Erzincan Üniversitesi
Fen Bilimleri Enstitüsü Dergisi
2019, 12(2), 759-770
ISSN: 1307-9085, e-ISSN: 2149-4584
Araştırma Makalesi

Erzincan University
Journal of Science and Technology
2019, 12(2), 759-770
DOI:10.18185/erzifbed.486806
Research Article

GP-Fuzzy Soft Groups

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Geliş / Received: 22/11/2018, Kabul / Accepted: 11/07/2019

Abstract

In this paper, the definition of a new concept which is called GP-fuzzy soft subgroup is introduced and some basic properties of the families of GP-fuzzy soft subgroups are examined.

Keywords: Groups, (Fuzzy) soft set, UP-fuzzy soft subset, GP-fuzzy soft subgroup

GP-Bulanık Esnek Gruplar

Öz

Bu makalede, GP-bulanık esnek altgrup olarak adlandırılan yeni bir kavram tanıtılmış ve GP-bulanık esnek altgrupların ailelerinin bazı temel özellikleri incelenmiştir.

Anahtar Kelimeler: Gruplar, (Bulanık) esnek kümeler, UP-bulanık esnek alt küme, GP-bulanık esnek altgrup

1. Introduction

The theory of soft sets originally is proposed by Molodtsov (1999) as a mathematical method to deal with uncertainties. In the sequel Maji et al. (2003) present some certain soft binary operations and some basic definitions on soft sets. Ali et al. (2009) discuss some results which are given by Maji et al and they give some new notions. Some potential of the soft sets such as the extension to some certain areas attract the attention of the researchers. Maji et al. (2001) combine the theory of soft set with the theory fuzzy set which was initiated by Zadeh in 1965. Their unified concept is referred as a fuzzy soft set. Yang et al. (2007) study on the notion of fuzzy soft sets. As a subsequent research, Ahmad and Kharal (2009) improve the studies on the fuzzy soft sets and define arbitrary fuzzy soft operations. The extension of fuzzy soft sets to algebraic structures has being interest since its introduction in 1999. Initially, Aygünoğlu and Aygün (2009) define the notion of fuzzy soft groups.

Akın and Karakaya (2018) have proposed a new concept which is called UP-fuzzy soft set recently. In their study they introduce the notion of SP-fuzzy soft subsemigroup. In this paper, we investigate the notion of UP-fuzzy soft set for groups.

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2. Preliminaries

A function f from a nonempty set A to the unit interval [0,1] is called a fuzzy subset of A (Zadeh, 1965). $F(A)$ denotes the set of all of the fuzzy subsets of A. Let f, g be fuzzy subsets of A, then $f \subseteq g$ means that $f(a) \leq g(a)$ for all $a \in A$. For any $\alpha \in [0,1]$, the set $f_\alpha = \{a \in A | f(a) \geq \alpha\}$ is called the α-level set of f. Let $\{f_i | i \in \Lambda\}$ be a family of the fuzzy subsets of A and $x \in A$. Then $\wedge$ and $\vee$ operations are defined by:

$$(\bigwedge_{i\in\Lambda} f_i)(x) = \bigwedge_{i\in\Lambda} f_i(x),$$

$$(\bigvee_{i\in\Lambda} f_i)(x) = \bigvee_{i\in\Lambda} f_i(x),$$

Let B be a subset of A, then χ_B denote the characteristic function of B and it is defined by

$$\chi_B(x) = \begin{cases} 1 \quad , & x \in B \\ 0 \quad , & x \notin B. \end{cases}$$

Let A, B be nonempty sets, $\mu \in F(A)$, $\nu \in F(B)$ and $f: A \to B$ be a function. Then $f(\mu) \in F(B)$ is defined by $f(\mu)(b) = \bigvee_{f(a)=b} \mu(a)$ for all $b \in B$ and $f^{-1}(\nu) \in F(A)$ is defined by $f^{-1}(\nu) = \nu(f)$ (Klir and Yuan, 1995).

2.1. Fuzzy Subgroups

Let G be group and $f, g \in F(G)$. Then the product operation of f and g is denoted by $f \cdot g$ and defined by $(f \cdot g)(x) = \bigvee_{x=ab} f(a) \wedge g(b)$ for all $x \in G$. $f^{-1} \in F(G)$ is defined by $f^{-1}(x) = f(x^{-1})$ for all $x \in G$. If, for all $x, y \in G$, $f(x) \wedge f(y) \leq f(xy)$ and $f(x) \leq f(x^{-1})$, then f is called a fuzzy subgroup of G.If a fuzzy subgroup f of G satisfies the condition $f(xy) = f(yx)$ for all $x, y \in G$, then it is called a normal fuzzy subgroup of G. f is a normal fuzzy subgroup of G if and only if $f(xy) = f(yx)$ for all $x, y \in G$. (Das,1981; Mordeson and Nair, 2001; Rosenfeld, 1971)

2.2. Soft Sets

Some known and useful definitions and notations on soft sets are given in the following.

Definition 2.1. Let U be an initial universal set and P be a set of parameters. Then the power set of U is denoted by $P(U)$. Let A be a subset of P. Then a pair (F, A) is called a soft set over U where F is a mapping given by $F: A \to P(U)$ (Molodtsov, 1999). The pair (U, P) denotes the collection of all soft sets on U with the attributes from P and (U, P) is called a soft class (Kharal and Ahmad, 2011). In this paper, we consider a soft class (G, P) with a group G as the initial universal set.

Definition 2.2. Let (F, A) and (G, B) be two soft sets over G, (F, A) is called a soft subset of (G, B), denoted by $(F, A) \subseteq (G, B)$, *if (i)*$A \subseteq B$, *(ii)*$F(x) \subseteq G(x)$ for each $x \in A$ (Molodtsov, 1999).

