



## Classroom Reflections of Model-Based Instruction: Ace Teaching Cycle\*

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### Abstract

Aim of this study was to reveal reflections of the instruction process based on ACE (Activity, Class Discussion, Exercises) teaching cycle. The study was carried out with 7<sup>th</sup> graders in a public middle school on equations in 2014-2015 academic year. This quasi-experimental study lasted for 20 hours in total. The whole implementation process was videotaped, and the records were transcribed by the researcher and was written as a document. The obtained documents were analyzed through content analysis. Consequently, it was revealed from the classroom reflections that the students could express their ideas easily, they could question their incorrect or incomplete ideas without hesitation, they learnt how to act with reference to these incomplete ideas, they perceived that incorrect ideas are as important as the correct ones, different approaches were precious and different approaches could emerge for all circumstances.

**Keywords:** Mathematical abstraction, APOS Theory, ACE Teaching Cycle, Mathematics teaching with model, Classroom reflections.

\* This study is a part of first author's PhD dissertation (Açıl, 2015) under the supervision of second author.

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# Model Temelli Öğretimin Sınıf İçi Yansımaları: Ace Öğretim Döngüsü\*

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## Öz

Bu çalışmanın amacı matematiksel soyutlama temelli geliştirilen ACE (Activity, Class Discussion, Exercises) öğretim modeline göre uygulanan öğretim sürecinin yansımalarını sunmaktır. Uygulama 2014-2015 eğitim-öğretim yılında Erzurum iline bağlı bir devlet ortaokulunun 7. sınıfında denklem alt öğrenme alanında gerçekleştirilmiştir. Yarı deneysel yöneme göre şekillendirilen bu çalışma, toplamda 20 saat süren bir uygulama sürecini kapsamaktadır. Uygulamanın tamamı video ile kayıt altına alınmış, daha sonra bu kayıtlar araştırmacı tarafından transkript edilerek yazılı dokümanlar haline getirilmiştir. Elde edilen dokümanlar içerik analizine tabi tutulmuştur. Araştırma sonunda sınıf içi yansımalar; öğrencilerin düşüncelerini rahatlıkla ifade edebildikleri, yanlış düşüncelerinin ya da eksik düşüncelerinin çekinilmeden sorgulayabildikleri, eksik düşüncelerden hareket etmeyi öğrendikleri, yanlış düşüncelerin en az doğru düşünceler kadar değerli olduğunu algıladıkları, farklı yaklaşımların değerli olduğu ve tüm durumlar için farklı yaklaşımların olabileceği gibi bazı çıkarımlar elde edilmiştir.

**Anahtar Kelimeler:** Matematiksel soyutlama, APOS Teorisi, ACE Öğretim Döngüsü, Modelle matematik öğretimi, Sınıf içi yansımalar.

\* Bu makale ikinci yazarın danışmanlığında yürütülen birinci yazarın doktora tezinin (Açıl, 2015) bir bölümüdür.

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## 1. Introduction

One of the most crucial characteristics of human beings is the ability to think. Thinking is described as a competence to make comparisons, to separate, to join, to comprehend connections and formations (TDK, 2018). This competence is a complex process that is necessary for people to adapt their surroundings. Especially, mathematical thinking becomes more complicated since both mathematics and thinking are quite rich. Mathematical thinking is a rich structure containing concepts such as guessing, induction, deduction, description, generalization, abstraction, exemplification and proving (Liu, 2003). Abstraction among these concepts is very important part of mathematical thinking process and it plays a key role in development of this process.

It can be told that there are a lot of descriptions about the concept of abstraction (Hampton, 2003; Hazzan and Zazkis, 2005; Noss and Hoyles, 1996). In this study, Dienes' description (1963) was taken into consideration. According to him, abstraction is the core of a thing which is common in different situations. When this and other descriptions are studied, it can be claimed that abstraction is a strategy of simplification.

Abstraction is a process that deals with cognitive activities of individuals like mathematics. This makes abstraction one of the important subjects of mathematics. There are several studies revealing the importance of abstraction in mathematics. While some researchers claim that abstraction is a basic process of math (Ferrari, 2003; Meel 2003; Yılmaz, 2011), some others think that it can lessen the complexity of mathematics (Mason and Pimm, 1984; McQuain and Keller, 2001; Mitchelmore and White, 2004a). Therefore, it is important to present the reflections of abstraction-based instruction.

There are perspectives that shape abstraction-based teaching and models developed from these perspectives. While APOS theory (Action-Process-Object-Shema) is a theory that deals with abstraction mechanism from a cognitive perspective, ACE teaching model is a pedagogical approach based on the theory. According to this model, lessons taught with traditional teaching model are re-arranged by getting divided into three parts which are purposeful (**activities-A**) prepared for the related subjects, (**class discussion-C**) during implementation of the activities, and extracurricular activities; that is homework (**exercises-E**), giving opportunity to the students to perform what they have learnt. It is significant what would instruction with a model bring both for the teachers and the students since a teaching model. Furthermore, the findings of this study will contribute about validity of the model. Therefore, mainly the following question was tried to be answered in the study:

What are the classroom reflections of instruction based on ACE teaching cycle?

