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Research Article

Efficiency Measurement with Network DEA: An Application to Sustainable Development Goals 4

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Relational network DEA, Sustainable development goals 4, Educational economy **Abstract:** Education is the core of the factors that improved people for a better lifestyle and increases the level of society' development. Quality education is one of the most vital goals of Sustainable Development Goals (SDGs) due to actualizing these factors. Using relational network data envelopment analysis (DEA), which have three interrelated substages, this current paper computes the educational economy efficiency of the Organisation for Economic Co-operation and Development (OECD) countries bearing in mind the characteristics related to SDGs. The contribution of our study is the use of a novel approach to computing the educational economy efficiency of the SDGs. Even though some interesting differences reveal in the efficiency of the countries, the findings show that countries with high-efficiency scores are clustered around countries like Latvia, Slovenia, and Korea.

1. INTRODUCTION

Performance evaluation is a crucial phenomenon for countries with regard to determining the current situation and finding an efficient process that ameliorated this situation. The good performance of countries on social, economic and health issues is possible through the acquisition of a quality education that influences directly the lives and sustainable development of human. Well-educated human capital can be considered of the engine of the production process for new discoveries, ideas, development and eventually new value-added productions.

In recent years, numerous studies have been examined as a result of the widespread interest in education. The United Nations emphasized that education for all is always an inseparable part of the agendas of both Millennium Development Goals (MDGs) and Sustainable Development Goals (SDGs) (United Nations, 2015). MDGs are expired at the end of 2015. SDGs is a new agenda integrated into MDGs and covering a 15 years period for the post-2015 with 17 goals and 169 interrelated targets in global developments efforts in social, economic and environmental areas (Griggs et al., 2013; Le Blanc, 2015). Quality Education, which is defined as *Ensure Inclusive and Equitable Quality Education and Promote Lifelong Learning Opportunities for All*, is the fourth goal of SDGs. Besides, SDGs have 10 targets comprising many different features of education, which aimed at educating the people for enhancing their

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individual well-being and socio-economic status. By 2030, these agendas pave the way of the creation of the societies with strong sustainable education and culture thanks to the effective and functional learning outcomes of these objectives (Hopkins & McKeown, 2002; Sterling, 2001).

Quality education is a cornerstone that ensures the human's sustainable development. Although the global awareness is existed for the importance of education, more than 265 million children are out of school and 22% of them are of primary school age and roughly the same number of them as will be out of school (UN, 2018; UNICEF, 2016). As a percentage of GDP, Latvia spends more on primary education than any other Organisation for Economic Co-operation and Development (OECD) country, followed closely by Slovenia and Poland. However, considering the education expenditure to secondary and tertiary education, Denmark takes the first place. Turkey spend the least as a percentage of their GDP on primary and secondary education. The share of public expenditure per student on tertiary education in Korea is also among the lowest, especially at the tertiary level. Estonia located one of the top performers in Programme for International Student Assessment (PISA) followed by Finland and Korea. In the light of these findings, the analysis of the educational economy efficiency of the OECD countries is crucial from the point of view of policymakers, officials and researchers who are concerned with education in both regional or worldwide (Abbott & Doucouliagos, 2003; Barra, Lagravinese & Zotti 2018; Worthington, 2001).

While evaluating the educational economy performance of the countries, any study which consider the linkages between substages of education could not be encountered. Besides, the lack of the efficient analysis or metrics in measuring the performance of the countries makes it difficult to carry out the comparisons of the countries. For intend to fill this gap, the efficiency of most OECD countries is measured towards to indicators based on education economics, employment and PISA (Programme for International Student Assessment) data. This measurement gives critical feedback to international studies whether the education system is quality and well design. Besides, an important focus of this study is that a newest developed theory (relational network DEA) is used for this intent. The relational network DEA can assess countries' education performance from a multistage efficiency and effectiveness perspective and further examine their education performance from a multidimensional viewpoint. Unlike previous studies, we not only directly investigate the relationship between inputs and outputs, but also consider the linkages between education substages (primary, secondary, and tertiary) by means of this methodology. Measuring the efficiency at these disaggregated levels is of great importance as it reflects realistically the concept of that education materializes at the student level (Ruggiero, 2006).

The structure of this paper is organized as follows: Section 2 discussed some relevant indicators of the educational economy. Section 3 describes the methodology. Section 4 presents the research findings obtained from running the GAMS codes. The final section presents a brief summary of the results.

1.1. Literature Review

Education efficiency assessment has become a core research to understand progress difficulties owing to the education services supported by government almost every country. To this end, different methodological approaches have showed up over the past few decades. Although the efficiency measurement techniques have been applied to many different types of institutions, but the studies on an international framework with whole countries as units of observation are rarely encountered (Afonso & Aubyn, 2006). These studies have generally concentrated on how to assign educational resource inputs to improve output performance efficiently. Moreover, it is well known that the education expenditure as input (see Afonso et al., 2005; Aubyn, 2003; Ciro & Garcia, 2018; Gupta & Verhoeven, 2001; Hanushek & Kimko, 2000; Lee & Barro,

2001), PISA score as output (see Afonso & Aubyn, 2005; Afonso & Aubyn, 2006; Aristovnik, 2012; Jafarov & Gunnarsson, 2008), and employment rate as output (see Afonso & Aubyn, 2006; Chen & Wu, 2007; Lavrinovicha et al., 2015) are the important factors of measurement of educational economy efficiency.