Definition 2.3. Let $\{(F_i, A_i) | i \in \Lambda\}$ be a family of soft sets in a soft class (G, P). Then

a) The *restricted intersection* of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $(\cap_r)_{i\in\Lambda}(F_i, A_i)$, is the soft set (F, A) defined as: $A = \bigcap_{i\in\Lambda} A_i$, $F(x) = \bigcap_{i\in\Lambda} F_i(x)$ *(*$\forall x \in A$*),*
b) The *extended intersection* of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $(\cap_e)_{i\in\Lambda}(F_i, A_i)$, is the soft set (F, A) defined as: $A = \bigcup_{i\in\Lambda} A_i$, $F(x) = \bigcap_{i\in\Lambda(x)} F_i(x)$ *(*$\forall x \in A$*)* where $\Lambda(x) = \{i | x \in A_i\}$,
c) The *restricted union* of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $(\cap_r)_{i\in\Lambda}(F_i, A_i)$, is the soft set (F, A)

defined as: $A = \bigcap_{i \in \Lambda} A_i$, $F(x) = \bigcup_{i \in \Lambda} F_i(x)$ ($\forall x \in A$).

d) The *extended union* of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $(\bigcap_e)_{i \in \Lambda}(F_i, A_i)$, is the soft set (F, A) defined as: $A = \bigcup_{i \in \Lambda} A_i$, $F(x) = \bigcup_{i \in \Lambda(x)} F_i(x)$ ($\forall x \in A$)

e) The $\wedge$-*intersection* of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $\wedge_{i \in \Lambda} (F_i, A_i)$, is the soft set (F, A) defined as: $A = \prod_{i \in \Lambda} A_i$, $F((x_i)_{i \in \Lambda}) = \bigcap_{i \in \Lambda} F_i(x_i)$ ($\forall (x_i)_{i \in \Lambda} \in A$),

f) The $\vee$-*union* of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $\vee_{i \in \Lambda} (F_i, A_i)$, is the soft set (F, A) defined as: $A = \prod_{i \in \Lambda} A_i$, $F((x_i)_{i \in \Lambda}) = \bigcup_{i \in \Lambda} F_i(x_i)$ ($\forall (x_i)_{i \in \Lambda} \in A$),

g) The *product* of the family $\{(F_i, A_i) | i \in \Lambda\}$, denoted by $\prod_{i \in \Lambda} (F_i, A_i)$, is the soft set (F, A) defined as: $A = \prod_{i \in \Lambda} A_i$, $F((x_i)_{i \in \Lambda}) = \prod_{i \in \Lambda} F_i(x_i)$ ($\forall (x_i)_{i \in \Lambda} \in A$).

(Ali et al., 2009; Çelik et al., 2011; Feng et al., 2008; Kazancı et al., 2010; Maji et al., 2003; Pei and Miao, 2005).

2.3. Fuzzy Soft Sets

In the following definitions we give some useful and known concepts of fuzzy soft sets.

Definition 2.4. Let U be an initial universal set and P be a set of parameters. A pair (f, E) is called a fuzzy soft set over U, where f is a mapping given by $f: E \to F(U)$ (Maji et al., 2001). The pair $\widetilde{(U, P)}$ denotes the collection of all fuzzy soft sets on U with the attributes from P and it is called a fuzzy soft class (Kharal and Ahmad, 2011). In this paper, we consider a fuzzy soft class $\widetilde{(G, P)}$ with a group G as the initial universal set.

Definition 2.5. Let (f, E) be a fuzzy soft set over U. For each $\alpha \in [0,1]$, the set $(f, E)_\alpha = (f_\alpha, E)$ is called an α-level set of (f, E), where $f_\alpha(a) = \{x \in G | f(a)(x) \geq \alpha\}$ for each $a \in E$. Obviously, $(f, E)_\alpha$ is a soft set over G (Aygünoğlu and Aygün, 2009).

Definition 2.6. Let (f, E) and (g, H) be two fuzzy soft sets over G, (f, E) is called a fuzzy soft subset of (g, H), denoted by $(f, E) \widetilde{\subseteq} (g, H)$, if (i) $E \subseteq H$, (ii) for each $a \in E$, $f(a) \leq g(a)$ (Maji et al., 2001).

Definition 2.7. Let $\{(f_i, E_i) | i \in \Lambda\}$ be a family of fuzzy soft sets in a fuzzy soft class $\widetilde{(G, P)}$. Then

a) The *restricted intersection* of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $\widetilde{\bigcap}^r_{i \in \Lambda}(f_i, E_i)$, is a fuzzy soft set (f, E), $E = \bigcap_{i \in \Lambda} E_i$ and for all $x \in E$, $f(x) = \wedge_{i \in \Lambda} f_i(x)$.

b) The *extended intersection* of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $\widetilde{\bigcap}^e_{i \in \Lambda}(f_i, E_i)$, is a fuzzy soft set (f, E), $E = \bigcup_{i \in \Lambda} E_i$ and for all $x \in E$, $f(x) = \wedge_{i \in \Lambda(x)} f_i(x)$ where $\Lambda(x) = \{i | x \in E_i\}$,

c) The *restricted union* of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $\widetilde{\bigcup}^r_{i \in \Lambda}(f_i, E_i)$, is a fuzzy soft set (f, E), $E = \bigcap_{i \in \Lambda} E_i$ and for all $x \in E$, $f(x) = \vee_{i \in \Lambda} f_i(x)$.

d) The *extended union* of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $\widetilde{\bigcup}^e_{i \in \Lambda}(f_i, E_i)$, is a fuzzy soft set (f, E), $E = \bigcup_{i \in \Lambda} E_i$ and for all $x \in E$, $f(x) = \vee_{i \in \Lambda(x)} f_i(x)$.

e) The *fuzzy* $\wedge$-*intersection* of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by