### 1.1. APOS Theory and ACE Teaching Cycle

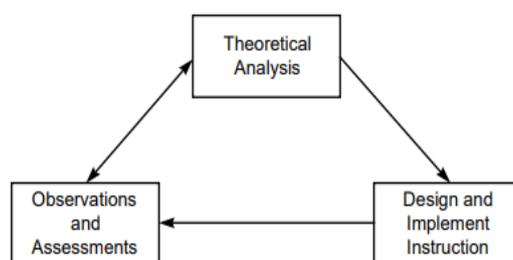
A model or a theory in mathematics can support prediction, have power of explanation, help individuals organize their thoughts about complex and interrelated phenomena, serve as a tool for data analysis and provide a language for communication of ideas about learning that go beyond superficial descriptions (Dubinsky & McDonald, 2001). Therefore, it can be claimed that using a model in teaching will support constructivist approach because of these contributions.

Using a model in teaching is available for various reasons such as to increase effectiveness of teaching, to empower created meaning, to help sustainability, to support conceptual learning or to reinforce teaching provided. The models used may vary according to theoretical framework on which they are based. ACE teaching model used in this study is a pedagogical approach based on APOS theory (Weller, Arnon and Dubinsky, 2009). APOS theory is a theoretical attempt created for explaining the mechanism of reflective abstraction which is used for describing development of logical thinking in children suggested by Piaget (Dubinsky, 1991).

Dubinsky (1991) stated that this theory arose from the hypothesis that "*mathematical knowledge consists in an individual's tendency to deal, in a social context, with perceived mathematical problem*

situations by constructing mental actions, processes, and objects and organizing them in schemas to make sense of the situations and solve the problems". This suggested theory has been called as APOS (Action-Process-Object-Schema) theory by referring to mental actions stated in the hypothesis. Several studies have been carried out on this theory. While some researchers used this theory as a research method (Cottrill, 1999; Montiel, Wilhelmi, Vidakovic and Elstak, 2009a; Kashefi, Ismail and Mohammed Yusof, 2010), some others tried to use it as an instructional model (Asiala, Cottrill, Dubinsky and Schwingendorf, 1997; Asiala, Dubinsky, Mathews, Morics and Oktaç, 1997; Çetin, 2009; Kathleen, 1999). On any ground, all of these studies revealed positive effects of the theory.

Asiala et al. (1997) suggested by regarding APOS theory that mathematical abstraction consisted of three components which are theoretical analysis, design and implement instruction and observations and assessments (Figure 1). ACE instructional model was suggested with reference to this structure. Components of the model:



**Figure 1. Research design based on APOS theory (Asiala et al., 1997)**

*A (Activities):* These are activities developed for improving abstraction skills of the students. As the goal is to improve cognitive thinking ways of the students, in activities, expressions requiring explanation and interpretation are preferred rather than expressions such as “What?” or “Find the solution.”

*C (Class Discussion):* It is providing students a setting where they can reflect their ideas and giving them opportunities to be active. What is important here is to carry out class discussion in an interactive way.

*E (Exercises):* These are exercises done outside the class with the aim of helping students use the mathematical concepts they learnt at school and of empowering the concepts learnt.

There are studies showing effects of ACE instructional model in the literature. These studies revealed that ACE instructional model is a model that provides information to be learnt by abstracting it, supports meaningful learning by affecting students’ perceptual structures, provides permanent learning and especially gives opportunity for students acquire mathematical skills (Asiala, Cottrill, Dubinsky and Schwingendorf, 1997; Asiala, Dubinsky, Mathews, Morics and Oktaç, 1997; Çetin; 2009; Maharaj, 2013; Tzirias, 2011; Weller et al., 2009). Additionally, effectiveness of ACE teaching cycle in helping students while learning mathematics and creating cognitive structures was highlighted in most of those studies.

## **2. Methodology**

### **2.1. Model of the Research**

This research is a quasi-experimental study intended to reveal classroom reflections of instruction implemented on a single group. It was formed according to experimental design without control group since there was only experimental group and no group for comparison. Design without control group is a plain and clear experimental research design in which empirical process is

carried out on a single group and which gives opportunity to observe changes formed by manipulation created by the researcher (Sönmez & Alacapınar, pp. 56-57, 2014).

## **2.2. Sample**

The experimental group consisted of 31 students, 16 of whom were male and 15 of whom were female. As aim of the research was to reveal how an implementation would shape the classroom atmosphere, the students' skills of expressing their ideas gained importance. Hence, it was necessary for group members to express themselves clearly. Moreover, it was obligatory rather than optional for the students to be objective while expressing their ideas. In this respect, the researcher's being applying teacher simultaneously contributed considerably. Furthermore, it was intended to put forward the position of the selected group compared to the other groups with some numeric data before intervention. With this aim, the students' scores of the readiness test in which questions that could be prerequisite for the subject of implementation were included and their achievement scores in mathematics in the previous academic semester were taken into consideration. As a result of the data obtained, it was understood that choosing this group was an accurate decision.