Generally, two type of decision making units (DMUs) have been used to assess the level of efficiency with respect to government expenditure on education in these studies. In the first group, the micro education level (university, school etc.) consider as DMUs. Furthermore, the macro level approaches in which countries are selected as DMUs are included in the second group.

There are various studies for assessing the education efficiency at micro level. Ramzi, Afonso and Ayadi (2016) used DEA for reveal the relationship between school resources and student performance. It is find that inefficiency in education was associated with the poverty in the governorates. Kashim et al. (2017) measured the efficiency of a university faculty in Malaysia by using a network DEA model. They selected several inputs including number of academicians (professors, associate professors, senior lecturers, lecturers, foreign academic staff, non-academic staff) and expenses. The outputs included number of graduates (from undergraduate program, master program, and Ph.D. program), publications, grants, main researchers based on different types of grants, expert lecturers, collaboration activities done under MoU/LoI. Qin and Du (2018) applied the network DEA approach to assess the effectiveness of the universities' research and development (R&D) performance.

Yang et al. (2018) investigated the inefficiency and productivity of Chinese universities, using two-stage network process over the period of 2010-2013. They used R&D funds, teaching and research staff, and government block funds as input and number of SCI/SSCI publications, the total number of students, patents, and the other intellectual property forms as output in the first stage. In the second stage, the number of patents and other intellectual properties, which is already used as output in the previous stage, and the staff of the application of R&D outputs and technology services were used as input; total income was used as output.

When viewed from macro level approaches, Afonso and Aubyn (2006) examined the efficiency of expenditure in education for the 25 mostly OECD countries by using a semiparametric model of a two-step DEA/Tobit analysis. PISA scores, education spending per student, number of teachers, and time spent at school were used as input. These indicators are similar to Sylwester (2002) which was revealed the government spends on education encourage income equality and Wasylenko and McGuire (1985) that the government expenditure on education increase the employment rate.

Guironnet and Peypoch (2018) seek an answer how institutional factors affect the productivity of university by using hierarchical DEA following distinctions: urban/rural areas and public/private universities. Ciro and Garcia (2018) emphasized that most discussions have concentrate on the importance of increasing public expenditure on education covers 37 countries. They measured the efficiency of public secondary education expenditure using a twostep semi-parametric DEA methodology. Private spending (%GDP), and government expenditures (%GDP per capita) were selected as input in the first model. Furthermore, the enrolment rates and PISA scores were used as outputs; the teacher-pupil ratio was used as input in second model.

It is important to note that PISA scores of the countries are a remarkable indicator in connection with the test is internationally validity. In this context, Aristovnik (2012) used the average data for 1999-2007 period to show that the long-term efficiency measures as the effects of ICT (Information and Communication Technology) are characterized by time lags. The study find

that ICT had a significant impact on education sector in the selected EU-27 and OECD countries.

2. METHOD

2.1. Relational Network DEA Model

The relational network DEA model accounts for both the efficiency of a system and the system's interrelated substages. Thus, the drawbacks of the traditional DEA models that neglects of interrelated substages can be eliminated. Besides, the overall steps of the traditional DEA models that are so-called "black box" can been made explicit.

This study uses the relational network DEA model is comprised of a series of three substages under the assumption of the constant return to scale and output-oriented. The fact that the aim is to increase output rather than input reduction in the educational economy efficiency reveals that the output-oriented model is the appropriate tool (Johnes, 2006).

Figure 1 presents the relational network DEA structure with inputs X_i , i = 1, 2, ..., m, intermediate products Z_p , p = 1, 2, ..., q and T_l , l = 1, 2, ..., d, and outputs Y_r , r = 1, 2, ..., s.



Figure 1. Relational network DEA structure

We define X_i and Y_r the *i*th input and *r*th output of the *j*th DMU by denoting the *i*, *r*, *j* indexes of input, output and DMU. The intermediate products 1, which is the outputs of the first stage and the inputs of the second stage, and the intermediate products 2, which is the outputs of the second stage and the inputs of the third stage, are represented by respectively Z_p and T_l by denoting the *p*, *l*, *j* indexes of intermediate products 1, intermediate products 2 and DMU.

The substages efficiency calculated by E_k^1 , E_k^2 and E_k^3 , and the overall efficiency of the system can be calculated by $E_k = E_k^1 \times E_k^2 \times E_k^3$. The linear program model of the overall efficiency and its constraint proposed by Kao (2009) is:

$$E_k = \max_{u_r, w_p, \gamma_l, \nu_i} \sum_r u_r Y_r$$

Subject to:

$$\begin{split} \sum_{i} v_{i} X_{ii} &= 1 \ \forall k, \\ \sum_{r} u_{r} Y_{r} &- \sum_{i} v_{i} X_{ii} \leq 0 \ \forall j, \\ \sum_{l} \gamma_{l} T_{li} &- \sum_{i} v_{i} X_{ii} \leq 0 \ \forall j, \\ \sum_{p} w_{p} Z_{p} &- \sum_{i} v_{i} X_{ii} \leq 0 \ \forall j, \\ \sum_{r} u_{r} Y_{r} &- \sum_{p} w_{p} Z_{p} \ \leq 0 \ \forall j, \end{split}$$

(1)

$$\begin{split} & \sum_{l} \gamma_{l} T_{l} - \sum_{p} w_{p} Z_{p} &\leq 0 \ \forall j, \\ & u_{r}, w_{p}, \gamma_{l}, v_{i} \geq 0 \ \forall r, p, l, i. \end{split}$$

