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Research Article

# New Wave Solutions of Time Fractional Kadomtsev-Petviashvili Equation Arising in the Evolution of Nonlinear Long Waves of Small Amplitude

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**Abstract**

The main aim of this paper is to obtain the travelling wave solutions of fractional Kadomtsev- Petviashvili(KP) Equation where the derivative is in conformable sense. For this aim the sub equation method is used with computer software called Mathematica. Then, solutions are investigated through the graphical representation for different cases of $\alpha$.

**Keywords:** Kadomtsev-Petviashvili Equation, Sub-Equation Method, Wave Solution; Conformable Fractional Derivative.

**Küçük Genlikli Lineer Olmayan Uzun Dalgaların Gelişiminde Türeyen Zaman Kesirli Kadomtsev-Petviashvili Denkleminin Yeni Dalga Çözümleri**

**Öz**

Bu makalenin asıl amacı, türevleri conformable cinsinden olan kesirli Kadomtsev-Petviashvili (KP) denkleminin dalga çözümlerini bulmaktır. Bu amaç için alt denklem metodu, Mathematica bilgisayar programı ile birlikte kullanılmıştır. Daha sonra elde edilen çözümlerin $\alpha$ ' nın değişik değerleri için grafiksel gösterimleri verilmiştir.

**Anahtar Kelimeler:** Kadomtsev-Petviashvili denklemi, Alt-denklem metodu, Dalga çözümü, Conformable kesirli türev.

## 1. Introduction

Nonlinear evolution equations (NLEEs) describe lots of natural phenomena in the fields of science, particularly in physics, engineering, chemistry and finance. Therefore, a diverse group of scientists are paying great attention to acquire the exact solutions of NLEEs [30,31]. For this purpose many different analytical methods are used such as the hyperbolic function method, Hirota method, new auxiliary equation method, the backland transformation, the modified simple equation method, the auxiliary equation method, algebraic method, extended simplest equation method, the sine-

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Gordon expansion method and improved generalized Jacobi elliptic function method [1-14].

Fractional calculus is the best tool to explain the complexity and nonlinearity of the nature. An arising interest to this subject grows day by day. To state the events that occurs in the different branches of science and technology, there are a few different definitions of derivatives and integrals with arbitrary order. Although each of them differs from the other one, generally these definitions include integral form such as Riemann-Liouville and Caputo. This integral form makes the definitions harder to understand and the calculations to obtain the analytical or approximate solutions of the equations become more complicated. In addition it is understood that these definitions have some flaws. For instance

1. The fractional derivative of a constant does not equals zero in Riemann-Liouville sense.
2. The formula for the derivative of the product of two functions is not satisfied by both Riemann-Liouville and Caputo derivative.
$$D_a^\alpha(fg) = gD_a^\alpha(f) + fD_a^\alpha(g).$$
3. The formula for the derivative of the quotient of two functions is not satisfied by both Riemann-Liouville and Caputo derivative.
$$D_a^\alpha\left(\frac{f}{g}\right) = \frac{fD_a^\alpha(f) - gD_a^\alpha(g)}{g^2}.$$
4. Both Riemann and Caputo derivative do not satisfy the chain rule.
$$D_a^\alpha(fog)(t) = f^\alpha(g(t))g^\alpha(t).$$
5. The property $D^\alpha D^\beta = D^{\alpha+\beta}$ is not satisfied by both Riemann-Liouville and Caputo derivative in general.
6. It is supposed that the function $f$ is differentiable in Caputo sense.

To overcome this flaws Khalil et. al. [1] expressed applicable definition of noninteger order differentiation and integration named conformable fractional derivative (CFD) and integral.

**Definition 1.1.** $f:[0,\infty)\to R$ be a function. The $\alpha^{th}$ order CFD of $f$ is defined by,

$$D_a^\alpha(f)(t) = \lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}$$

for all $t>0, \alpha\in(0,1)$.

**Definition 1.2.** Let $f$ be $\alpha$ differentiable in some $(0,a)$, $a>0$ and $\lim_{t\to 0^+} f^{(\alpha)}(t)$ exist then let $f^{(\alpha)}(0) = \lim_{t\to 0^+} f^\alpha(t)$. The conformable fractional integral of a function $f$ starting from $a\geq 0$ is mentioned as:

$$I_a^\alpha(f)(t) = \int_a^t f(x)d_\alpha x = \int_a^t \frac{f(x)}{x^{1-\alpha}}dx$$

that the integral denotes Riemann improper integral and $\alpha\in(0,1]$.