## **2.3. Implementation and Data Collection Process**

The data of the study were collected from a camera positioned by sighting the whole class. Videotape is a method that provides the environment analyzed to be observed again and again (Goodnough, 2011). Additionally, the researcher finds opportunity to analyze both his/her and the participants' behaviors during the research, and to see special cases that he/she missed. The research lasted for 20 hours of class time. The students' desks were in U-shape. Thus, the students could express their ideas interactively. The researcher functioned as a coordinator during the process and contributed the process by working as a manager about disagreements. No other intervention was applied. Mainly a process in which the students were active was dominant. The students discussed in groups or in whole class related to the activities they did during the class. Aim of the discussion was to create mental structures of the students (action, process, object and schema). After the discussion about the activities, the students were given homework to reinforce the subject they have learnt. The process continued as a cycle in this way. This cycle is referred as ACE teaching cycle.

Some books taught at home and abroad were used for preparing the activities applied in the study (Abels, de Jong, Dekker, Meyer, Shew, Burrill and Simon, 2006; Kindt, Roodhardt, Wijers, Dekker, Spence, Simon, Pligge and Burrill, 2006; Kindt, Wijers, Spence, Brinker, Pligge, Burrill and Burrill, 2006; Kindt, Dekker and Burrill, 2006; Wijers, Roodhardt, van Reeuwijk, Dekker, Burrill, Cole and Pligge, 2006). Aim of the activities was to help the students do reflective abstractions about the related subject. In addition to this, it was paid attention to encourage the students to think during the activities. Therefore, completely or partly wrong cases were given in some of the activities to steer the students to question their learning and to improve their causal thinking. Nineteen activities in total were used during the study.

## **2.4. Data Analysis**

The video records of 20 class hours of the research were listened by the researcher carefully and transcribed. Some of the records chosen randomly were also watched and transcribed by another mathematics teacher and a mathematics educator. Content analysis was made for the records transcribed into written texts. Aim of content analysis is to reach individuals who can explain the data collected (Yıldırım & Şimşek, 2011, p. 227). For reliability of the analysis the colleague

approval and expert opinion were applied. Compatibility between the analyzers was found 86%. Consensus was built on the topics that were unaccountable.

## **2.5. Validity and Reliability of the Research**

It was important to videotape all phases of the process since aim of the research was to reflect the classroom atmosphere. Existence of a material may distract students' attention and may cause them to display extraordinary behaviors. Particularly, individuals who know that they are videotaped may perform unusual behaviors. Videotaping the process with a camera provide a basis for such problems. These problems may be reduced if the students get accustomed to the camera. Therefore, using a camera before starting the real implementation can be useful. In this study, the camera was installed in the classroom 2 weeks earlier before the implementation, and the pre-process was shot. Thus, negative effects to be felt by the students caused by the camera were taken under control.

Reliability of the activities prepared was important for the study, too. After the learning outcomes were determined, a range of activities were prepared by making use of domestic and international resources. These activities were analyzed by the experts, and the ones to be used in the study were decided meticulously. The experts and experienced mathematics teachers were asked for checking suitability and sufficiency of these activities. Moreover, the pilot scheme gave the researcher feedback about structural suitability of the activities.

The records were watched and transcribed by another researcher for reliability of the data obtained. Transcriptions made by researchers in different places were compared to each other. Additionally, the researcher analyzed the same records two weeks later for confirmation. After confirmation, the same process was followed for content analysis.

## **3. Findings**

In this section, 4 samples which were considered to represent reflections of the research were presented. The samples that show how the students coped with the activities are as follows:

### ***Sample Implementation-1***

In this implementation, the students were given a figure pattern some boxes of which were partly shaded, and some questions regarding this figure were asked: 1- finding the general rule giving number of shaded boxes, 2- finding the general rule giving number of unshaded boxes, 3- finding the general rule giving number of all boxes, 4- explaining the relationship (if available) between these general rules (Appendix 1). The conversation among the students about this activity is as follows:





**Figure 2. Sample Implementation-1 case representation**

**Aleyna:** 1 is shaded in the first, 4, 9, 16 are shaded in the second. The first increased 3, the second 5 and the third 7; that is, increased by odd numbers.

**Taha:** I found the same result, too. It increased two by two.

**Aziz:** Teacher, let me say the rule  $2n$ .

**Samet:** I think Aziz's explanation is wrong because  $2n$  isn't appropriate even for the first row. I think general rule for this pattern is  $n^2$  because these are square of step numbers.

**Rüveyda:** Teacher, there are squares in each step. Number of the shaded squares is the same as area of this square.

**Rabia:** That is internal square, not outside.

**Rüveyda:** Therefore, number of shaded square at the row of  $n$ . becomes area of the square with  $n$ -sided square.

**Aziz:** It is  $n \cdot n$ , teacher.

**Teacher:** Okay. What can we say for the unshaded boxes?

**Elif:** The rule here should be  $4n + 4$ . Firstly, I wrote the number of boxes at each row; 8, 12, 16, 20. That is to say, it should be  $4n$  as a rule since it increased four by four. I multiplied 4 by 1 to find the first one, that is 4. To find 8, I should add 4, so it should be  $4n + 4$ .

**Dilara:** But we should check the others. The first step is not enough; we already know that it is applicable for other steps.