The aim of the optimal multipliers u_r , w_p , γ_l , v_i is unique, in the first instance the overall efficiency namely Model (1) is calculated. Then the efficiencies of the substages must be calculated. In this study, after the measurement of the overall efficiency, the second and third stage will be calculated. With the help of the $E_k = E_k^1 \times E_k^2 \times E_k^3$, the efficiency of the first stage can be obtained as $E_k^1 = E_k/(E_k^2 \times E_k^3)$. Model (2) shows the linear program of the efficiency of the third stage and its constraint:

$$E_k^3 = \max_{u_r, w_p, \gamma_l, \nu_i} \sum_{r=1}^s u_r Y_r$$

Subject to:

$$\begin{split} \sum_{l=1}^{t} \gamma_{l} H_{j} &= 1, \\ \sum_{r} u_{r} Y_{r} &- E_{k} \sum_{i} v_{i} X_{i} &= 0 \ \forall j, \\ \sum_{r} u_{r} Y_{r} &- \sum_{i} v_{i} X_{ij} \leq 0 \ \forall j, \\ \sum_{l} \gamma_{l} T_{l} &- \sum_{i} v_{i} X_{i} &\leq 0 \ \forall j, \\ \sum_{p} w_{p} Z_{p} &- \sum_{i} v_{i} X_{i} &\leq 0 \ \forall j, \\ \sum_{r} u_{r} Y_{r} &- \sum_{p} w_{p} Z_{p} &\leq 0 \ \forall j, \\ \sum_{l} \gamma_{l} T_{l} &- \sum_{p} w_{p} Z_{p} &\leq 0 \ \forall j, \\ u_{r}, w_{p}, \gamma_{l}, v_{i} \geq 0 \ \forall r, p, l, i. \end{split}$$

$$(2)$$

If the first constraint and the objective function of Model (2) is expressed as $\sum_{p=1}^{q} w_p Z_p = 1$ and $\sum_{l=1}^{t} \gamma_l T_l$, the efficiency of the second stage can be obtained. Otherwise, if the first constraint and the objective function of Model (2) is expressed as $\sum_{i=1}^{m} v_i X_{ii} = 1$ and $\sum_{l=1}^{t} w_p Z_p$, the efficiency of the first stage can be obtained.

2.2. Data

In this study, the data that express the sub-objectives of the SDG 4 and can be used to measure the educational economy efficiency of the OECD countries are taken into consideration. The 30 OECD countries are the DMUs in the analysis. Besides, the inputs, intermediate products and outputs of the network structure are expressed as Table 1:

Table 1. The inputs, intermediate products and outputs[†].

X_1 :	Government expenditure per primary student (% of GDP per capita)	2013-14
Z_1 :	PISA science performance (mean)	2015
Z_2 :	Government expenditure per secondary student (% of GDP per capita)	2013-14
T_1	Employment rate for upper secondary level (% of 25-64 year-olds)	2013-15
T_2 :	Government expenditure per tertiary student (% of GDP per capita)	2013-14
Y_1	Employment rate for tertiary level (% of 25-64 year-olds)	2013-15

These indicators based on the levels defined by International Standard Classification of Education (ISCED) are taken are will be used in the substages of the network structure. The

⁺ The data that cover 2013-2015 period were collected from the database of OECD.

reason is that these standard international education levels provide unity for measuring the performance of the students (Johnes et al., 2017). PISA are used in the analysis that is the reason why the quality of education can be measured by the achievement of students via the scores on international test represented the cross-country variations in cognitive skills of the students and thereby the differences in the quality of the future labour force (Lee & Barro, 2001). Besides, the government expenditure that placed in primary, secondary and tertiary levels is selected in relation to the country's sources of educational finance (Riddell, 1993). Figure 2 presents the framework of the network structure modeled as a three-stage process.



Figure 2. Relational network DEA structure for SDG 4

Table	e 2. 1	Data
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DMU	Country	X1	Z_1	Z_2	T_1	T_2	Y_1
1	Australia	18.5932	509.9939	16.7917	77.6700	22.5295	83.1767
2	Austria	23.4497	495.0375	27.3519	75.9433	36.1737	85.4733
3	Belgium	22.4726	501.9997	25.7987	72.8533	33.0074	84.4900
4	Chile	15.0754	446.9561	15.1517	71.7000	17.3957	84.1950
5	Czech Republic	15.5312	492.8300	23.5477	77.7067	21.6123	84.7400
6	Denmark	25.6117	501.9369	28.2276	79.6433	44.6629	86.2533
7	Estonia	21.4939	534.1937	20.1925	75.7733	27.7667	84.2533
8	Finland	21.0185	530.6612	27.2000	73.2000	35.4852	83.3400
9	France	18.0165	494.9776	26.8059	72.8867	35.0586	84.0300
10	Germany	17.9128	509.1406	23.4938	79.4667	37.5885	87.9900
11	Hungary	18.4491	476.7475	19.5469	71.5033	23.8811	81.6667
12	Iceland	24.4524	473.2301	18.3453	87.0433	25.6424	91.2133
13	Ireland	16.7971	502.5751	21.6000	67.6133	25.2509	81.1100
14	Israel	21.4471	466.5528	16.9659	72.4467	19.4927	85.9100
15	Italy	21.3017	480.5468	23.0962	69.8567	26.1989	78.1000
16	Korea	23.9656	515.8099	23.3231	71.9867	13.7440	77.4733
17	Latvia	31.2500	490.2250	29.6703	70.9233	22.9545	85.1000
18	Mexico	14.8592	415.7099	16.4109	70.6100	40.4662	80.3367
19	New Zealand	18.6102	513.3035	22.3160	80.9400	27.9934	86.8167
20	Norway	19.9855	498.4811	24.3500	81.2633	38.0238	89.5500
21	Poland	26.4191	501.4353	21.7531	66.1867	24.8147	86.0367
22	Portugal	23.6089	501.1001	15.1691	77.4400	25.4516	82.2067
23	Slovak Republic	19.4372	460.7749	18.7881	71.2133	20.7731	79.9067
24	Slovenia	28.8696	512.8636	25.5421	69.5467	21.1539	83.8000
25	Spain	17.7485	492.7861	22.4697	66.0233	22.6820	77.3800
26	Sweden	25.3405	493.4224	24.6998	84.1767	43.4855	89.1367
27	Switzerland	25.3405	505.5058	25.4500	81.2233	38.1637	88.1067
28	Turkey	13.3391	425.4895	14.7689	61.8800	24.2958	76.4267
29	United Kingdom	22.8298	509.2215	22.6604	79.5733	37.0920	84.9200
30	United States	19.8534	496.2424	22.5901	68.0967	24.6532	80.5533