Following theorem [1] expresses some basic properties satisfied by CFD.

**Theorem 1.1.** Let $\alpha\in(0,1]$ and $f,g$ are $\alpha$-differentiable functions at point $t>0$, then

**1.** $T_\alpha(mf+ng) = mT_\alpha(f) + nT_\alpha(g)$ for all $m,n\in R$

**2.** $T_\alpha(t^p) = pt^{p-\alpha}$ for all $p$

**3.** $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f)$

**4.** $T_\alpha\left(\frac{f}{g}\right)=\frac{gT_\alpha(f)-fT_\alpha(g)}{g^2}$

**5.** $T_\alpha(c)=0$ for all constant functions $f(t)=c$

**6.** Also, let $f$ be a differentiable function, then $T_\alpha(f)(t)=t^{1-\alpha}\frac{df(t)}{dt}$.

After existence of these definitions huge amount of applications are made by many scientists [15-26] In this work is going to shed some light on conformable time fractional KP equation [34]

$$\frac{1}{4}\frac{\partial^4 u}{\partial x^4}+\frac{3}{2}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+\frac{3}{4}\frac{\partial^2 u}{\partial y^2}+\frac{\partial}{\partial x}\frac{\partial^\alpha}{\partial t^\alpha}(u)=0 \quad (0.1)$$

The KP equation aroused as generalization of KDV equation [32] and used as a model for shallow long water waves. Also this equation is an integrable equation and used for modelling waves in ferromagnetic media [33].

## 2. Basics of Sub-Equation Method

In this section, we mention a short description of sub equation method [27]. Regard the conformable fractional partial differential equation (CFPDE) as

$$P\left(u,D_t^\alpha u,D_x u,D_y u,D_t^{2\alpha}u,D_x^2 u,D_y^2 u...\right)=0 \quad (2.1)$$

where $D_t^\alpha u$ denotes CFD of $u(x,y,t)$ and $D_t^{2\alpha}$ shows two times CFD of the function $u(x,y,t)$. By using the conformable wave transform [15]

$$u(x,y,t)=U(\xi),\xi=mx+ny+p\frac{t^\alpha}{\alpha} \quad (2.2)$$

and the chain rule [28] Eq. (2.1) becomes to a differential equation

$$G\left(U.U',U'',...\right)=0. \quad (2.3)$$

In Eq. (2.3) prime indicates the derivative with respect to $\xi$. Also $m,n,p$ are arbitrary constants to be evaluated later. It is assumed that the solution of Eq. (2.3) can be given as

$$U(\xi)=\sum_{i=0}^{N}a_i\varphi^i(\xi),a_N\neq 0, \quad (2.4)$$

that the constants $a_i\,(0\le i\le N)$ are going to be utilized. Also positive integer $N$ can be evaluated with the help of balancing procedure [29] in Eq. (2.3) and $\varphi(\xi)$ is the solution of the equation

$$\varphi'(\xi)=\sigma+(\varphi(\xi))^2 \quad (2.5)$$

where $\sigma$ is a constant. Some of the solutions of Eq. (2.5) can be obtained as follows.

$$\phi(\xi)=\begin{cases}-\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\xi\right),\sigma<0\\ -\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\xi\right),\sigma<0\\ \sqrt{\sigma}\tan\left(\sqrt{\sigma}\xi\right),\sigma>0\\ -\sqrt{\sigma}\cot\left(\sqrt{\sigma}\xi\right),\sigma>0\\ -\dfrac{1}{\xi+\varpi},\varpi\ is\ a\ cons.,\sigma=0\end{cases} \tag{2.6}$$

Subrogating Eqs. (2.4) and (2.5) in Eq. (2.3), a polynomial of t$\varphi(\xi)$arises. Equating the coefficients of $\varphi^i(\xi)(i=0,1,...,N)$ to zero led to an equation system with respect to $k,a_i(i=0,1,...,N)$. Solving the system we obtain $k,p,a_i(i=0,1,...,N)$. Subrogating the acquired values of the constants and the solution set of Eq. (2.5) into Eq. (2.4) with the aid of (2.6) we handle the analytical solutions for Eq. (2.1).

