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6th, 7th and 8th Grade Students' Misconceptions about the Order of Operations*

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Abstract: The aim of this research is to determine the misconceptions among 6th, 7th and 8th grade students about the order of operations in line with arithmetic expressions and posing and solving problems related to arithmetic expressions. The research has a mixed-method research design with concurrent-triangulation design. The study group for the research comprised a total of 240 students with 78 from 6th grade, 80 from 7th grade and 82 from 8th grade chosen with the simple random sampling method from schools located in a city in the Eastern Black Sea Region in Turkey. The research used a two-tier diagnostic test developed by the researcher to determine the misconceptions of students. According to the results of the research, it was determined that 6th, 7th and 8th grade students performed arithmetic expressions from left to right without paying attention to order of operations, stated that only operations in brackets need to be performed first in terms of order of operations, did not pay attention to order of operation when writing or solving an arithmetic expression equal to a given number, and that students had difficulty posing a problem sentence involving arithmetic expressions and requiring order of operations.

Keywords: *Misconceptions, mathematics education, order of operations.*

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Introduction

In mathematics education, it is very important to know the arithmetic operations and the characteristics of the operations. All arithmetic operations are related to each other and have similar and different aspects. Multiplication can be stated as repeated addition; division can be seen as multiplying by the inverse of a number, and subtraction can be considered addition with a negative number. In mathematics teaching, it is important to abide by certain rules while calculating arithmetic operations. One of these rules is order of operations. The order of operations in arithmetic is used when a number or equation in an arithmetic equation has both priority and binary operations (Peterson, 2000). According to the order of operations, firstly arithmetic operations within the brackets should be performed, then exponents, multiplication-division and addition-subtraction. For example, the equation 62 – 15 x 3 firstly has multiplication performed according to the order of operations (15 x 3) and then subtraction (62 – 45) in order to obtain the result (17).

When arithmetic operations become complicated, in other words when two or more arithmetic operations occur together, awareness of students about the order of operation in this complicated structure will contribute to effective use of the student's mathematical skills. The order of operations which supports the effective development of these skills is given as an outcome of calculating arithmetic operations in Turkey (MNE, 2005). The order of operations is included in the 2013 and 2018 middle school mathematics curriculum (5th -8th grade) as operation priority within the Numbers and Operations learning domain outcomes beginning in 6th class (MNE, 2013; 2018).

If the order of operations rule, which plays an important role in gaining mathematical skills for arithmetic operations, is not correctly configured or care is not taken about the rule, students may encounter the problem of having more than one correct answer when solving a problem or calculating arithmetic operations. In such a situation, based on the

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equation given above as 62 – 15 x 3, the student may first subtract 15 from 62 and then multiply the result (47) by 3 and obtain the result 141. Just as in this example, when the student calculates based on performing operations from left to right, more than one solution may be obtained for the arithmetic operation. This situation was revealed in research results from abroad. Research observed that students encounter difficulties in more advanced learning based on the order of operations. The research results support the above example with students revealed not to pay attention to priority operations in calculating arithmetic operations and solving problems, to be unable to make sense of the order of operations rule, and generally to attempt to reach the solution by performing operations from left to right (Blando, Kelly, Schneider & Sleeman, 1989; Linchevski & Livneh, 1999; Pappanastos, Hall & Honan, 2002; Vanderbeek, 2007; Wu, 2007; Yenilmez & Coksoyler, 2018).

The difficulties that students experience include the fact that the mnemonic devices used in teaching the order of operations do not involve the priority of arithmetic operations from left to right because they contain only the names of arithmetic operations; do not support conceptual learning; activities that are not connected with prior knowledge of the students are included in the teaching of the order of operations and that real-life problems involving this concept are not used in teaching (Joseph, 2014). In this respect, it can be said that students' misconceptions about the order of operations stem from their learning at the level of procedural knowledge.