In stage 1, government expenditure for primary level and PISA scores are taken into consideration. In stage 2 and stage 3, government expenditures, employment rates respectively for upper secondary and tertiary levels are taken into consideration. In this case, we can measure the efficiency of stage 1 of each OECD country among the set of DMUs using X_1 as input and Z_1, Z_2 as outputs. The efficiencies of the stage 2 can be measured using Z_1, Z_2 as input and T_1, T_2 as outputs. Similarly, the efficiencies of the stage 3 can be measured using T_1, T_2 as input and Y_1 as outputs. The overall efficiency is also measured using X_1 as input and Y_1 as outputs with Model (1). Table 2 presents the network structure for the implementation of the SDG 4 and the data.

3. RESULT and FINDINGS

As mentioned in the literature section, a number of studies have shown that there is a positive link between government expenditure on education and employment. This study reveals that the network DEA model can be used with the aim of measuring the efficiency of the OECD countries from an educational economy perspective. The overall and substages efficiencies are calculated for each country with the Model (1) and Model (2) using the GAMS code in Appendix A and Appendix B.

After running the GAMS codes, the efficiency scores and the rank of countries are shown in Table 3 and Figure 3. The ranking at the overall efficiency scores (E_k) shows that Latvia, Slovenia and Korea at the top-three countries in terms of network structure's indicators. Additionally, Turkey, Chile and Czechia are found as the lowest three countries. Broadly speaking, the findings are verified that the developed countries in data set are carried to an upper order in the ranking. Conversely, the developing countries such as Turkey, Chile and Mexico are located at the lower in the ranking. Amazingly, Korea is located at third place notwithstanding the country is developing.

Table 3 shows that Sweden, Iceland, United Kingdom and Portugal are at the highest order of ranking in stage-1. At first look the rank of Portugal are demonstrating encouraging results. But, Portugal is one of the countries that has made the fastest progress in improving educational attainment such as PISA scores (OECD, 2012). In this context, there is no doubt that an increase in PISA scores and government expenditure for related level will lead to an enhancement in the efficiency of stage-1 due to the impact of the output-oriented model in the analysis.

In stage-2, Slovenia has been found an efficient country with regard to indicators of the educational economy. Besides, Latvia, and Spain have an efficiency score that is very close to 1.0000. However, Iceland, Portugal, and Mexico have the lowest efficiency scores. Considering the structure of Figure 3, high investments in education accelerates the growth of countries, and the growth of the country maintains the employment rates (Domar, 1946; Landau, 1983).

In stage-3, Latvia has the highest efficiency score ($E_k = 1.0000$). Following this country, it is seen that Portugal and Iceland have respectively 0.9998 and 0.9726 efficiency score. The fact that those countries are in the top three can be owing to having high employment rate and budgeting high government expenditure per student. On the other hand, Czechia, France, and Ireland have respectively 0.5411, 0.5983 and 0.6066 efficiency scores. This indicates that these countries need improvement in the indicators that used for the network structure model.