## 3. Solution of Time Fractional KP Equation

Considering time fractional KP equation (1.1) then using the chain rule [28] and conformable wave transform (2.2) the Eq. (1.1) changes into following differential equation

$$\frac{1}{4}m^4U^{(v)}(\xi)+\frac{3}{2}m^3U'(\xi)U''(\xi)+\frac{3}{4}n^2U''(\xi)+mpU''(\xi)=0. \tag{3.1}$$

Integrating the above equation once and making some algebraic calculations led to

$$m^4U'''(\xi)+3m^3\left(U'(\xi)\right)^2+3n^2U'(\xi)+4mpU'(\xi)=0. \tag{3.2}$$

Assuming the solution of Eq. (3.2) is denoted by the following series

$$U(\xi)=\sum_{i=0}^{N}a_i\phi^i(\xi),a_N\neq 0 \tag{3.3}$$

where $\varphi(\xi)$is the analytical solutions of differential Eq. (2.5) given in (2.6). With the aid of balancing principle [29], we get N = 1. When substituting the obtained data into (3.1), led to an equation system.

$$\begin{aligned}&3a_1^2m^3\sigma^2+2a_1m^4\sigma^2+4a_1mp\sigma+3a_1n^2\sigma=0,\\&6a_1^2m^3\sigma+8a_1m^4\sigma+4a_1mp+3a_1n^2=0,\\&3a_1^2m^3+6a_1m^4=0.\end{aligned} \tag{3.4}$$

The solution set for system (3.4) can be expressed as

$$a_0=a_0,a_1=-2m,p=\frac{4m^4\sigma-3n^2}{4m}.$$

For $\sigma > 0$

$$u_1(x,y,t) = a_0 + 2m\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\left(mx + ny + \frac{4m^4\sigma - 3n^2}{4m}\frac{t^\alpha}{\alpha}\right)\right),$$

$$u_2(x,y,t) = a_0 + 2m\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\left(mx + ny + \frac{4m^4\sigma - 3n^2}{4m}\frac{t^\alpha}{\alpha}\right)\right),$$

For $\sigma > 0$

$$u_3(x,t) = a_0 - 2m\sqrt{\sigma}\tan\left(\sqrt{\sigma}\left(mx + ny + \frac{4m^4\sigma - 3n^2}{4m}\frac{t^\alpha}{\alpha}\right)\right),$$

$$u_4(x,t) = a_0 + 2m\sqrt{\sigma}\cot\left(\sqrt{\sigma}\left(mx + ny + \frac{4m^4\sigma - 3n^2}{4m}\frac{t^\alpha}{\alpha}\right)\right),$$

For $\sigma = 0$

$$u_5(x,t) = a_0 + \frac{2m}{\varpi + mx + ny - \frac{3n^2}{4m}\frac{t^\alpha}{\alpha}}.$$

Now let's give some graphical representations of obtained results. In Figure 1 represents the travelling wave solution, Figures 2 and Figure 4 show the solitary wave solution of the obtained results.