When literature related to the order of operations, which is frequently used in arithmetic operations and in solving problems, is examined, it is seen that there are mnemonic devices aimed at permanently structuring this rule for students (Oksuz, 2009; Uca, 2010). A study aimed to evaluate students' problem-building skills for arithmetic expressions by taking into consideration the order of operations (Ocal, Ipek, Ozdemir & Kar, 2018) and research examined whether the order of operations of university students is structured correctly (Glidden, 2008; Pappanastos, Hall & Honan, 2002). Research about the difficulties faced by sixth grade students regarding the order of operations was completed (Yenilmez & Coksoyler, 2018) and question structures prepared for the order of transactions in the textbooks were examined (Lee, 2000). According to the results of the research about order of operations, textbooks caused misconceptions in relation to the meaning of order of operations (Glidden, 2008; Lee, 2000, Pappanastos, Hall & Honan, 2002); it was found that students could not use the order of operations meaningfully and correctly (Glidden, 2008, Joseph, 2014; Pappanastos, Hall & Honan, 2002; Yenilmez & Coksoyler, 2018).

Glidden (2008) emphasizes the necessity of meaningful learning of order of operations in recent years and states that order of operations is an important concept used at all grade levels. According to Glidden (2008), the order of operations should be learned significantly by students from the elementary level. If the order of operations is learned conceptually by the students in these years, students will not have difficulty with this subject in the following years. From this point stated by Glidden (2008) and the fact that the studies in the literature were conducted with university students in general, it is thought that determining the misconceptions of middle school 6th, 7th and 8th grade students regarding the order of operations is a priority study. Moreover, it is thought that the results of the research will be a resource for research to be done to eliminate the misconceptions of students regarding the order of operations.

In this study, where the misconceptions of middle school 6th, 7th and 8th grade students regarding the order of operations were examined, the answers to the following questions were sought:

1. What are the students' misconceptions about arithmetic operations that require order of operations?
2. What are the students' misconceptions about problems that require order of operations?

Methodology

Research Model

The research was designed with the concurrent-triangulation design from mixed-method research designs. The concurrent-triangulation design collects and analyzes quantitative and qualitative data together. Analysis of data is generally completed separately with the data combined when making interpretations (Creswell, 2003). In line with this, the quantitative dimension of the research was designed as a survey model. For this, an attempt was made to identify the misconceptions of 6th, 7th and 8th grade students with a two-tier diagnostic test and to reveal the current situation of students in relation to operations and problems requiring the order of operations rule. The qualitative dimension of the study involved data about the reasons students gave for their answers, collected and analyzed in accordance with qualitative research method principles. An attempt was made to reveal in detail the misconceptions and possible reasons for these misconceptions.

Study Group

The study group comprised 6th, 7th, and 8th grade students from schools located in a city in the Eastern Black Sea Region in Turkey. Taking note of statistical data from the MNE, schools were divided into four regions based on success lists and two middle schools were selected from each region with the simple random sampling method. From these schools, one class each of 6th grade (N=78), 7th grade (N=80) and 8th grade (N=82) were selected. In line with this, the study group for the research comprised a total of 240 students.

Within the scope of the research, as there is no outcome or explanation about the order of operations topic on the 5th grade mathematics curriculum in Turkey, and students do not have any activities about operation skills related to this topic, so 5th grade students were not included in the study group for the research.

Data Collection Tool

The study developed a two tier diagnostic test to determine student misconceptions, based on the "Order of Operations Success Test" developed by Uca (2010), as a data collection tool with the aim of determining misconceptions of students about the order of operations. The questions on the two-tier diagnostic test were developed based on the outcomes in the Numbers and Operations learning domain on the middle school mathematics curriculum (5th and 8th grade) in Turkey. When developing this tool, the decision was made to perform the research with a single scale tool considering that it would provide more effective results in revealing whether or not students applied order of operations, instead of developing the tool separately for each grade level.

In order to ensure the content validity of the test, the views of three faculty members and two mathematics teachers who are experts in mathematics education were obtained. Necessary corrections were made to the test according to expert opinions. Within the scope of the validity and reliability studies of the two-tier diagnostic test, a pilot study was conducted with 100 6th, 7th, and 8th grade students who were not included in the study group. The data obtained from the pilot study were analyzed using ITEMAN. The item distinctiveness in the test ranged between .10 and .58 and difficulty level of items ranged between .10 and .68. The Cronbach Alpha internal consistency coefficient of the test was determined as .85.

Different types and levels of questions were used on the two-tier diagnostic test. Instead of questions relying on operational knowledge, questions requiring students to use conceptual knowledge and mathematical reasoning were chosen. Accordingly, the two-tier diagnostic test comprised a total of 25 questions including the question types of symbolic expressions (13 questions), confirmatory expressions (4 questions), true-false questions (4 questions), open-ended questions (1 question), translating symbolic expressions to problem statements (1 question), and problem statements (2 questions). In addition to the questions on the two-tier diagnostic test, for each diagnostic question the students were asked to write an answer to the statement *"Write why you think this"* after each question to determine the reason for the student's answers. A blank space was left for the students to explain how they solved the problem and whether they paid attention to order of operations while solving the problem.