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DMU	Country	E_k	E_k^1	E_k^2	E_k^3
1	Australia	0.5305 (22)	0.9720 (7)	0.7274 (26)	0.7503 (17)
2	Austria	0.6532 (9)	0.9401 (14)	0.8988 (11)	0.7731 (14)
3	Belgium	0.6307 (13)	0.9123 (20)	0.9137 (10)	0.7566 (16)
4	Chile	0.4269 (29)	0.8875 (23)	0.7157 (27)	0.6721 (21)
5	Czechia	0.4358 (28)	0.9542 (10)	0.8441 (14)	0.5411 (30)
6	Denmark	0.7074 (5)	0.9799 (4)	0.8651 (13)	0.8345 (9)
7	Estonia	0.6087 (14)	0.9494 (12)	0.8266 (15)	0.7756 (13)
8	Finland	0.5996 (15)	0.9294 (17)	0.9610 (4)	0.6713 (22)
9	France	0.5100 (23)	0.9214 (18)	0.9252 (9)	0.5983 (29)
10	Germany	0.4859 (26)	0.9594 (9)	0.8103 (19)	0.6250 (27)
11	Hungary	0.5353 (20)	0.9150 (19)	0.8194 (16)	0.7140 (18)
12	Iceland	0.6397 (12)	1.0000(1)	0.6577 (30)	0.9726 (3)
13	Ireland	0.4917 (25)	0.8758 (25)	0.9256 (8)	0.6066 (28)
14	Israel	0.5920 (16)	0.8758 (26)	0.7649 (25)	0.8837 (6)
15	Italy	0.6491 (10)	0.9433 (13)	0.8926 (12)	0.7709 (15)
16	Korea	0.7357 (3)	0.9519 (11)	0.9514 (5)	0.8124 (10)
17	Latvia	0.8770 (1)	0.8778 (24)	0.9991 (2)	1.0000(1)
18	Mexico	0.4382 (27)	0.9298 (16)	0.7049 (28)	0.6686 (23)
19	New Zealand	0.5093 (24)	0.9751 (5)	0.8125 (18)	0.6428 (24)
20	Norway	0.5327 (21)	0.9653 (8)	0.8032 (20)	0.6871 (20)
21	Poland	0.7279 (4)	0.8096 (29)	0.9476 (6)	0.9488 (4)
22	Portugal	0.6843 (6)	0.9925 (3)	0.6896 (29)	0.9998 (2)
23	Slovak Republic	0.5784 (18)	0.9330 (15)	0.7885 (22)	0.7862 (12)
24	Slovenia	0.8180 (2)	0.8650 (27)	1.0000(1)	0.9457 (5)
25	Spain	0.5449 (19)	0.8978 (21)	0.9632 (3)	0.6301 (25)
26	Sweden	0.6744 (8)	1.0000(1)	0.7784 (23)	0.8664 (7)
27	Switzerland	0.6804 (7)	0.9740 (6)	0.8160 (17)	0.8561 (8)
28	Turkey	0.4154 (30)	0.8534 (28)	0.7737 (24)	0.6291 (26)
29	United Kingdom	0.6407 (11)	0.9927 (2)	0.8001 (21)	0.8067 (11)
30	United States	0.5851 (17)	0.8880 (22)	0.9379 (7)	0.7025 (19)

 Table 3. Efficiency scores*

* The values in parentheses are rank values of the countries.

Country	Ek	E_k^1	E_k^2	E_k^3
Australia	0.5305	0.972	0.7274	0.7503
Austria	0.6532	0.9401	0.8988	0.7731
Belgium	0.6307	0.9123	0.9137	0.7566
Chile	0.4269	0.8875	0.7157	0.6721
Czechia	0.4358	0.9542	0.8441	0.5411
Denmark	0.7074	0.9799	0.8651	0.8345
Estonia	0.6087	0.9494	0.8266	0.7756
Finland	0.5996	0.9294	0.961	0.6713
France	0.51	0.9214	0.9252	0.5983
Germany	0.4859	0.9594	0.8103	0.625
Hungary	0.5353	0.915	0.8194	0.714
Iceland	0.6397	1	0.6577	0.9726
Ireland	0.4917	0.8758	0.9256	0.6066
Israel	0.592	0.8758	0.7649	0.8837
Italy	0.6491	0.9433	0.8926	0.7709
Korea 🗾	0.7357	0.9519	0.9514	0.8124
Latvia	0.877	0.8778	0.9991	1
Mexico	0.4382	0.9298	0.7049	0.6686
New Zealand	0.5093	0.9751	0.8125	0.6428
Norway	0.5327	0.9653	0.8032	0.6871
Poland	0.7279	0.8096	0.9476	0.9488
Portugal	0.6843	0.9925	0.6896	0.9998
Slovak Republic	0.5784	0.933	0.7885	0.7862
Slovenia	0.818	0.865	1	0.9457
Spain	0.5449	0.8978	0.9632	0.6301
Sweden	0.6744	1	0.7784	0.8664
Switzerland	0.6804	0.974	0.816	0.8561
Turkey	0.4154	0.8534	0.7737	0.6291
United Kingdom	0.6407	0.9927	0.8001	0.8067
United States	0.5851	0.888	0.9379	0.7025

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To sum up, the reason for the high ranks of the countries is that the employment rate can be increased by the awareness of the quality education. Within this framework, these countries can enhance their efficiency score by designing educational economy policies toward strengthening the employment rate per each stage.

4. DISCUSSION and CONCLUSION

The government expenditure on education can be regarded as one of the most important indicators influence on increasing employment growth. On a priori grounds, it is not always possible to measure the efficiency of the countries on how government expenditure affects the employment rate. However, it is noteworthy that the evaluation of the countries as a whole in terms of educational economy for academic literature and policy-making studies. Besides, indicators such as government expenditures on education and employment rates at educational levels determined by the OECD can be used directly in the evaluation of the educational performance of countries. However, these indicators used alone are not sufficient to determine the educational economy performance of a country. In this context, situations related to the different economic, social and cultural conditions of the nations at the micro level should be taken into consideration while a combination of official statistics should be used at the macro

Figure 3. Efficiency scores of the countries

level. In this way, it is possible to evaluate the multi-dimensional concepts of quality education (SDG 4) together.

This paper has desired to find an answer to the question of whether the government expenditure on education affects the employment rates at ISCED education levels. To tackle this issue, we examine the educational economy efficiency of OECD countries using the relational network DEA, which is a sub-branch of the network DEA model, in order to provide support to policymakers, international education statistics users and academic studies and to determine the indicators that affect quality education. Traditional DEA models perform better than parametric methods in the performance measurement of individual decision-making units. For this reason, it is more accurate to use the traditional DEA based approaches in the research of regional and national education systems and in measuring the performance of the educational economy. However, traditional DEA models are not suitable for measuring the efficiency of substages structures because the performance of interactive substages is neglected. In contrast to traditional DEA models, the relational network DEA can present a systematic view which reflects the countries' correct rank, and provide information about the countries' positioning with regard to indicators used. This analysis shed new light on measuring the educational economy efficiency by taking into consideration indicators on the substages. In this context, we have investigated multistage efficiency scores across the OECD countries by assessing the outputs PISA science performance (stage-1), government expenditure per secondary student (stage-1), employment rate for upper secondary level (stage-2), government expenditure per tertiary student (stage-2), employment rate for tertiary level (stage-3) against inputs directly used in the education system (Government expenditure per primary student (stage-1), PISA science performance (stage-2), Government expenditure per secondary student (stage-2), employment rate for upper secondary level (stage-3), government expenditure per tertiary student (stage-3). By means of having the efficiency of the substages, it was also possible to examine the effects of the indicators used in each substages on the overall educational economy efficiency.