$\widetilde{\wedge}_{i\in\Lambda}(f_i, E_i)$, is the soft set (f, E) defined as: $E = \prod_{i\in\Lambda} E_i$, $f((x_i)_{i\in\Lambda}) = \wedge_{i\in\Lambda} f_i(x_i)$ $(\forall (x_i)_{i\in\Lambda} \in E)$,

f) The *fuzzy* $\vee$-*union* of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $\widetilde{\vee}_{i\in\Lambda}(f_i, E_i)$, is the soft set (f, E) defined as $E = \prod_{i\in\Lambda} E_i$, $f((x_i)_{i\in\Lambda}) = \vee_{i\in\Lambda} f_i(x_i)$ $(\forall (x_i)_{i\in\Lambda} \in E)$.

g) The *product* of the family $\{(f_i, E_i) | i \in \Lambda\}$, denoted by $\widetilde{\prod}_{i\in\Lambda}(f_i, E_i)$, is a fuzzy soft set (f, E), $E = \prod_{i\in\Lambda} E_i$ and $f((x_i)_{i\in\Lambda}) = \vee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\wedge_{j\in J} f_j(x_j))$.

(Ahmad and Kharal, 2009; Çelik et al., 2013; Maji et al., 2001)

Definition 2.8. Let $(f, E_1), (g, E_2)$ be fuzzy soft sets in a fuzzy soft class $(\widetilde{G, P})$. Then the fuzzy *product* of them, denoted by $(f, E_1) \widetilde{\times} (g, E_2)$, is the soft set (h, C) defined as $C = E_2 \times E_1$, $h(a, b) = f(a) \cdot g(b)$ for all $a \in E_1, b \in E_2$ (Çelik et al., 2013).

2.4. Soft Groups and Fuzzy Soft Groups

Some known and useful definitions on soft and fuzzy soft groups are shown forth as follows.

Definition 2.9. Let (F, E) be a soft set over G, (F, E) is called a soft group over G if and only if $F(a)$ is a subgroup of G for each $a \in E$ (Aktaş and Çağman, 2007; Aslam and Qurashi, 2012; Yin and Liao, 2013).

Definition 2.10. Let (F, E) be a soft group over G, (F, E) is called a normal soft group over G if and only if $F(a)$ is a normal subgroup of G for each $a \in E$ (Aktaş and Çağman, 2007; Aslam and Qurashi, 2012).

Definition 2.11. Let (f, E) be a fuzzy soft set over G, (f, E) is called a fuzzy soft group if and only if $f(a)$ is a fuzzy subgroup over G for each $a \in E$ (Aygünoğlu and Aygün, 2009).

Definition 2.12. Let (f, E) be a fuzzy soft set over G, (f, E) is called a normal fuzzy soft group if and only if $f(a)$ is a normal fuzzy subgroup over G for each $a \in E$ (Aygünoğlu and Aygün, 2009).

Definition 2.13. Let (f, E) be a fuzzy soft group over G and $\alpha, \in [0,1]$. Then (f, E) is said to be a α-identity fuzzy soft group over G if, for all $a \in E, x \in G$, $f(a)(x) = \begin{cases} \alpha, & if\ x = e \\ 0, & otherwise \end{cases}$ (Aygünoğlu and Aygün, 2009).

Definition 2.14. Let $(U_1, P_1), (U_2, P_2)$ be soft classes and $\varphi: U_1 \to U_2, \psi: P_1 \to P_2$ be functions. Then the pair (φ, ψ) is called a fuzzy soft function from U_1 to U_2. (Aygünoğlu and Aygün, 2009).

Definition 2.15. Let (f, A) and (g, B) fuzzy soft sets on the classes (U_1, P_1) and (U_2, P_2), respectively and let (φ, ψ) be called a fuzzy soft function from U_1 to U_2.Then

a) The image of (f, A) under the soft function (φ, ψ), denoted by $(\varphi, \psi)(f, A)$, is the fuzzy soft set on the class (U_2, P_2) defined by $(\varphi, \psi)(f, A) = (\varphi(f), \psi(A))$, where $\varphi(f)(b)(y) = \begin{cases} \vee_{\varphi(x)=y} \vee_{\psi(a)=b} f(a)(x), & if\ \exists x \in \varphi^{-1}(y) \\ 0 & , otherwise \end{cases}$, $(\forall b \in \psi(A)$ and $\forall y \in U_2)$.

b) The pre-image of (g, B) under the fuzzy soft function (φ, ψ), denoted by $(\varphi, \psi)^{-1}(g, B)$, is defined by $(\varphi, \psi)^{-1}(g, B) = (\varphi^{-1}(g), \psi^{-1}(B))$, where $\varphi^{-1}(g)(a)(x) = g(\psi(a))(\varphi(x))$, $(\forall a \in \psi^{-1}(B), \forall x \in U_1)$

(Aygünoğlu and Aygün, 2009).

Definition 2.16. Let (φ, ψ) be a fuzzy soft function from G to H. If φ is a homomorphism from G to H then (φ, ψ) is said to be fuzzy soft homomorphism (Aygünoğlu and Aygün, 2009).

2.5. UP-Fuzzy Soft Sets

Akın and Karakaya (2018) propose a fuzzy soft set of a crisp soft set as new concept of a member of the class $(\widetilde{U, P})$. They give the following definitions.

Definition 2.17. Let (F, A) be a soft set in a soft class (U, P) and (f, E) be a fuzzy soft set in the fuzzy soft class $(\widetilde{U, P})$. Then (f, E) is said to be a UP-fuzzy soft subset of (F, A), denoted by $(f, E) \subseteq_{UP} (F, A)$, if $E \subseteq A$ and $f(x)$ is a fuzzy subset of $F(x)$ for all $x \in E$.

Definition 2.18. Let (f, E) be a UP-fuzzy soft subset of (F, A). (F_α, E) called α-level soft subset of (f, E), where $F_\alpha : E \to P(U)$ is defined by $F_\alpha(x) = \{a \in F(x) | f(x)(a) \geq \alpha\} = f(x)_\alpha$ for all $x \in E$.