**Teacher:** Let's go on.

**Sefa:** Numbers of boxes are 9, 16, 25, 36, respectively. The rule enabling this is  $7n + 2$ .

**Samet:** Impossible! This formula is not valid for the third row.

**Alparslan:** Can it be  $4n + 4$ ?

**İrem:** It is not valid even for the first row. This is not possible.

**Yiğit:** I think  $(n + 2) \cdot (n + 2)$  (meanwhile, he writes this statement on the board).

**Sefa:** Hmm. Then it is  $n^2 + 4$ .

**Fatih:** No it is not possible, either.

**Tuana:** This is a perfect square because it multiplied two same statements.

**Yiğit:** Yes, we can write as  $(n+2)^2$ . Because, if we do this statement for each row, we can find the number of all boxes.

**Taha:** That is the number of both shaded and unshaded boxes.

**Teacher:** What can you tell about the statements you found respectively?

**Yasin:** There is  $n$  in all statements, and operations like addition and multiplication.

**Burcu:** Sum of these is, we wrote  $(n + 2) \cdot (n + 2)$  finally, and we can find something if we do this (she wants to write what she thinks on the board. She adds  $n \cdot n$  to  $(n + 2) \cdot (n + 2)$ , and tries to reach statement of  $4n + 4$ . While doing this, she explains that ' $n \cdot n = 2n$ ,  $2 \cdot 2 = 4$ ,  $n \cdot n = 2n$  in the beginning and the sum is  $4n + 4$ ).

**Yiğit:** I thought in the same way, but  $+4$  is not convenient.

**Yasin:** I think so.

**Ömer:** I think it is not problematic.

**İrem:** I think so. Shouldn't we already add  $n \cdot n$  to  $4n + 4$ ? It should be  $n \cdot n + 4n + 4 = n^2 + 4n + 4$ , but why did I do like this?

**Yiğit:** There is  $(n + 2)^2$  here. Teacher, sum of the shaded and unshaded boxes just give the total. It means there are two formulas in this statement. There is already  $n^2$  here. I found 4 in the second. There is  $2n$  here, but we need one more  $2n$ . I also found it, one minute.  $(n + 2) \cdot (n + 2)$  I divided this (he is doing operations). Teacher, here are the first two formulas.

**Ömer and Ayşegül:** We didn't understand what Yiğit did.

**Sefa:** I got it, and want to explain. When the second statement analyzed, the first two formulas are already in it. Because this statement gives us all boxes, and we needed these two rules.

**Fatih:** I want to state my opinion, too. If  $n^2$  is subtracted from  $(n + 2) \cdot (n + 2)$ , we can find  $4n + 4$ .

All the results obtained during the activity were elaborated with necessary explanations until all students understood them. In the previous activities, the students were helped to realize that sum or difference of two patterns is still a pattern. In this activity, they were also encouraged to visualize the situation in their minds. Therefore, time was allocated also for these explanations following the activity. It was highlighted that the students were required to think each activity independent from the previous ones and to structure them by associating them with each other.

As it can be seen in the conversation above, the students questioned how development of a perfect square statement is with reference to pattern rule. Although they used some incorrect statements in the beginning, they learnt the correct ones over time. The important issue here is that they tried to work in cooperation. In addition to this, participation of the class in the activities was quite high. An activity starting with confusion may make more participation essential.

### **Sample Implementation-2**

In this section, an activity related to the learning outcome "...is able to solve first degree equations with one variable." With this aim, the students were given two different patterns, and they were asked to question if the correlation (ascending, descending) of each term of these patterns at the same row would continue in the following steps or not. Furthermore, it was aimed to reveal if this circumstance would be significant (Appendix 2). The class discussion related to one part of this activity is as follows:



Figure 3. Sample Implementation-2 case representation

**Yasin:** Each of them proceeds according to a pattern, so the result cannot change. Hence, R's values are always greater than B's values.

**Aziz:** I think it's not true because I checked the difference between each term. The difference is 25 between the first terms, 24 between the second terms and 23 between the third terms. Therefore, it won't continue like this. It proceeds descendingly. I think it becomes lesser when it proceeds too much.

**Teacher:** What do you mean by "proceeds too much"?

**Aziz:** In my opinion, it will proceed to 1, 2, and then the second pattern will be greater.

**Tuana:** The first pattern increases by five and the second by six. It will go on like this and won't change. R would be greater, and B would be less.

**Teacher:** What can you tell about general terms of the patterns?

**Alparslan:** The first should be  $5n$  and the second should be  $6n$ .

**Yusuf:** We should add 29 to the statement of  $5n$  in the first pattern.

**Ayşegül:** 3 should be added for the second statement; that is,  $6n + 3$  (The students make the necessary mathematical operations while finding the general rule).

**Beyza:** Terms of the pattern will proceed in the same way. Both R and B are increasing; however, as the rise in R is more than the rise in B, R will always be greater than B.

**Sema:** The way of the pattern will be the same, and nothing will change. R will be greater than B. However, the difference will be zero as it will gradually diminish, and they will be equal at zero... I think it will change when the difference is balanced.

**İrem:** I think you misunderstand the question. While one is ascending by 5, the other is ascending by 6. It will get ahead upon closing the gap.

**Elif:** I also think that it will get ahead. For example, B will get ahead of R in the 27<sup>th</sup> term because when B becomes 164, R will be 163.