As a consequence of the relational network DEA model's solution, a low-efficiency score is assigned to inadequate units, namely countries, and a high-efficiency score is assigned to adequate units. This efficiency scores reflect the distance to other units in the efficient border estimated during the performance evaluation phase. Thus, the minimum proportional decrease in the inputs or the maximum proportional increase in the outputs of the efficient units can be determined. The empirical results demonstrate that the countries with high-efficiency scores are clustered around countries like Latvia, Slovenia, Korea, and Poland in both overall efficiency and the substages efficiency. In other respect, the countries with low-efficiency scores are clustered around a small number of core countries like Czechia, Mexico, Turkey, and Chile. Therefore, the current paper points out that the relational network DEA can be applied for measuring the educational economy efficiency of the countries due to the capability of providing realistic findings in the country assessment. Besides, it can be said that the relational network DEA models, which provide a scientifically objective analysis and capture the performance complexity of the units dealt with by their nature, are used as an important tool in making international comparisons of country performance in specific areas such as competitiveness, globalization, innovation, and sustainable development. Considering the efficiency scores obtained with this model, the substages efficiencies of the countries define the performance of macroeconomic indicators affecting the education economy at a disaggregated level and enables the analysis of policy areas. On the other hand, the overall efficiency scores of the countries can help determine the policy priorities by determining the extent to which the national performance expectation is met through an international comparison. In this context, network DEA models analyze economic performance beyond simple one-dimensional models that allow analysis between different areas.

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APPENDIX

Appendix A. GAMS code to calculate the overall eff	iciency
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SETS

SE1S				
j 'number o	f DMUS' /DMU1*DMU30/			
i 'number of	f inputs' /X1/			
r 'number o	f outputs' /Y1/			
p 'number o	of intermediates1'/Z1, Z2/			
1 'number of	f intermediates2' /T1, T2/;			
TABLE X(j, i) "input matrix"			
	X1			
DMU1	18.5932			
DMU2	23.4497			
DMU3	22.4726			
DMU4	15.0754			
DMU5	15.5312			
DMU6	25.6117			
DMU7	21.4939			
DMU8	21.0185			
DMU9	18 0165			
DMU10	17 9128			
DMU11	18 4401			
DMU12	24 4524			
DMU12	24.4324			
DMU15	10.7971			
DMU14	21.4471			
DMU15	21.3017			
DMU16	23.9656			
DMU17	31.2500			
DMU18	14.8592			
DMU19	18.6102			
DMU20	19.9855			
DMU21	26.4191			
DMU22	23.6089			
DMU23	19.4372			
DMU24	28.8696			
DMU25	17.7485			
DMU26	25.3405			
DMU27	25.3405			
DMU28	13.3391			
DMU29	22.8298			
DMU30	19.8534;			
TABLE Y((j, r) "output matrix"			
DIG	Y1			
DMU1	83.1/6/			
DMU2	85.4733			
DMU3	84.4900			
DMU4	84.1950			
DMU5	84.7400			
DMU6	86.2533			
DMU7	84.2533			
DMU8	83.3400			
DMU9	84.0300			
DMU10	87.9900			
DMU11	81.6667			
DMU12	91.2133			
DMU13	81.1100			
DMU14	85.9100			
DMU15	78.1000			

1	DMU16	77.4733	
	DMU17	85,1000	
	DMU18	80.3367	
	DMU19	86 8167	
	DMU20	89 5500	
	DMU20	86.0267	
	DMU21	80.0507	
	DMU22	82.2067	
	DMU23	/9.906/	
	DMU24	83.8000	
	DMU25	77.3800	
	DMU26	89.1367	
	DMU27	88.1067	
	DMU28	76.4267	
	DMU29	84.9200	
	DMU30	80.5533;	
	TABLE Z	i. p) "interme	diate1 matrix"
		Z 1	72
	DMI1	500 0030	16 7917
	DMU2	495 0375	27 3519
		501 0007	25 7087
		JUI.7777/	15 1517
	DMU4	440.9301	13.1317
	DMU5	492.8300	25.5477
	DMU6	501.9369	28.2276
	DMU7	534.1937	20.1925
	DMU8	530.6612	27.2000
	DMU9	494.9776	26.8059
	DMU10	509.1406	23.4938
	DMU11	476.7475	19.5469
	DMU12	473.2301	18.3453
	DMU13	502.5751	21.6000
	DMU14	466.5528	16.9659
	DMU15	480.5468	23.0962
	DMU16	515.8099	23.3231
	DMU17	490.2250	29.6703
	DMU18	415,7099	16.4109
	DMU19	513 3035	22 3160
	DMU20	/98/1811	24.3500
	DMU20	501 4353	21.7531
	DMU22	501.4001	15 1601
	DMU22	460 7740	18 7881
		400.1149 510 9696	25 5421
		J12.0030	23.3 4 21 22.4607
	DIVIU25	492./801	22.407/ 24.2009
	DMU20	495.4224	24.0998
	DMU27	505.5058	25.4500
	DMU28	425.4895	14.7689
	DMU29	509.2215	22.6604
	DMU30	496.2424	22.5901;
	TABLE T(j, l) "intermed	tiate2 matrix"
		T1	T2
	DMU1	77.6700	22.5295
	DMU2	75.9433	36.1737
	DMU3	72.8533	33.0074
	DMU4	71.7000	17.3957
	DMU5	77.7067	21.6123
	DMU6	79.6433	44.6629
	DMU7	75,7733	27.7667
	DMU8	73.2000	35,4852
	DMI	72.8867	35.0586
		. =	·····