Definition 2.19. Let (f, E) be a UP-fuzzy soft subset of (F, A). Then the UP-fuzzy soft subset $(f, E)^c := (f^c, E)$ of (F, A) is called the complement of (f, E), where for any $x \in E$, $f^c(x): F(x) \to [0,1]$ is defined by $f^c(x)(a) = 1 - f(x)(a)$ for all $a \in F(x)$.

Definition 2.20. Let S be a semigroup and let (F, A) be a soft subsemigroup in a soft class (S, P) and (f, E) be a fuzzy soft set in the fuzzy soft class $(\widetilde{S, P})$. (f, E) is called a SP-fuzzy soft subsemigroup of (F, A) if $E \subseteq A$ and $f(x)$ is a fuzzy subsemigroup of $F(x)$ for all $x \in E$.

3. GP-Fuzzy Soft Groups

In this section we give a definition of a new concept which is called GP-fuzzy soft group, where G is a group. Then we investigate some properties of GP-fuzzy soft group. Throughout this section G, H, N will be considered as groups.

Definition 3.1. Let (F, A) be a soft group in a soft class (G, P) and (f, E) be a fuzzy soft set in the fuzzy soft class $(\widetilde{G, P})$. (f, E) is called a GP-fuzzy soft group of (F, A) if $E \subseteq A$ and $f(x)$ is a fuzzy subgroup of $F(x)$ for all $x \in E$.

Example 3.2.

a) Let G be the group $(\mathbb{Z}_6, +)$ and $P = \{x_1, x_2, x_3\}$. Let $A = \{x_1, x_2\}$ and $F(x_1) = \{\bar{0}\}$ and $F(x_2) = \{\bar{0}, \bar{2}, \bar{4}\}$. Let $E = \{x_2\}$ and (f, E) be defined by $f(x_2)(\bar{a}) = \begin{cases} 1, & if\ \bar{a} = \bar{0}, \\ 0, & otherwise \end{cases}$ for all $\bar{a} \in F(x_2)$. Then (f, E) is a GP-fuzzy soft group of (F, A).
b) Let G be the general linear group of matrices in $\mathbb{R}$ of type 2×2 and $P = \{x_1, x_2, x_3\}$. Let $A = \{x_1, x_2\}$ and $F(x_1) = \{[\delta_{ij}]\}$ and $F(x_2)$ be the special linear group of matrices in $\mathbb{R}$ of type 2×2, where $[\delta_{ij}]$ is unit matrix. Let $E = \{x_2\}$ and (f, E) be defined by $f(x_2)([a_{ij}]) = \begin{cases} 1, & if\ a_{ij} = \delta_{ij}, \\ 0, & otherwise \end{cases}$, $(\forall [a_{ij}] \in F(x_2))$. Then (f, E) is a GP-fuzzy soft group of (F, A).

Remark. Let (f, E) be a fuzzy soft set in the fuzzy soft class $(\widetilde{G,P})$. If (f, E) is a fuzzy soft group of G then it is a GP-fuzzy soft group of (F, A). Therefore the class of GP-fuzzy soft groups of a soft group covers the class of fuzzy soft groups and the inverse of this is not true in general. For instance, in Example 3.2 (b), the function $f(x_2)$ is not defined for the matrices which have the determinant different from 1.

Theorem 3.3. Let (F, A) be a soft group over G and (f, E) be a GP-fuzzy soft subset of (F, A). Then (F_α, E), if $f(x)_\alpha \neq \emptyset$ for all $x \in E$ and for any $\alpha \in (0,1]$, is a soft group over G for all $\alpha \in [0,1]$ if and only if (f, E) is GP-fuzzy soft subgroup of (F, A).

Proof. Suppose that (F_α, E) is a soft group for all $\alpha \in [0,1]$. Let $x \in E$ and $\alpha := f(x)(a) \wedge f(x)(b)$ for any $a, b \in F(x)$. So $f(x)(a) \geq \alpha$ and $f(x)(b) \geq \alpha$. Thus $a, b \in F_\alpha(x)$. $ab^{-1} \in F_\alpha(x)$ since $F_\alpha(x)$ is a subgroup of G for all $x \in E$. So $f(x)(ab^{-1}) \geq \alpha$, i.e, $f(x)(ab^{-1}) \geq f(x)(a) \wedge f(x)(b)$. Hence $f(x): F(x) \to [0,1]$ is fuzzy subgroup for all $x \in E$. Therefore (f, E) is a GP-fuzzy soft group of (F, A). On the contrary, let $a, b \in F_\alpha(x)$ for any $\alpha \in [0,1]$. Hence $f(x)(a) \geq \alpha$ and $f(x)(b) \geq \alpha$. Thus $f(x)(a) \wedge f(x)(b) \geq \alpha$. Thus $f(x)(ab^{-1}) \geq \alpha$ since $f(x)$ is a fuzzy subgroup of $F(x)$ for all $x \in E$. So $ab^{-1} \in F_\alpha(x)$. Therefore (F_α, E) is a soft group for all $\alpha \in [0,1]$.

Theorem 3.4. Let (f_i, E_i) be GP-fuzzy soft group of (F_i, A_i) for all $i \in \Lambda$. Then

a) $\widetilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is GP-fuzzy soft group of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$.
b) $\widetilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i)$ is GP-fuzzy soft group of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.
c) $\widetilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is GP-fuzzy soft group of $(\bigcap_e)_{i\in\Lambda}(F_i, A_i)$.
d) $\widetilde{\prod}_{i\in\Lambda}(f_i, E_i)$ is a GP-fuzzy soft group of $\prod_{i\in\Lambda}(F_i, A_i)$.