Data Analysis

Analysis of quantitative data in the research is given as frequency and percentage of student answers given to each question type in the first stage. The point values for each response in this section were calculated as true (2), false (1) and blank (0), and an attempt was made to reveal participant success on the test.

Data obtained from the qualitative dimension of the research were interpreted based on the content analysis method. The reasons given by students in the second section of the test were investigated in detail. With the aim of determining the preconceptions of students about operations and problems requiring order of operations, the reasons for solving each problem type were investigated, especially based on incorrect answer percentages for each question type. The basic reason for analyzing the reasons students gave for incorrect answers based on the percentages of incorrect answers is that when data are investigated, students answering correctly stated they paid attention to the order of operations in their reasons for answering.

Results

In this section, data obtained in the research are separately analyzed for each of the six question types included on the Order of Operations Two-Tier Diagnostic Test. Firstly the percentage and frequency values for responses to the question types by students in the quantitative dimension of the research are given. Then, in the qualitative dimension of the research, findings obtained from quantitative data are given along with the reasons students gave for incorrect answers based on question type. Sample student responses are also given.

The responses given to the question type involving symbolic expressions to determine the arithmetic operations and properties between operations taking note of the order of operations are given in Table 1.

Table 1. Student Responses to Symbolic Expressions Question Types

Problem	Correct Answers		Incorrect Answers		Blank		Total	
	f	%	f	%	f	%	f	%
1) 6 + 5 × 2 =?	45	19	195	81	-	-	240	100
2) 25 × 4 – 38 =?	203	85	37	15	-	-	240	100
3) 30 + 6 × 4 – 5 =?	48	20	192	80	-	-	240	100
4) 265 – 12 × 4 + 68 =?	46	19	194	81	-	-	240	100
5) 68 + 34 – 20 × 4 =?	52	21	188	79	-	-	240	100
6) 60 ÷ 4 + 23 =?	222	93	18	7	-	-	240	100
7) 89 – 60 ÷ 3 =?	46	19	194	81	-	-	240	100
8) 69 + 65 ÷ 5 – 18 =?	37	15	203	85	-	-	240	100
9) 185 + 89 – 56 ÷ 4 =?	33	14	194	81	13	5	240	100
10) 196 ÷ 14 + 56 – 25 = ?	198	82	42	18	-	-	240	100
11) 27 + 5 × 16 – 8 ÷ 2 = ?	34	14	188	78	18	8	240	100
12) 9 + 6 × (11 – 7) ÷ 4 = ?	27	11	191	80	22	9	240	100
13) 37 + 8 – (9 × 9) ÷ 3 = ?	25	10	182	76	33	14	240	100

When Table 1 is investigated, to determine the students' knowledge of arithmetic operations and characteristics between operations, for double (e.g. 6 + 5 × 2), triple (e.g. 30 + 6 × 4 – 5) and quadruple (e.g. 27 + 5 × 16 – 8 ÷ 2) arithmetic operations with the order of operations required, arithmetic operations beginning with multiplication and division (e.g., 60 ÷ 4 + 23 equation) had high correct answer percentages. Questions beginning with addition and subtraction operations (e.g., 69 + 65 ÷ 5 – 18 equation) appeared to have higher incorrect answer percentages. Additionally, it appeared students had higher incorrect answer percentages for questions involving arithmetic expressions with brackets.

Within the scope of the qualitative dimension of the study, the findings obtained from quantitative data about the reasons students gave for incorrect answers to this question type and example student solutions are given in Table 2.