**Teacher:** Why in the 27<sup>th</sup> term?

**Elif:** The difference is 1 in the 27<sup>th</sup> term. They become equal when it is set to zero, but I could not count in which term.

**Sefa:** The terms get equal in the 26<sup>th</sup> term.

**Yiğit:** I think first 11 terms of the patterns were given, and the difference between the last terms is 15. It means that the difference will be 0 after 15 steps. That is 26<sup>th</sup> term as Sefa said. Even their equality proves that it doesn't proceed like that. We don't have to check the next step because it was always greater, but became equal in the 26<sup>th</sup> term.

**Teacher:** Alright. Are there any easier ways to do this?

**Yiğit:** We can do by using the pattern rules...

**Dilara:** Teacher, as the difference is 0 in the 26<sup>th</sup> term, the terms are equal here...

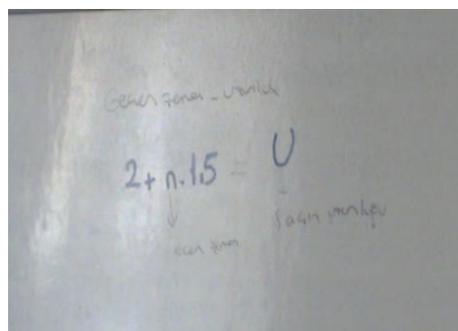
**Elif:** I think we can say briefly that the first amount of decrease is 25, and the differences become equal in the next step; that is,  $25+1=26^{\text{th}}$  term.

**Teacher:** Try to think regarding the pattern rules...

As it can be seen in this activity, the students' answers were not different from their previous statements in the process. They could not generalize the situation that they could express as a term like the 26<sup>th</sup> term's equalization mathematically such as equalization of general terms. In other words, the students could not consider the state of terms' being equal as equalization of two general terms. The teacher guided the students for helping them make generalizations like this. What is important here is that not only did the students express their opinions, but they also searched for a basis; that is, justified their statements. For instance, Yiğit stated that the fact that the differences between the terms were equal was an evidence that both of the two patterns could not proceed increasingly.

### **Sample Implementation-3**

The students were given a number pattern within bounds to find its general term. This pattern was illustrated in a table, and the students were asked to interpret this table. In the activity in which various questions related to the scenario were addressed, the students were also encouraged to design graphics. They were helped to generalize correlations they obtained via number pattern algebraically; to visualize with graphics and to correlate them (Appendix 3). The video part related to this activity is as follows:





**Figure 4. Sample Implementation-3 case representation**

**Ayşegül:** The first box in the table gives first length of Hasan's hair; that is to say, it was 2 cm when he went to the hairdresser's.

**Teacher:** Do you think how Ayşegül could find this?

**Taha:** Teacher, time when month was 0 in the table gives hair length in the beginning. In the part of 1 month, length is 3,5 cm. This means elapsed time, but when it was 2 cm no time passed. Then this was the first hair length.

**Teacher:** Alright! The hair length after 6 months is asked.

**Bezza:** As it grows 1,5 cm monthly, it becomes 11 cm after 6 months.

**Teacher:** Is it easy to count this?

**Rüveyda:** Because the amount of growing is 1,5. It means that there is a pattern, and it can be found easily by adding.

**Yiğit:** In other words, there is a rule.

**Buse:** Since there is a table, it is easier.

**Samet:** Is it easy because it is the 6<sup>th</sup> step?

**Teacher:** Why do you think Samet thinks so?

**Nergis:** Because 6 is a small number.

**Yusuf:** It is also a step which is empty box in the table.

**Teacher:** What if it was a big number?

**Fatih:** Then it couldn't be counted easily.

**Teacher:** What can we do then?

**Yiğit:** Let's write the rule so that we can find it directly. However, the first step here is 0.

**Samet:** Some patterns would start from zero. We can then change the general term.

**Yiğit:** Then the general term should be  $1,5 \cdot n + n$ .

**Berat:** It does not provide, at zero order is 0, but here is 2.

**İrem:** Then it should be +2.

**Yiğit:** Yes, I misunderstood it. This should be 2 not  $n$  (he edits his statement).

**Dilara:** Thus, we can find his hair length after 12 months easily. Let's write 12 instead of  $n$  in this formula. The answer is 20 cm.

**Teacher:** Can you interpret this statement that you found?

**Sema:** Amount of growing plus elapsed time is equal to hair length.

**Dilara:** Can I tell, too?

**Sema:** One minute, I will revise. The beginning amount plus growing amount times difference between them equals hair length.

**Teacher:** I don't ask you to re-read the explanation here, but to interpret it by considering the question.

**Ayşegül:** Here, initial and final states of hair are associated, but I can't express exactly.

**Ömer:** Hair length is one and a half times of elapsed time plus 2.

**Sefa:** It is overall length of 2 cm hair which grows 1,5 cm each month.

.....  
**Teacher:** Alright. How can we state these values and hair length changing with elapsed time in a different way?

**Ömer:** It is monthly in the table. Maybe we can edit it as every two months, so we get rid of the decimal number.

**Sema:** We indicated in the table.

**Taha:** We also found the equation showing the correlation between them.

**Yiğit:** I combine the table and the formula. In other words, I point out each value in one step by inserting all values in the table into the equation.