Proof.

a) Let $\widetilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_r)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E = \bigcap_{i\in\Lambda} E_i \subseteq \bigcap_{i\in\Lambda} A_i = A$. Let $a, b \in F(x)$ for any $x \in E$. $f(x)(ab^{-1}) = (\bigwedge_{i\in\Lambda} f_i(x))(ab^{-1}) = \bigwedge_{i\in\Lambda}(f_i(x)(ab^{-1})) \geq \bigwedge_{i\in\Lambda}(f_i(x)(a) \wedge f_i(x)(b)) = \bigwedge_{i\in\Lambda}(f_i(x)(a)) \wedge \bigwedge_{i\in\Lambda}(f_i(x)(b)) = f(x)(a) \wedge f(x)(b)$ for all $a, b \in F(x)$ since $F(x) = \bigcap_{i\in\Lambda} F_i(x)$. We obtain that $f(x)$ is fuzzy subgroup of $F(x)$ for all $x \in E$. Thus $\widetilde{\bigcap}^r_{i\in\Lambda}(f_i, E_i)$ is a GP-fuzzy soft group of $(\bigcap_r)_{i\in\Lambda}(F_i, A_i)$.
b) Let $\widetilde{\bigcap}^e_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\bigcap_e)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly, $E = \bigcup_{i\in\Lambda} E_i \subseteq \bigcup_{i\in\Lambda} A_i = A$. Let $i \in \Lambda$ be arbitrary and constant, and $J = \{j \in \Lambda | x \in E_j, i \neq j\}$, and $x \in E$. If $x \in E_i$, then there are two cases: $x \in E_i \setminus \bigcup_{i\neq j} E_j$ or $J \neq \emptyset$. If $x \in E_i \setminus \bigcup_{i\neq j} E_j$, then $F(x) = F_i(x)$ and $f(x) = f_i(x)$, and since $f(x)$ is a fuzzy subgroup of $F(x)$, then (f, E) is GP-fuzzy soft group of (F, A). If $J \neq \emptyset$, then $F(x) \subseteq (\bigcap_{j\in J} F_j(x)) \cap F_i(x)$ and $f(x) = (\bigwedge_{j\in J} f_j(x)) \wedge f_i(x)$. Hence (f, E) is GP-fuzzy soft group of (F, A) by similar way in (i).
c) Let $x \notin E_i$. Then $F(x) \subseteq \bigcap_{j\in J} F_j(x)$ and $f(x) = \bigwedge_{j\in J} f_j(x)$. Hence (f, E) is GP-fuzzy soft group of (F, A) by similar way in (i).

d) Let $\widetilde{\cap}^r_{i\in\Lambda}(f_i,E_i) = (f,E)$ and $(\cap_e)_{i\in\Lambda}(F_i,A_i) = (F,A)$. Then $E = \cap_{i\in\Lambda} E_i \subseteq \cap_{i\in\Lambda} A_i \subseteq \cup_{i\in\Lambda} A_i = A$. Let $x \in E$. Thus $x \in \cap_{i\in\Lambda} A_i$. Hence $f(x) = \wedge_{i\in\Lambda} f_i(x)$ and $F(x) = \cap_{i\in\Lambda} F_i(x)$. Since $\wedge_{i\in\Lambda} f_i(x)$ is a fuzzy subgroup of $\cap_{i\in\Lambda} F_i(x)$, (f,E) is a GP-fuzzy soft subgruop of (F,A).

e) Let $\widetilde{\prod}_{i\in\Lambda}(f_i,E_i) = (f,E)$ and $\prod_{i\in\Lambda}(F_i,A_i) = (F,A)$. Clearly, $E = \prod_{i\in\Lambda} E_i \subseteq \prod_{i\in\Lambda} A_i = A$. Let $a,b \in F((x_i)_{i\in\Lambda})$ for any $(x_i)_{i\in\Lambda} \in E$. Then

$$f((x_i)_{i\in\Lambda})(a) \wedge f((x_i)_{i\in\Lambda})(b) = \Big(\bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)\big)\Big)(a) \wedge \Big(\bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)\big)\Big)(b)$$

$$= \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)(a)\big) \wedge \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)(b)\big)$$

$$= \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} \big(\bigwedge_{i\in J} (f_i(x_i)(a) \wedge f_i(x_i)(b))\big)$$

$$\leq \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} \big(\bigwedge_{i\in J} (f_i(x_i)(ab^{-1}))\big) = \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} \big(\bigwedge_{i\in J} f_i(x_i)\big)(ab^{-1})$$

$$= f((x_i)_{i\in\Lambda})(ab^{-1}).$$

Thus $\widetilde{\prod}_{i\in\Lambda}(f_i,E_i)$ is a GP-fuzzy soft group of $\prod_{i\in\Lambda}(F_i,A_i)$ since $f((x_i)_{i\in\Lambda})$ is a fuzzy subgroup of $F((x_i)_{i\in\Lambda})$.

Definition 3.5.

a) Let (f,E) be a fuzzy soft group over G and $\alpha,\beta \in [0,1]$, $\beta \leq \alpha$. Then (f,E) is said to be a (α,β)-identity fuzzy soft group over G if, for all $a \in E, x \in G$, $f(a)(x) = \begin{cases} \alpha, & if\ x = e; \\ \beta, & otherwise. \end{cases}$

b) Let (f,E) be a GP-fuzzy soft group of (F,A) and $\alpha,\beta \in [0,1]$, $\beta \leq \alpha$. Then (f,E) is said to be a (α,β)-identity fuzzy soft group of (F,A) if, for all $a \in E, x \in F(a)$, $f(a)(x) = \begin{cases} \alpha, & if\ x = e; \\ \beta, & otherwise. \end{cases}$

Theorem 3.6. Let φ be a homomorphism from $F(G)$ to $F(H)$ and (f,E) is a fuzzy soft group over G defined by $f(a)(x) = \begin{cases} \alpha, & if\ x \in Ker\ \varphi; \\ \beta, & otherwise. \end{cases}$, for all $a \in E$ and $x \in G$. Then $(\varphi(f),E)$ is a (α,β)-identity fuzzy soft group over H.