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DMU10	79.4667	37.5885
DMU 11	71.5033	23.8811
DMU12	87.0433	25.6424
DMU13	67.6133	25.2509
DMU14	72.4467	19.4927
DMU15	69.8567	26.1989
DMU16	71.9867	13.7440
DMU17	70.9233	22.9545
DMU18	70.6100	40.4662
DMU19	80.9400	27.9934
DMU20	81.2633	38.0238
DMU21	66.1867	24.8147
DMU22	77.4400	25.4516
DMU23	71.2133	20.7731
DMU24	69.5467	21.1539
DMU25	66.0233	22.6820
DMU26	84.1767	43.4855
DMU27	81.2233	38.1637
DMU28	61.8800	24.2958
DMU29	79.5733	37.0920
DMU30	68.0967	24.6532;

parameters

Xo(i) "input vector of DMUo" Yo(r) "outputput vector of DMUo"

Zo(p) "intermediate1 vector of DMUo"

To(l) "intermediate2 vector of DMUo";

variables

thetaall "efficiency score all"
v(i) "input weights"
u(r) "output weights"
w(p) "intermediate1 weights"
q(l) "intermediate2 weights";

free variables

thetaall;

positive variables

v(i) u(r) w(p)

q(l);

equations

EQA EQB EQC EQD EQE EQF EQG OBJ;

```
\begin{split} & EQA.. \; \textbf{SUM} \; (i, v(i) * Xo(i)) = E = 1; \\ & EQB \; (j).. \; \textbf{SUM} \; (r, u(r) * Y(j, r)) - \textbf{SUM} \; (i, v(i) * X(j, i)) = L = 0; \\ & EQC \; (j).. \; \textbf{SUM} \; (l, q(l) * T(j, l)) - \textbf{SUM} \; (i, v(i) * X(j, i)) = L = 0; \\ & EQD(j).. \; \textbf{SUM} \; (p, w(p) * Z(j, p)) - \textbf{SUM} \; (i, v(i) * X(j, i)) = L = 0; \\ & EQE \; (j).. \; \textbf{SUM} \; (r, u(r) * Y(j, r)) - \textbf{SUM} \; (p, w(p) * Z(j, p)) = L = 0; \\ & EQF \; (j).. \; \textbf{SUM} \; (r, u(r) * Y(j, r)) - \textbf{SUM} \; (l, q(l) * T(j, l)) = L = 0; \end{split}
```

EQG (j).. **SUM** (l, q(l) * T(j, l)) -**SUM** (p, w(p) * Z(j, p)) = L = 0;OBJ.. thetaall =E= **SUM** (r, u(r) * Yo(r)); * * overall efficiency score *_____ model overall / EQA EQB EQC EQD EQE EQF EQG OBJ /; ALIAS (j,o); LOOP (o, **LOOP** (i, Xo(i) = X(o, i));**LOOP** (r, Yo(r) = Y(o, r));**LOOP** (l, To(l) = T(o, l));**LOOP** (p, Zo(p) = Z(o, p));**SOLVE** overall USING LP maximizing thetaall;);

Appendix B. GAMS code to calculate the substages efficiencies

SETS	
j 'number of	f DMUS' /DMU1*DMU30/
i 'number of	f inputs' /X1/
r 'number of	f outputs' /Y1/
p 'number o	of intermediates1'/Z1, Z2/
1 'number of	f intermediates2' /T1, T2/
m 'number	of theta3'/thetaall/;
TABLE X((j, i) "input matrix"
	X1
DMU1	18.5932
DMU2	23.4497
DMU3	22.4726
DMU4	15.0754
DMU5	15.5312
DMU6	25.6117
DMU7	21.4939
DMU8	21.0185
DMU9	18.0165
DMU10	17.9128
DMU 11	18.4491
DMU12	24.4524
DMU13	16.7971
DMU 14	21.4471
DMU15	21.3017
DMU16	23.9656