Table 2. Student Reasons and Sample Solutions for Symbolic Expressions

Problem	Student reason	Example student solution
1) 6 + 5 × 2 =?	Performing operations from left to right	6 + 5 × 2 = 11× 2 =22
2) 25 × 4 – 38 =?	Performing operations from left to right	25 × 4 – 38 = 100– 38 = 62
3) 30 + 6 × 4 – 5 =?	Performing operations from left to right	30 + 6 × 4 – 5 = 36 × 4 – 5 = 144 – 5 =
4) 265 – 12 × 4 + 68 =?	Performing operations from left to right	265 – 12 × 4 + 68 = 253 × 4 + 68 = 1012 + 68 = 1080
5) 68 + 34 – 20 × 4 =?	Performing operations from left to right	68 + 34 – 20 × 4 = 102 – 20 × 4 = 82 × 4 = 328
6) 60 ÷ 4 + 23 =?	Performing operations from left to right	60 ÷ 4 + 23 = 15 + 23 = 38
7) 89 – 60 ÷ 3 =?	Performing operations from left to right	89 – 60 ÷ 3 = 29 ÷ 3 = 9
8) 69 + 65 ÷ 5 – 18 =?	Performing operations from left to right	69 + 65 ÷ 5 – 18 = 134 ÷ 5 – 18 = 26– 18 = 8
9) 185 + 89 – 56 ÷ 4 =?	Performing operations from left to right	185 + 89 – 56 ÷ 4 = 274– 56 ÷ 4 = 218 ÷ 4 = 54
10) 196 ÷ 14 + 56 – 25 =?	Performing operations from left to right	196 ÷ 14 + 56 – 25 = 14 + 56 – 25 = 70 – 25 = 45

Table 2. Continued

Problem	Student reason	Example student solution
11) 27 + 5 × 16 – 8 ÷ 2 =?	Performing operations from left to right	27 + 5 × 16 – 8 ÷ 2 = 32 × 16 – 8 ÷ 2 = 512 – 8 ÷ 2 = 504 ÷ 2 = 252
12) 9 + 6 × (11 – 7) ÷ 4 = ?	First performing operation in brackets, then operations from left to right	9 + 6 × (11 – 7) ÷ 4 = 9 + 6 × 4 ÷ 4 = 15 × 4 ÷ 4 = 60 ÷ 4 = 15
13) 37 + 8 – (9 × 9) ÷ 3 = ?	First performing operation in brackets, then operations from left to right	37 + 8 – (9 × 9) ÷ 3 = 37 + 8 – 81 ÷ 3 = 45 – 81 ÷ 3 = 36 ÷ 3 = 12

When reasons for students' solutions to symbolic expressions are investigated, students performed operations from left to right and did not take care about characteristics between operations and order of operations. When performing operations, they only knew operations in brackets were performed first and firstly completed the operation in brackets and then continued with left to right operations afterwards.

The responses to confirmatory expression problems requiring students consider brackets, addition, subtraction, multiplication and division operations and the rule about order of operations are given in Table 3.

Table 3. Student Responses to Confirmatory Expressions Question Type

Problem	**Correct Answer**		**Incorrect Answer**		**Blank**		**Total**	
	f	**%**	**f**	**%**	**f**	**%**	**f**	**%**
14) 13 × 4 + 15 ÷ 5 = 55	173	72	25	10	42	18	240	100
15) 50 × 5 ÷ 5 – 30 = 20	202	85	13	5	25	10	240	100
16) 60 ÷ 4 + 25 × 5 – 11 = 129	197	82	8	3	35	15	240	100
17) 120 × 9 ÷ 3 + 6 – 5 = 361	218	91	5	2	17	7	240	100

When Table 4 is investigated, students used multiplication-addition-subtraction-division operations and brackets with the order of operations rule to reach the desired confirmatory expressions and this question type had higher correct answer percentages than all other problems.

Within the scope of the qualitative dimension of the research, the reasons students gave for incorrect answers in findings obtained from quantitative data to this type of question and sample student solutions are given in Table 4.

Table 4. Student Reasons and Sample Solutions for Confirmatory Expressions.

Problem	**Student reason**	**Example student solution**
14) 13 × 4 + 15 ÷ 5 = 55	Performing operations from left to right Checking the accuracy of the operation Inserting brackets by trial and error	15÷ 5 = 3 13 × 4 = 52 52 + 3 = 55 13 × 4 + (15 ÷ 5) = 55
15) 50 × 5 ÷ 5 – 30 = 20	Performing operations from left to right Inserting brackets by trial and error Checking the accuracy of the operation	(50 × 5) ÷ 5 – 30 = 20 50 × 5 = 250 250 ÷ 5 = 50 50 – 30 = 20
16) 60 ÷ 4 + 25 × 5 – 11 = 129	Performing operations from left to right Checking the accuracy of the operation Inserting brackets by trial and error	60 ÷ 4 = 15 25 × 5 = 125 125 + 15 = 140 140 – 11 = 129 (60 ÷ 4) + (25 × 5) – 11 = 129
17) 120 × 9 ÷ 3 + 6 – 5 = 361	Performing operations from left to right	120 × 9 ÷ 3 + 6 – 5 = 361

When student reasons for solving confirmatory expressions problems are examined, students firstly solved the equations, then inserted brackets based on trial and error and reached their answer by solving the problem from left to right.