Proof. $\varphi(f)(a)(e_H) = \vee_{\varphi(x)=e_H} f(a)(x) = \vee_{x\in Ker\varphi} f(a)(x) = \alpha$.

If $y \neq e_H$ then $\varphi(f)(a)(y) = \beta$. Therefore $(\varphi(f),E)$ is a (α,β)-identity fuzzy soft group over H.

Theorem 3.7. Let φ be a homomorphism from$F(G)$ to $F(H)$ and (f,E) be a GP-fuzzy soft group of (F,A) defined by $f(a)(x) = \begin{cases} \alpha, & if\ x \in Ker\ \varphi; \\ \beta, & otherwise. \end{cases}$, for all $x \in F(a)$ for any $a \in E$. Then$(\varphi(f),E)$ is a (α,β)-identity fuzzy soft group of $(\varphi(F),A)$.

Proof. It is straightforward.

Theorem 3.8. Let (f,E) be a GP_1-fuzzy soft group of (F,A). If (φ,ψ) is a fuzzy soft homomorphism then $(\varphi,\psi)(f,E)$ is a HP_2-fuzzy soft group of $(\varphi(F),\psi(A))$.

Proof. Let $y_1, y_2 \in \varphi(F)(b)$ for any $b \in \psi(E)$. Suppose that there exist $x_1, x_2 \in G$ such that $\varphi(x_1) = y_1$ and $\varphi(x_2) = y_2$. Then

$$\varphi(f)(b)(y_1 y_2^{-1}) \geq \bigvee_{\varphi(x_1)=y_1} \bigvee_{\psi(a)=b} f(a)(x_1) \wedge \bigvee_{\varphi(x_2)=y_2} \bigvee_{\psi(a)=b} f(a)(x_2)$$
$$= \varphi(f)(b)(y_1) \wedge \varphi(f)(b)(y_2)$$

since the inequality

$$\varphi(f)(b)(y_1 y_2^{-1}) = \bigvee_{\varphi(x)=y_1 y_2^{-1}} \bigvee_{\psi(a)=b} f(a)(x) \geq \bigvee_{\psi(a)=b} f(a)(x_1 x_2^{-1})$$
$$\geq \bigvee_{\psi(a)=b} f(a)(x_1) \wedge f(a)(x_2) = \bigvee_{\psi(a)=b} f(a)(x_1) \wedge \bigvee_{\psi(a)=b} f(a)(x_2)$$

is satisfied for each $x_1, x_2 \in G$ such that $\varphi(x_1) = y_1$ and $\varphi(x_2) = y_2$. Thus $(\varphi, \psi)(f, E)$ is a HP_2-fuzzy soft group of $(\varphi(F), \psi(A))$.

Theorem 3.9. Let (g, B) be a HP_2-fuzzy soft group of (T, K). If (φ, ψ) is a fuzzy soft homomorphism then $(\varphi, \psi)^{-1}(g, B)$ is a GP_1-fuzzy soft group of $(\varphi^{-1}(T(\psi)), \psi^{-1}(K))$.

Proof. Let $x_1, x_2 \in \varphi^{-1}(T)(a)$ for all $a \in \psi^{-1}(B)$. Then

$$\varphi^{-1}(g)(a)(x_1 x_2^{-1}) = g\big(\psi(a)\big)\big(\varphi(x_1 x_2^{-1})\big) = g\big(\psi(a)\big)\big(\varphi(x_1)\varphi(x_2^{-1})\big)$$
$$\geq g\big(\psi(a)\big)\big(\varphi(x_1)\big) \wedge\ g\big(\psi(a)\big)\big(\varphi(x_2)\big) = \varphi^{-1}(g)(a)(x_1) \wedge \varphi^{-1}(g)(a)(x_2).$$

Thus$(\varphi, \psi)^{-1}(g, B)$ is a GP_1-fuzzy soft group of $(\varphi^{-1}(T(\psi)), \psi^{-1}(K))$.

Definition 3.10. Let (f, E) be GP-fuzzy soft group of (F, A). (f, E) is called a GP-normal fuzzy soft group of (F, A) if and only if $f(a)$ is a normal fuzzy subgroup over $F(a)$ for each $a \in E$.

Example 3.11. In Example 3.2 (a), (f, E) is a GP-normal fuzzy soft group of (F, A). In Example 3.2 (b), (f, E) is not a GP-normal fuzzy soft group of (F, A).

Theorem 3.12. Let (F, A) be a soft group over G and (f, E) be a GP-fuzzy soft subset of (F, A).

a) If (F_α, E) is a normal soft group for all $\alpha \in [0,1]$ then (f, E) is a GP-normal fuzzy soft group of (F, A).
b) Let (F, A) be a normal soft subgroup over G. If (f, E) is a GP-normal fuzzy soft group of (F, A) then (F_α, E) , if $f(x)_\alpha \neq \emptyset$ for all $x \in E$ and for any $\alpha \in (0,1]$, is a normal soft group of (F, A) for all $\alpha \in [0,1]$.

Proof.

a) Let $a, b \in F(x)$ for all $x \in E$ and $\alpha \coloneqq f(x)(b)$. Then $b \in F_\alpha(x)$. Hence $aba^{-1} \in F_\alpha(x)$ since (F_α, E) is a soft normal subgroup for all $\alpha \in [0,1]$. So $f(x)(aba^{-1}) \geq \alpha$. Thus $f(x)(aba^{-1}) \geq f(x)(b)$. Therefore

(f, E) is a GP-normal fuzzy soft subgroup of (F, A).

b) Let $\alpha \in [0,1]$; $a \in F_\alpha(x)$ and $b \in F(x)$. Thus $f(x)(bab^{-1}) \geq f(x)(a) \geq \alpha$ since (f, E) is a GP-normal fuzzy soft group of (F, A). Hence $bab^{-1} \in F_\alpha(x)$. Therefore (F_α, E) is a normal soft group of (F, A) for all $\alpha \in [0,1]$ from Theorem 3.3.