**Ayşegül:** Can we show in graphics?

**Teacher:** Sure. Can everybody try to do?

During the activity, while the students designed graphics, the teacher observed them by walking around the classroom. Most of the mistakes were made in designing graphics because of zero point. Afterwards, they completed graphic design altogether in a class discussion. Furthermore, they interpreted type of the graphic they designed (linear and ascending). Then they assessed the things done under the guidance of the teacher, and tried to explain the correlation among the table, the equation and the graphic.

In this activity, it was understood that the students could not explain what the mathematical statement meant and so they preferred to read it like reading a Turkish text (Sema and Ömer). Buse and Samet were aware of the fact that some small things provide convenience for bigger things, and Yiğit had an idea about explanation of this situation. This activity is a good example for revealing that proceeding by cooperating provides more effective results.

**İrem:** Teacher, this is really easy.

**Dilara:** Yes. I thought it was difficult.

**Samet:** I think we will start coordinate system following this subject.

We understand from the lines right above that the students became aware of the subject studied and which subject was based on it (Samet). This is directly related to predictive skill.

#### **Sample Implementation-4**

In this application, students were given problem situations related to the path taken by two frogs. It was questioned whether students could show the same algebraic expressions in different ways. They were asked to advance according to the scenario of the problem and to move from each other's thoughts (Appendix 4). The class discussion related to one part of this activity is as follows:

**Yiğit:** One jumps 5 times and the other 3 times. If distance is the same, it should be like this (he wants to draw a template about his statement).

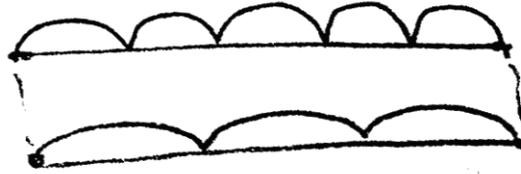


Figure 5. Yiğit's case representation

**Yusuf:** But the distances covered by the frogs' jumps are equal, and it can't be like that.

**Sema:** Teacher, I drew this. While one jumps 5 times, the other jumps three times, and they cover the same distance. However, distances they first cover are different, which is important. The difference is 2 jumps, and with these two jumps 8 dm is ahead of 18 dm. If we subtract 8 from 18, difference is 10 dm. If this difference is equal to two jumps, we divide this value by 2. That is to say, each jump is 5 dm.

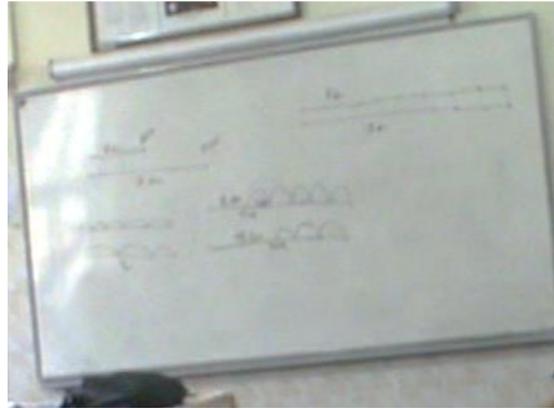


Figure 6. Sema's case representation

**İrem:** I think the first jumps 8 by 8, and the other jumps 18, don't they?

**Sema:** No. It just shows how far they are from the pathway at first. Number of each jump is not clear yet...

**Samet:** How can they be equal if each jump is 5 dm?

**Sema:** Initial distances are important.

**Yiğit:** Now that each jump is 5 dm, how many times did they jump until arriving here that they are at the distance of 8 and 18 dm? Then it needs to be sum times?

**Sema:** Maybe they jumped 2dm near the pathway, it doesn't concern us. What is important here is it is given beginning from the pathway.

**Nergis:** Maybe they didn't come by jumping. The question was started there, so it is not important.

**Aleyna:** Here we will find how long each jump is.

**Dilara:** I want to say something about accuracy of Sema's notes. I think both her drawings and values are correct. Because if we add 8 to 25, we find 33 dm which is total distance. On the other hand,  $18 + 15 = 33$  for Fred. Two values are the same, and it is stated in the question that they covered equal distance. Thus, the answer is true.

**Ömer:** But we haven't written it algebraically. This is the result and we verified it, but can we write it algebraically, too?

**Nergis:** We can write an equation...  $x \cdot 5 = 25$ , here we can find  $x$  is 5. But let's write  $x \cdot 5$ , I mean it should be  $x$  here (she notes on the template).

**Yiğit:** Shouldn't we write an algebraic statement here? The first way should be  $8 + 5x$ , and the second should be  $18 + 3x$ .

**Beyza:** An equal sign should be added since these two ways are equal.

**Nergis:** Yeah. Then let me write (she writes  $8 + 5x = 18 + 3x$  on the board).

**Fatih:** We can create one more equation (he writes  $8 + (5x - 3x) = 18$  on the board).

**Teacher:** What can you tell about Nergis and Fatih's statements?

**Fatih:** Actually, both statements are the same. Nergis used equality of the ways, and I wrote so.

**Elif:** Fatih, I think you made your operation by looking at Alice and Fred's jumping difference. Infact, both cases are the same.