The responses to true-false question problems to determine whether students took note of order of operations for arithmetic operations and operations within brackets with solutions provided in the question are given in Table 5.

Table 5. Student Responses to True-False Question Type

Problem	Correct Answer		Incorrect Answer		Blank		Total	
	f	%	f	%	f	%	f	%
18) 6 × (12 × 5) – (9 × 3) = equation is the same as 6 × 12 × 5 – 9 × 3 = equation	33	14	207	86	-	-	240	100
19) 6 + 5 × 3 – 9 = operation is the same as 5 × 3 – 9 + 6 operation	41	17	199	83	-	-	240	100
20) The solution to 9 + 6 × 2 ÷ 3 = is as follows 9 + 6 × 2 ÷ 3 = 15× 2 ÷ 3 = 30 ÷ 3 = 10.	51	21	189	79	-	-	240	100
21) 9 × (6 – 2) ÷ (7 – 4) = 12 is this answer true or false?	218	91	22	9	-	-	240	100

When Table 5 is investigated, where students used multiplication-addition-subtraction operations and brackets with the order of operations rule, there appear to be higher incorrect answer percentages for problems that distinguish whether the answer is given to arithmetic equations including multiplication-addition-subtraction operations together with addition-multiplication-division operations. Operations side-by-side are completed with the solution given stage by stage to reveal whether the order of operation rule is used or not. Additionally, in questions that were asked in order to reveal strategies about whether brackets are used when two of the same operations are present in multiplication and subtraction operations, the students were determined to have higher correct answer percentages.

Within the scope of the qualitative dimension of the research, the reasons students gave for incorrect answers in findings obtained from quantitative data to this type of question and sample student solutions are given in Table 6.

Table 6. Student Reasons and Sample Solutions for True-False Questions.

Problem	Student reason	Example student solution
18) 6 × (12 × 5) – (9 × 3) = equation is the same as 6 × 12 × 5 – 9 × 3 = equation	Solving the problem Firstly doing operations in the brackets	1st operation: 6 × (12 × 5) – (9 × 3) = 12 × 5 = 60 9 × 3 = 27 60 – 27 = 33 33 × 6 = 198 2nd operation: 6 × 12 × 5 – 9 × 3 = 6 × 12 = 72 72 × 5 = 360 9 × 3 = 27 360 – 27 = 333
19) 6 + 5 × 3 – 9 = operation is the same as 5 × 3 – 9 + 6 operations	Solving the problem Performing operations left to right	1st operation:6 + 5 × 3 – 9 = 6 + 5 = 11 11 × 3 = 33 33 – 9 = 24 2nd operation: 5 × 3 – 9 + 6 5 × 3 = 15 15 – 9 = 6 6 + 6 =12
20) The solution to 9 + 6 × 2 ÷ 3 = equation is as follows 9 + 6 × 2 ÷ 3 = 15× 2 ÷ 3 = 30 ÷ 3 = 10	Checking the accuracy of the operation Performing operations from left to right	9 + 6 × 2 ÷ 3 = 9 + 6 = 15 15 × 2 = 30 30 ÷ 3 = 10
21) 9 × (6 – 2) ÷ (7 – 4) = 12 is this answer correct or incorrect?	Solving the problem Firstly doing operations in the brackets Performing operations from left to right	9 × (6 – 2) ÷ (7 – 4) = 12 6 – 2 = 4 7 – 4 = 3 9 × 4 = 36 36 ÷ 3 = 12

When reasons students gave for solving this type of problem are investigated, students solved the equations given in the question, compared their own solutions and answered the question in this way. They completed the operations in brackets first and they solved the operations from left to right to reach the answer.

The answers to the open-ended question type to determine whether students used the order of operations rule in writing arithmetic equations using arithmetic operations equal to a given number are given in Table 7.