Theorem 3.13. Let (f_i, E_i) be GP-normal fuzzy soft group of (F_i, A_i) for all $i \in \Lambda$. Then

a) $\tilde{\cap}^r_{i\in\Lambda}(f_i, E_i)$ is GP-normal fuzzy soft group of $(\cap_r)_{i\in\Lambda}(F_i, A_i)$.

b) $\tilde{\cap}^e_{i\in\Lambda}(f_i, E_i)$ is GP-normal fuzzy soft group of $(\cap_e)_{i\in\Lambda}(F_i, A_i)$.

c) $\tilde{\cap}^r_{i\in\Lambda}(f_i, E_i)$ is GP-normal fuzzy soft group of $(\cap_e)_{i\in\Lambda}(F_i, A_i)$.

d) $\tilde{\prod}_{i\in\Lambda}(f_i, E_i)$ is a GP-normal fuzzy soft group of $\prod_{i\in\Lambda}(F_i, A_i)$.

Proof.

a) Let $\tilde{\cap}^r_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $(\cap_r)_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly $E = \cap_{i\in\Lambda} E_i \subseteq \cap_{i\in\Lambda} A_i = A$. Let $a, b \in F(x)$ for any $x \in E$. $f(x)(ab) = \left(\wedge_{i\in\Lambda} f_i(x)\right)(ab) = \wedge_{i\in\Lambda}\left(f_i(x)(ab)\right) = \wedge_{i\in\Lambda}\left(f_i(x)(ba)\right) = \wedge_{i\in\Lambda}\left(f_i(x)(ba)\right) = \left(\wedge_{i\in\Lambda} f_i(x)\right)(ba) = f(x)(ba)$ for all $a, b \in F(x)$ since $F(x) = \cap_{i\in\Lambda} F_i(x)$. We obtain that $f(x)$ is fuzzy normal subgroup of $F(x)$ for all $x \in E$ with Theorem 3.4. Thus $\tilde{\cap}^r_{i\in\Lambda}(f_i, E_i)$ is a GP-normal fuzzy soft group of $(\cap_r)_{i\in\Lambda}(F_i, A_i)$.

b) It is straightforward.

c) It is straightforward.

d) Let $\tilde{\prod}_{i\in\Lambda}(f_i, E_i) = (f, E)$ and $\prod_{i\in\Lambda}(F_i, A_i) = (F, A)$. Clearly $E = \prod_{i\in\Lambda} E_i \subseteq \prod_{i\in\Lambda} A_i = A$. Let $a, b \in F((x_i)_{i\in\Lambda})$ for any $(x_i)_{i\in\Lambda} \in E$. Then

$$f((x_i)_{i\in\Lambda})(ab) = \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\bigwedge_{i\in J} f_i(x_i)))(ab) = \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\bigwedge_{i\in J} f_i(x_i)(ab))$$

$$= \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\bigwedge_{i\in J} f_i(x_i)(ba) = \bigvee_{\substack{J\subseteq\Lambda \\ J \text{ is finite}}} (\bigwedge_{i\in J} f_i(x_i)))(ba) = f((x_i)_{i\in\Lambda})(ba).$$

Thus $\tilde{\prod}_{i\in\Lambda}(f_i, E_i)$ is a GP-normal fuzzy soft group of $\prod_{i\in\Lambda}(F_i, A_i)$ with Theorem 3.5.

Definition 3.14. Let $(f, E_1), (g, E_2)$ be fuzzy soft sets in a fuzzy soft class $\widetilde{(G, P)}$. Then the fuzzy *intersection product* of them, denoted by $(f, E_1) \cdot (g, E_2)$, is the soft set (h, C) defined as $C = E_1 \cap E_2$, $h(x) = f(x) \cdot g(x)$ for all $x \in E_1 \cap E_2$.

Definition 3.15. Let (f, E_1) be GP-fuzzy soft group of (F, A_1) and (g, E_2) be GP-fuzzy soft group of (G, A_2). Then the fuzzy *intersection product* of them, denoted by $(f, E_1) \tilde{\circ} (g, E_2)$, is the soft set (h, C) defined as $C = E_1 \cap E_2$, $h(x) = f(x) \circ g(x)$ for all $x \in E_1 \cap E_2$, where

$$(f(x) \circ g(x))(s) = \begin{cases} \bigvee_{s=pq} f(x)(p) \wedge g(x)(q), \exists p \in F(x), q \in G(x): s = pq; \\ 0 \qquad , \quad otherwise. \end{cases}$$

Theorem 3.16. Let (G, A_2) be a normal soft group over G and let (f, E_1) be GP-fuzzy soft group of (F, A_1) and (g, E_2) be GP-normal fuzzy soft group of (G, A_2). Then $(f, E_1) \tilde{\circ} (g, E_2)$ is a GP-fuzzy soft group of $(\mathrm{F}, A_1) \cap (\mathrm{G}, A_2)$.

