compensator. The reference tracking for the position of the top stage in shake table system should be carried out with minimum tracking error. Hence, the improvement of 1% in reference tracking performance of this system is thought to be important and useful. Finally, it is illustrated that the friction is highly effective when the stage moves with low velocity such as the peak point of the sinusoidal signal given in Figure 6.

Here, since the cost function is structured in frequency domain via HOSIDF theory and FFT method, the improvement in the harmonic plots is more significant than the time-domain results.

Another important point to be emphasized for the simulation studies is that sensor noise is applied to the system during the simulation as given in Figure 4. Here, in order to simulate the measurement noise which is important for experimental studies, uniform random signal with small-scaled amplitude and high frequency value is applied to the system. By the implementation of the statistical equations given in Section 2 into the cost function, the feedforward compensator design procedure is not affected from the noisy sensor data. It is also possible to illustrate this result via the harmonic plot given in Figure 5.

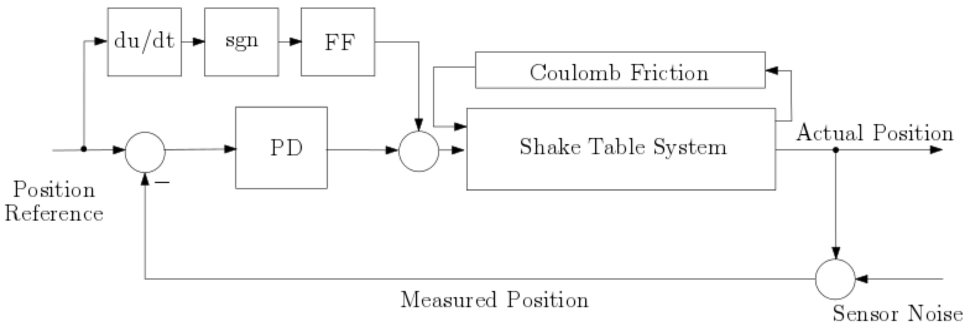

Figure 4. Block diagram representation of closed-loop system

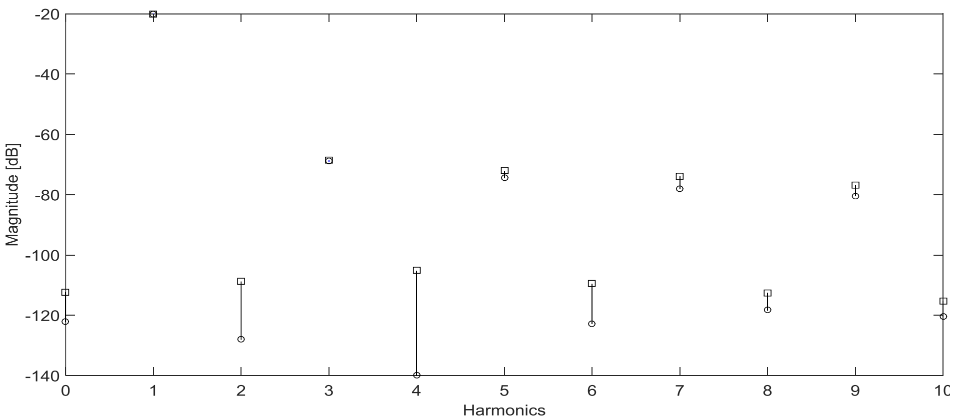

Figure 5. Harmonic plots for uncompensated (square) and compensated (circle) cases

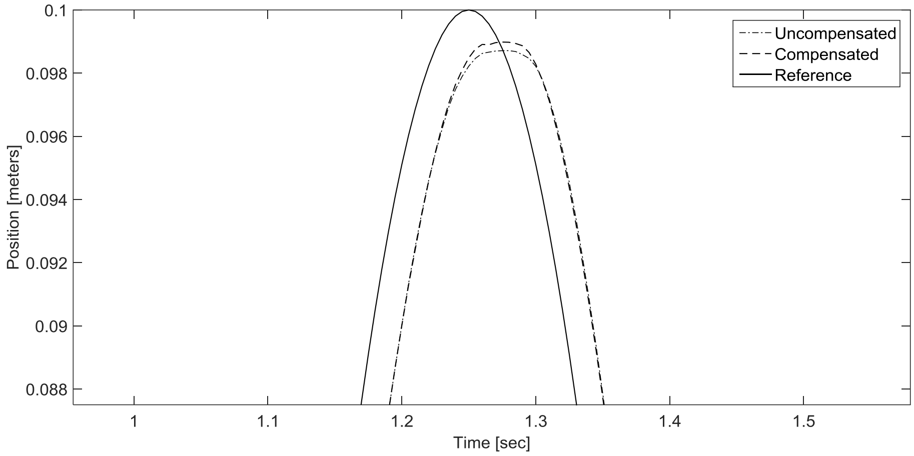

Figure 6. Time domain plot of system output

6. CONCLUSION

The proposed compensator in order to reduce the effect of the friction to the output of the shake table system, is designed in this study. In order to illustrate the achievement and effectiveness of the proposed compensator, the time-domain and frequency domain plots are given and analyzed. The given plots illustrate that the additional feedforward compensator increases the reference tracking performance of the overall system when it is compared to the situation where only PD controller is implemented in the system. The future studies of the research involve the dynamic compensator design for the systems involving different non-linear structures such as actuator saturation and time-delay.

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