Table 7. Student Responses to Open-Ended Question Type

Problem	Correct Answer		Incorrect Answer		Blank		Total	
	f	%	f	%	f	%	f	%
22) Using the following operations and determining the order of operations yourself, write an arithmetic equation equal to the number 24. a. Addition and multiplication b. subtraction and division c. addition and division d. subtraction and multiplication	37	15	161	67	42	18	240	100

When Table 7 is investigated, students appear to have higher incorrect answer percentages for this question type.

Within the scope of the qualitative dimension of the research, the reasons students gave for incorrect answers to this type of question in findings obtained from quantitative data and sample student solutions are given in Table 8.

Table 8. Student Reasons and Sample Solutions for Open-Ended Questions.

Problem	Student reason	Example student solution
22) Using the following operations and determining the order of operations yourself, write an arithmetic equation equal to the number 24. a. Addition and multiplication b. subtraction and division c. addition and division d. subtraction and multiplication	Writing arithmetic equations with operations from left to right	a) 3 + 1 =4 4 × 6 = 24 b) 36 – 12 = 24 24 ÷ 1 = 24 c) 24 + 0 = 24 24 ÷ 1 = 24 d) 72 – 60 = 12 12 × 2 = 24

When student reasoning for solutions to open-ended question types is investigated, students appear to use the operations as they were given in the question (addition and multiplication, subtraction and division, addition and division, subtraction and multiplication), did not use operations based on priority and performed operations from left to right.

When students were asked to use arithmetic operations to translate a symbolic expression into a problem sentence with regard to the order of operations rule, the answers given to this type of question are shown in Table 9.

Table 9. Student Responses to Translating Symbolic Expressions to Problem Statements Question Type

Problem	Correct Answer		Incorrect Answer		Blank		Total	
	f	%	f	%	f	%	f	%
23) Write a problem sentence in accordance with the symbolic equation 6 × 2 + 5 × 3 = 27	58	24	75	31	107	45	240	100

When Table 9 is investigated, students generally left this type of question blank. As a result, it can be said they could not create a problem sentence using the order of operations rule from these symbolic expressions using arithmetic operations.

Within the scope of the qualitative dimension of the research, the reasons students gave for incorrect answers to this type of question in findings obtained from quantitative data and sample student solutions are given in Table 10.

Table 10. Student Reasons and Sample Solutions for Translating Symbolic Expressions to Problem Statement

Problem	Student reason	Example student solution
23) Write a problem sentence in accordance with the symbolic equation 6 × 2 + 5 × 3 = 27	Writing problem sentence that does not abide by the symbolic equation	"Ali goes to the shop. He buys 12 eggs. Six eggs are rotten and 2 are good. His mother sends him to the shop again. Ali buys 5 eggs. Then 3 eggs are good. If Ali used multiplication on the eggs, in other words if he multiplied them, how many would it be?"
	Not writing an appropriate problem sentence for the arithmetic equation using order of operations	

When student reasons for answering this type of question are investigated, students could not write an appropriate problem sentence for the arithmetic expressions involving addition and multiplication. Students who did write problem sentences did not correctly state the problem sentence.

The responses of students when requested to symbolically state a problem sentence with arithmetic operations using the order of operations rule are given in Table 11.

Table 11. Student Responses to Problem Statement Question Type

Problem	Correct Answer		Incorrect Answer		Blank		Total	
	f	%	f	%	f	%	f	%
24) In the market there are 35 boxes with six packs and 15 boxes with four packs of fruit juice. a. Using the numbers 35, 6, 15 and 4 in the question, write the symbolic equation to find the total number of fruit juice in the market b. Solve this problem	221	92	19	8	-	-	240	100
25) A greengrocer has 30 bananas in each banana crate, 16 apples in each apple crate, and 8 peaches in each peach crate. The greengrocer only had 30 bananas, 3 crates of apple and 8 peaches. Half of the 8 peaches are rotten. a. Write a symbolic equation to find the number of fruits in the greengrocers b. Solve this problem.	232	97	8	3	-	-	240	100

When Table 11 is investigated, students appeared to have high percentages of correct answers to this type of problem.

Within the scope of the qualitative dimension of the research, the reasons students gave for incorrect answers to this type of question in findings obtained from quantitative data and sample student solutions are given in Table 12.

Table 12. Student Reasons and Sample Solutions for Problem Statement.