Proof. Let $(f, E_1) \tilde{\circ} (g, E_2) = (h, C)$ and $(\mathrm{F}, A_1) \cap (\mathrm{G}, A_2) = (H, D)$. Then $C \subseteq D$. Let $a, b \in H(x)$ for any $x \in E_1 \cap E_2$. Thus $h(x)$ is a fuzzy subgroup of $H(x)$ since

$h(x)(a) \wedge h(x)(b) = \big(f(x) \circ g(x)\big)(a) \wedge \big(f(x) \circ g(x)\big)(b) = \big(\bigvee_{a=pt} f(x)(p) \wedge g(x)(t)\big) \wedge (\bigvee_{b=uv} f(x)(u) \wedge g(x)(v)) = \bigvee_{\substack{a=pt \\ b=uv}}\big(f(x)(p) \wedge f(x)(u) \wedge g(x)(t) \wedge g(x)(v)\big) \leq \bigvee_{\substack{a=pt \\ b^{-1}=v^{-1}u^{-1}}} \big(f(x)(pu^{-1}) \wedge g(x)(tv^{-1})\big) = \bigvee_{\substack{a=pt \\ b^{-1}=v^{-1}u^{-1}}} (f(x)(pu^{-1}) \wedge g(x)(utv^{-1}u^{-1})) \leq \bigvee_{ab^{-1}=ptv^{-1}u^{-1}}(f(x)(pu^{-1}) \wedge g(x)(utv^{-1}u^{-1}))$.

Definition 3.17. Let (f, E_1) be GP-fuzzy soft group of (F, A_1) and (g, E_2) be GP-fuzzy soft group of (G, A_2). Then the fuzzy *product* of them, denoted by $(f, E_1) \tilde{*} (g, E_2)$, is the soft set (h, C) defined as $C = E_2 \times E_1$, $h(a, b) = f(a) * g(b)$ for all $a \in E_1, b \in E_2$, where

$$(f(a) * g(b))(s) = \begin{cases} \bigvee_{s=pq} f(a)(p) \wedge g(b)(q), \exists p \in F(a), q \in G(b): s = pq; \\ 0 \qquad , \qquad otherwise. \end{cases}$$

Theorem 3.18. Let (G, A_2) be a normal soft group over G and let (f, E_1) be GP-fuzzy soft group of (F, A_1) and (g, E_2) be GP-normal fuzzy soft group of (G, A_2). Then $(f, E_1) \tilde{*} (g, E_2)$ is a GP-fuzzy soft group of $(\mathrm{F}, A_1) \wedge (\mathrm{G}, A_2)$.

Proof. Let $(f, E_1) \tilde{*} (g, E_2) = (h, C)$ and $(\mathrm{F}, A_1) \wedge (\mathrm{G}, A_2) = (H, D)$. Then $C \subseteq D$. Let $a, b \in H(x, y)$ for any $(x, y) \in E_1 \times E_2$. Thus $h(x, y)$ is a fuzzy subgroup of $H(x, y)$ since

$h(x, y)(a) \wedge h(x, y)(b) = \big(f(x) * g(y)\big)(a) \wedge \big(f(x) * g(y)\big)(b) = \big(\bigvee_{a=pt} f(x)(p) \wedge g(y)(t)\big) \wedge (\bigvee_{b=uv} f(x)(u) \wedge g(y)(v)) = \bigvee_{\substack{a=pt \\ b=uv}}\big(f(x)(p) \wedge f(x)(u) \wedge g(y)(t) \wedge g(y)(v)\big) \leq \bigvee_{\substack{a=pt \\ b^{-1}=v^{-1}u^{-1}}} \big(f(x)(pu^{-1}) \wedge g(y)(tv^{-1})\big) = \bigvee_{\substack{a=pt \\ b^{-1}=v^{-1}u^{-1}}} (f(x)(pu^{-1}) \wedge g(y)(utv^{-1}u^{-1})) \leq \bigvee_{ab^{-1}=ptv^{-1}u^{-1}}(f(x)(pu^{-1}) \wedge g(y)(utv^{-1}u^{-1}))$.

Theorem 3.19. Let (f, E) be a GP_1-normal fuzzy soft group of (F, A). If (φ, ψ) is a fuzzy soft homomorphism then $(\varphi, \psi)(f, E)$ is a HP_2-normal fuzzy soft group of $(\varphi(F), \psi(A))$.

Proof. Let $y_1, y_2 \in \varphi(F)(b)$ for any $b \in \psi(E)$. Suppose that there exist $x_1, x_2 \in G$ such that $\varphi(x_1) = y_1$ and $\varphi(x_2) = y_2$. Then

$$\varphi(f)(b)(y_1 y_2) \geq \bigvee_{\varphi(x)=y_2 y_1} \bigvee_{\psi(a)=b} f(a)(x) = \varphi(f)(b)(y_2 y_1)$$

since the inequality

$$\varphi(f)(b)(y_1y_2) = \bigvee_{\varphi(x)=y_1y_2} \bigvee_{\psi(a)=b} f(a)(x) \geq \bigvee_{\psi(a)=b} f(a)(x_1x_2) = \bigvee_{\psi(a)=b} f(a)(x_2x_1)$$

is satisfied for each $x_1, x_2 \in G$ such that $\varphi(x_1) = y_1$ and $\varphi(x_2) = y_2$. Thus $(\varphi,\psi)(f,E)$ is a HP_2-normal fuzzy soft group of $(\varphi(F),\psi(A))$.

Theorem 3.20. Let (g,B) be a HP_2-normal fuzzy soft group of (T,K). If (φ,ψ) is a fuzzy soft homomorphism then $(\varphi,\psi)^{-1}(g,B)$ is a GP_1-normal fuzzy soft group of $(\varphi^{-1}(T(\psi)),\psi^{-1}(K))$.

Proof. Let $x_1, x_2 \in \varphi^{-1}(T)(a)$ for all $a \in \psi^{-1}(B)$. Then

$$\varphi^{-1}(g)(a)(x_1x_2) = g(\psi(a))(\varphi(x_1x_2)) = g(\psi(a))(\varphi(x_1)\varphi(x_2)) = g(\psi(a))(\varphi(x_2)\varphi(x_1)) = g(\psi(a))(\varphi(x_2x_1)) = \varphi^{-1}(g)(a)(x_2x_1).$$

Thus$(\varphi,\psi)^{-1}(g,B)$ is a GP_1-normal fuzzy soft group of $(\varphi^{-1}(T(\psi)),\psi^{-1}(K))$.

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