**Rabia:** In my opinion, Fatih parenthesized the unknowns. There is  $3x$  on the one side; it is moved to the other side as  $-3x$ .

**Ayşegül:** As the unknowns are on one side in Fatih's statement and parenthetical is first to be done, we can find the value of  $x$ . We cannot find in Nergis's statement since there is  $x$  on both sides.

**Yiğit:** Teacher, both equations are the same. Just Fatih wrote the next step; in other words, we can say that the second equation was simplified.

**Yasin:** Fatih gathered unknowns together to one side, teacher. It looks more organized.

**Yiğit:** Teacher, then let's write:  $18 - 8 = 5x - 3x$ .

It can be seen that participation in the activity above was high. Therefore, these students' success states varied. It was observed that especially the students with low classroom performance and participation could state their opinions and criticize their friend's interpretations. On the other hand, it was understood from the students' responses that they frequently reasoned (such as Yiğit, Sema and Nergis). This is an indicator confirming that the activity setting motivated the students and helped them feel free. Additionally, in view of the classroom atmosphere observed by the teacher, it can be claimed that the activities prepared helped the students keep their attention on the related subjects.

#### **4. Discussion and Conclusion**

Aim of the research was to inform the readers about reflections of an implementation carried out based on ACE teaching cycle. With reference to the findings we obtained, it can be stated that revealing students' ideas is the first step of meaningful learning. Hence, situations like learning by questioning, focusing on mislearning and trying to find the correct way were encountered in all implementation samples. These are the key concepts for describing meaningful learning.

It was concluded from the study that all students, either active and successful or not, tried to be included in the process and placed more importance on each other's' opinions. It can be claimed that the activities leded the students, who were generally reluctant to participate in classes, to state their own ideas. Moreover, the fact that the students were encouraged to control themselves kept their motivation strong during the implementation. Providing such classroom settings can enable quality learning. This is an important issue which was also revealed some other previous studies (Akkaya, 2010; Sezgin-Memnun, 2011).

Another matter attracting attention in teaching with ACE teaching cycle was that interaction among students is a crucial element of learning process. It is believed that group work, peer discussion on a subject and acting on each other's' opinions contribute to students' own

development. This belief was observed in the first and last of the video records obtained during the process. In the beginning, the students behaved mostly independently, and so comprehension was more difficult. However, in the following implementations they improved their opinions by benefitting from each other's ideas and helped each other. Furthermore, the students realized in time that stating their opinions easily was an ordinary behavior, and they tried to understand that showing evidence and reason was important. For instance, Yiğit's explanation '...even its being equal proves that it cannot continue like this...' is the best example for this. Students' learning by justification always triggers more quality learning. Several studies can be found in the literature on this topic (Blair & Johnson, 1987; Driver, Newton & Osborne, 2000; Serin & Korkmaz, 2018).

Although the students apply skills such as justifying, proving and associating, they always behave timidly about generalization. This study also revealed that the students applied skills such as associating, reasoning, predicting, justifying and proving, but they behaved timidly on generalization. Actually, students' timidity on generalizations is not worth worrying because it is clear that easy generalizations generally cause mislearning. This may be an indicator of that students are more attentive on learning concepts. Hence, Akkan and Çakıroğlu (2012) and Soylu (2006) stated in their studies that problems caused by incorrect generalizations are too difficult to overcome. It can be deduced from this study that the students comprehended the fact that generalization is not so easy, and for achieving this, it is important to create meaning by associating the concept, by supporting with different ideas and by enriching with various situations. These have the characteristics to be prior conditions for abstraction of information. According to Ferrari (2003), abstraction is a main process for improvement of mathematical thinking, and it is associated with creation of new mathematical concepts.

It can be suggested that the implementation process gave opportunity to the students who were less successful than the others for closing the gap with their friends. For instance, Tuana was among the less successful students. However, in the first implementation she confidently said '...This is a perfect square because it multiplied two same statements...'. It is really interesting that a student with low success rate intervened in the process by explaining with mathematical expressions. The fact that the students found correct answers by benefitting from each other's ideas helped them realize that every idea may be important regardless of its accuracy. This became meaningful especially on the matter of integrating less successful students into the teaching process.

Finally, the students' concept usage was taken into consideration, too. It was seen that the students did not avoid using mathematical concepts, and they used these concepts accurately. Concept usage is important in view of mathematical communication. National Council of Teachers of Mathematics [NCTM] (2000) emphasizes that "all students from Pre-K to 12<sup>th</sup> grade need to convey their mathematical thoughts by using mathematical language actively" as well as regarding mathematical communication as one of the five process standards (p. 60). Therefore, it would be natural to talk about the benefits of such teaching.