Problem	Student reason	Example student solution
24) In the market there are 35 boxes with six packs and 15 boxes with four packs of fruit juice. a. Using the numbers 35, 6, 15 and 4 in the question, write the symbolic equation to find the total number of fruit juice in the market b. Solve this problem	Solve the problem Complete the operations in brackets	b) 35 × 6 = 190 15 × 4 = 60 190 + 60 = 250 a) (35 × 6) + (15 × 4) =
25) A greengrocer has 30 bananas in each banana crate, 16 apples in each apple crate, and 8 peaches in each peach crate. The greengrocer only had 30 bananas, 3 crates of apple and 8 peaches. Half of the 8 peaches are rotten. a. Write a symbolic equation to find the number of fruits in the greengrocers b. Solve this problem.	Solve the problem Complete the operations in brackets	b)16×3 = 48 8 ÷ 2 = 4 30 + 48 +4 =82 a)(30 + 16 ×3) +8 ÷ 2 =82

When the reasons students gave for answering this type of questions are investigated, students provided appropriate solutions for the problem sentences given in the question, first solved the problem and then did the operations in brackets and used the order of operations.

Conclusion and Discussion

When the quantitative and qualitative findings obtained in the research are combined and assessed, the conclusion given in Figure 1 was reached.

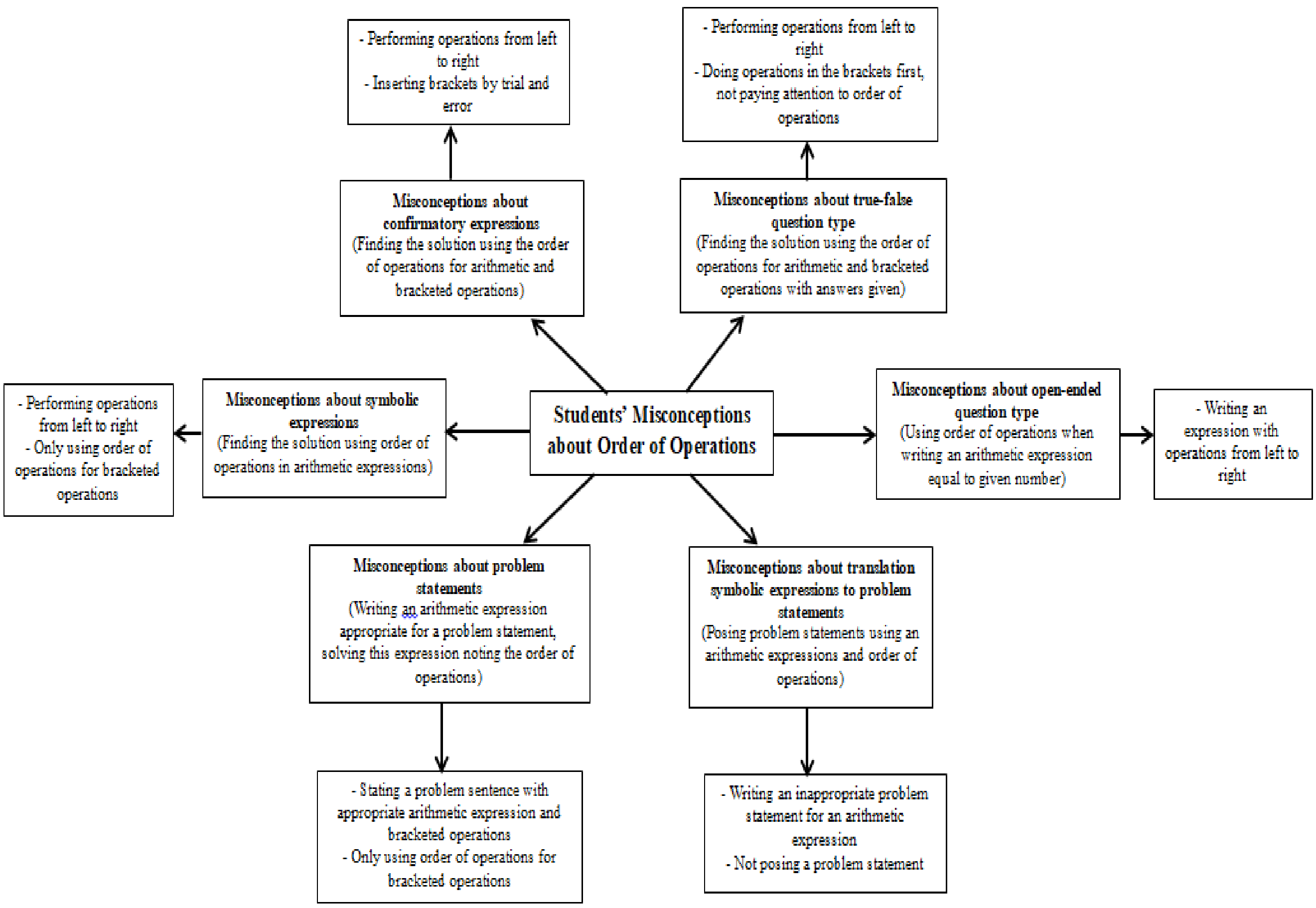

Figure 1. Students Misconceptions about Order of Operations

Based on Figure 1, generally 6th, 7th and 8th grade students did not pay attention to order of operations in arithmetic expressions but performed operations from left to right. They only stated that operations in brackets needed to be completed first in terms of order of operations and used this when solving problems. They did not pay attention to order of operations when writing an arithmetic expression equal to a given number but wrote and performed operations from left to right. The students could not pose a problem sentence, or had difficulty doing so, containing arithmetic operations and requiring the order of operations and did not pay attention to order of operations when solving the problem.

In this study, the aim was to determine misconceptions of middle school 6th, 7th and 8th grade students about order of operations. For this purpose, students' misconceptions about arithmetic operations and problems that require order of operations were examined.

First of all, students' misconceptions about symbolic expressions, confirmatory expressions, true-false questions, and open-ended questions that require order of operations in arithmetic were determined. In this respect, it was determined that students did not pay attention to the necessity of performing multiplication and division before addition and subtraction in all the mentioned question types and they found the result of the arithmetic expression by performing operations from left to right without paying attention to the order of operations. It is seen that this situation was revealed in other studies conducted about order of operations (Blando, Kelly, Schneider & Sleeman, 1989; Linchevski & Livneh, 1999; Pappanastos, Hall & Honan, 2002; Vanderbeek, 2007; Wu, 2007; Yenilmez & Coksoyler, 2018)