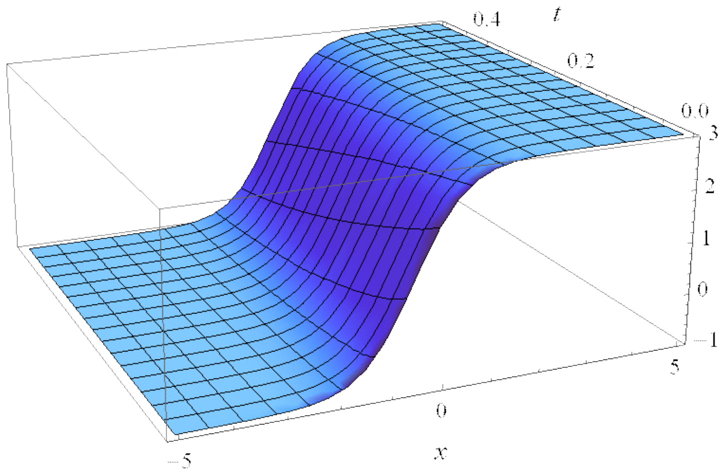


**Figure 1:** The surface plot of $u_1(x,y,t)$ for $m=1, n=1, \sigma=-1, a_0=1, \alpha=0.8, y=5.$

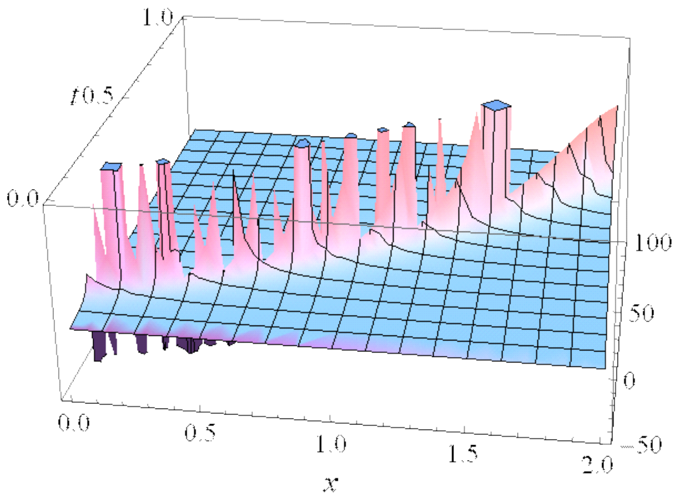


**Figure 2:** The graphical illustration of $u_2(x,y,t)$ for $m=1, n=1, \sigma=-1, a_0=1, \alpha=0.8, y=5.$

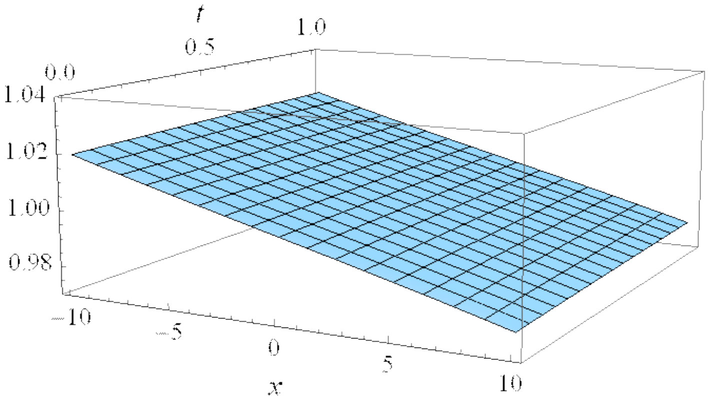


**Figure 3:** The graphical representation of $u_3(x,y,t)$ for $m=0.1, n=0.1, \sigma=0.1, a_0=1, \alpha=0.8, y=0.5.$

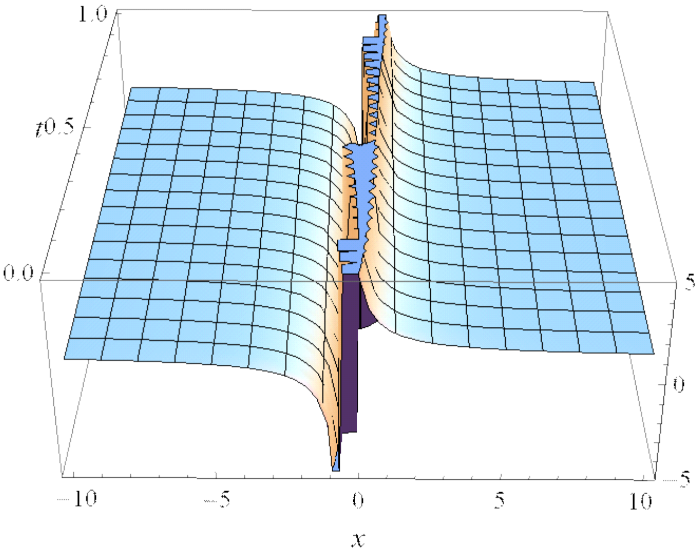


**Figure 4:** 3D graphical simulation of $u_4(x,y,t)$ for $m=0.1, n=0.1, \sigma=0.1, a_0=1, \alpha=0.8, y=0.5.$

## 4. Results

The sub-equation method is employed to acquire the new exact solutions for the fractional KP equation. It is seen that the considered method is efficient and effective mathematical tool to handle conformable fractional nonlinear evolution equations that arises in many sciences. The CFD is well behaved and applicable derivative. Also this derivative has some advantages over other popular definitions. Especially obtaining the exact results of CFPDEs using the chain rule and wave transform is the most important property of CFD definition.

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