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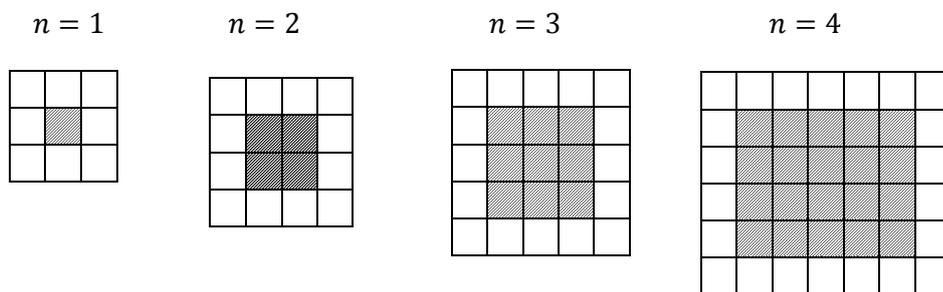
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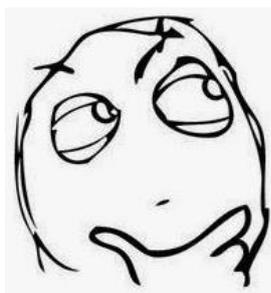
**Appendix 1.**

**PATTERNS AND RELATIONSHIPS**



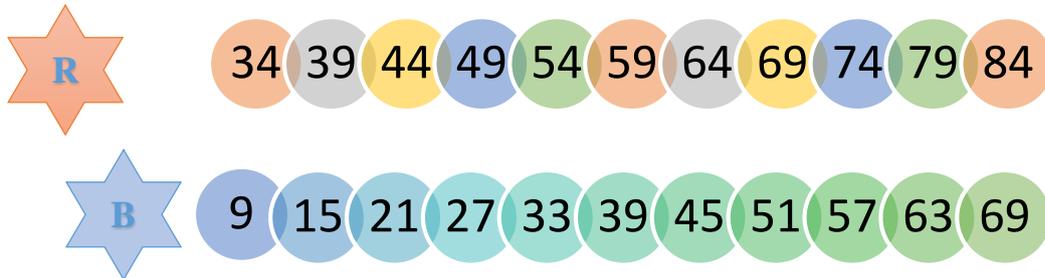
There is a pattern given above with the first four steps. According to this;

- 1) Let's write the relation that gives the number of the hatched boxes in any row ( $n$ ).
- 2) Let's write the relation that gives the number of non-hatched boxes in any row ( $n$ ).
- 3) Let's write the relation that gives the number of all the boxes in any row ( $n$ ).
- 4) Let's relate the relationships we have found in questions 1, 2 and 3.



Appendix 2.

EQUATIONS



When we compare the numbers in the above R and B schemes with each other;

$34 > 9, 39 > 15, 44 > 21...$

→ So if this comparison is continued as long as you want it, will it always give the same result?

Let's discuss.



**Appendix 3.**

**LINEAR EQUATIONS**



Hasan goes to the barbershop to cut his hair on Sunday. When she comes home she looks in the mirror and realizes that her hair is pretty short. He takes a decision not to take a haircut for a long time and decides to measure the speed of hair growth during this time. The following table shows the measurements that Hasan made every month.

Time(month)	0	1	2	3	4	5	6	7
Length(cm)	2	3.5	5	6.5				

→ After the haircut, how many centimeters was the length of Hasan's hair?

→ How many centimeters will be the length of Hasan's hair in 6 months?

→ Is it easy to calculate this length? Why?

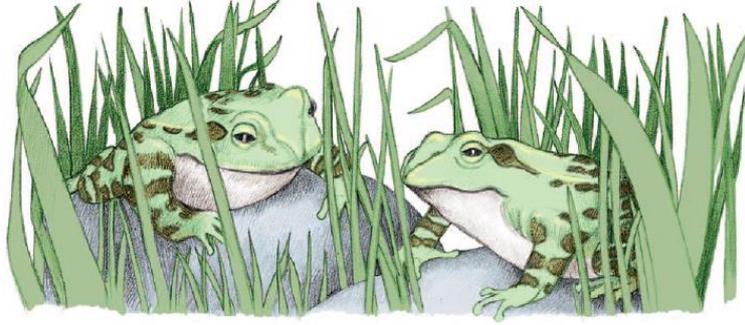
→ If Hasan never cuts his hair in one year, how many centimeters is the length of his hair?

→ Can you write a formula that related to time and length in the above table ?

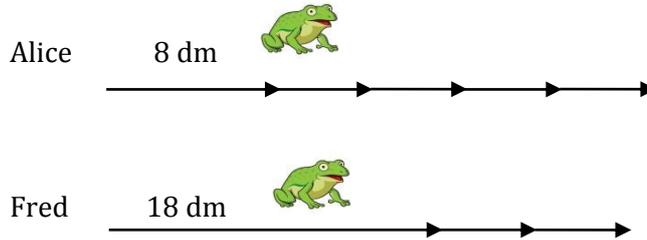
→ Can the relationship between length of the hair with passing time be shown different?

Appendix 4.

EQUATIONS



Alice and Fred are two frogs that are on a path in the forest. One day they suddenly hear footsteps and jump to get away from danger. Alice and Fred start to jump in the same direction from the pathway, respectively at a distance of 8 dm and 18 dm. They stop after they jump a few times.



→ Suppose that Alice jumps 5 times, Fred jumps 3 times and they take the same distance on the pathway. Farz edelim ki Alice 5 kez, Fred ise 3 kez zıplıyor ve patikada aynı mesafeyi alıyorlar. How many decimeters is the length of each jump? (**Note:** The length of Alice and Fred's jump is equal)

→ How can you be sure the answer is correct?