Also, it was observed that the students only performed the operation in parentheses, and when they reached the result by performing the operation from left to right without paying attention to the necessity of multiplication and division before addition and subtraction. Therefore, it can be said that the students have information about the order of operations, so that only the parentheses should be done first. This result proves that students make the order of operations more meaningful at the procedural level. According to Glidden (2008), Joseph (2014), Lee (2000) and Pappanastos, Hall and Honan (2002), students do not know the order of operations at the conceptual level and this situation stems from the teaching of this concept. Therefore, instead of memorizing the rule and using mnemonic devices while teaching order of operations, it is necessary to support with arithmetic expressions that students will connect with their prior knowledge and learn this concept based on real-life problems.

Also, students' misconceptions about posing and solving problems that require order of operations were determined. In the research, it appears students could not write or had difficulty writing arithmetic expressions and problems using arithmetic operations and order of operations. Similar situations were revealed in research by Ocal, Ipek, Ozdemir and Kar (2018), Uca (2010) and, Yenilmez and Coksoyler (2018). Gardella (2009) and Joseph (2014) state that this situation arises from the fact that arithmetic expressions cannot be correlated effectively with real-life situations, and more procedural knowledge is included in mathematics teaching. As a result, in terms of ensuring students can form problems related to arithmetic expressions and can solve problems, it is thought that teaching will be more effective if it is based on the links between mathematical statements and real-life.

According to the results of the research, 6th, 7th and 8th grade students do not make sense of the concept of order of operations at a conceptual level. This result confirms that the order of operations expressed by Glidden (2008) is an important concept for students at every grade level. In Turkey, in the mathematics curriculum the concept of order of operation is included within the scope of the 6th grade level. At this point, it is seen that the students do not learn this concept, and they know the order of operations only in operations where the brackets are used. This can be said to be due to the teaching of this concept and the use of only textbooks in teaching (Lee, 2000). Consequently, further research can focus on teachers' knowledge of order of operations and how they teach this concept.

In this study, it was determined that middle school 6th, 7th and 8th grade students have misconceptions about the order of operations. In future research, experimental studies in which real-life problems that require order of operations are taught should be completed in order to avoid these misconceptions.

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