DMU17	31.2500	
DMU18	14.8592	
DMU19	18.6102	
DMU20	19.9855	
DMU21	26 4191	
DMU22	23 6089	
DMU22	10 4372	
DMU24	19.4572 28.8606	
DMU24	20.0090	
DMU25	25 2405	
DMU20	25.5405	
DMU27	23.3403	
DMU28	13.3391	
DMU29	22.8298	
DMU30	19.8534;	
TABLE Y	(j, r) "output r	natrix"
	Y1	
DMU1	83.1767	
DMU2	85.4733	
DMU3	84.4900	
DMU4	84.1950	
DMU5	84.7400	
DMU6	86.2533	
DMU7	84.2533	
DMU8	83.3400	
DMU9	84.0300	
DMU10	87.9900	
DMU11	81.6667	
DMU12	91.2133	
DMU13	81.1100	
DMU14	85.9100	
DMU15	78,1000	
DMU16	77.4733	
DMU17	85 1000	
DMU18	80 3367	
DMU19	86 8167	
DMU20	89 5500	
DMU21	86.0367	
DMU22	82 2067	
DMU23	79 9067	
DMU24	83 8000	
DMU24	77 3800	
DMU25	20 1267	
DMU20	09.1307 99.1067	
DMU27	00.1007 76.4267	
DMU28	70.4207	
DMU29	84.9200	
DMU30	80.5533;	
	(*	The section sector II
IABLE Z	(J, p) interme	
DMUI	Z1	L2
DMUI	509.9939	16./91/
DMU2	495.0375	27.3519
DMU3	501.9997	25.7987
DMU4	446.9561	15.1517
DMU5	492.8300	23.5477
DMU6	501.9369	28.2276
DMU7	534.1937	20.1925
DMU8	530.6612	27.2000
DMU9	494.9776	26.8059
DMU10	509.1406	23.4938

DMU11	476.7475	19.5469
DMU12	473.2301	18.3453
DMU13	502.5751	21.6000
DMU14	466 5528	16 9659
DMU15	480 5468	23,0962
DMU15	515 2000	23.0702
DMU10	313.8099	25.5251
DMU17	490.2250	29.6703
DMU18	415.7099	16.4109
DMU19	513.3035	22.3160
DMU20	498.4811	24.3500
DMU21	501.4353	21.7531
DMU22	501.1001	15.1691
DMU23	460.7749	18.7881
DMU24	512.8636	25.5421
DMU25	492.7861	22.4697
DMU26	493.4224	24.6998
DMU27	505.5058	25.4500
DMU28	425 4895	14 7689
DMU29	509 2215	22 6604
DMU30	496 2424	22.5004
DIVICSO	470.2424	22.3901,
TABLE I	(j, l) "interme	diate2 matrix
	TI	12
DMU1	77.6700	22.5295
DMU2	75.9433	36.1737
DMU3	72.8533	33.0074
DMU4	71.7000	17.3957
DMU5	77.7067	21.6123
DMU6	79.6433	44.6629
DMU7	75.7733	27.7667
DMU8	73.2000	35.4852
DMU9	72.8867	35.0586
DMU10	79 4667	37 5885
DMU11	71 5033	23 8811
DMU12	87 0/33	25.6011
DMU12	67 6133	25.0424
DMU13	72 4467	10 4007
DMU14	/2.440/	19.4927
DMU15	69.8567	26.1989
DMU16	/1.986/	13.7440
DMU17	70.9233	22.9545
DMU18	70.6100	40.4662
DMU19	80.9400	27.9934
DMU20	81.2633	38.0238
DMU21	66.1867	24.8147
DMU22	77.4400	25.4516
DMU23	71.2133	20.7731
DMU24	69.5467	21.1539
DMU25	66.0233	22.6820
DMU26	84.1767	43.4855
DMU27	81.2233	38.1637
DMI128	61.8800	24,2958
DMI129	79 5733	37 0920
DMU20	68 0067	24 6532.
00000	00.0707	24.0332,
	ataall(i m) "	ffician are soone mathin!
	thotas 11	enciency score matrix"
DMU		
	0.0538	
DMU2	0.0426	
DMU3	0.0445	
DMU4	0.0663	

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DMU5	0.0644
DMU6	0.0390
DMU7	0.0465
DMU8	0.0476
DMU9	0.0555
DMU10	0.0558
DMU11	0.0542
DMU12	0.0409
DMU13	0.0595
DMU14	0.0466
DMU15	0.0469
DMU16	0.0417
DMU17	0.0320
DMU18	0.0673
DMU19	0.0537
DMU20	0.0500
DMU21	0.0379
DMU22	0.0424
DMU23	0.0514
DMU24	0.0346
DMU25	0.0563
DMU26	0.0395
DMU27	0.0395
DMU28	0.0750
DMU29	0.0438
DMU30	0.0504;

parameters

Xo(i) "input vector of DMUo" Yo(r) "outputput vector of DMUo" Zo(p) "intermediate1 vector of DMUo" To(1) "intermediate2 vector of DMUo" thetaallo(m) "efficiency score vector of DMUj";

variables

theta3 "efficiency score of subprocess 3"
v(i) "input weights"
u(r) "output weights"
w(p) "intermediate1 weights"
q(1) "intermediate2 weights";

free variables

theta3;

positive variables v(i) u(r) w(p) q(l); equations EQA

EQA EQB EQC EQD EQE EQF EQG EQH OBJ;

```
EQA..SUM (p, w(p) * Zo(p)) =E= 1;
EQB (m).. SUM (r, u(r) * Yo(r)) – (thetaallo(m) * SUM (i, v(i) * Xo(i))) =E= 0;
EQC (j).. SUM (r, u(r) * Y(j, r)) – SUM (i, v(i) * X(j, i)) =L= 0;
EQD (j).. SUM (l, q(l) * T(j, l)) – SUM (i, v(i) * X(j, i)) =L= 0;
EQE (j).. SUM (p, w(p) * Z(j, p)) – SUM (i, v(i) * X(j, i)) =L= 0;
EQF (j).. SUM (r, u(r) * Y(j, r)) – SUM (l, q(l) * T(j, l)) =L= 0;
EQG (j).. SUM (r, u(r) * Y(j, r)) – SUM (p, w(p) * Z(j, p)) =L= 0;
EQH (j).. SUM (l, q(l) * T(j, l)) - SUM (p, w(p) * Z(j, p)) = L = 0;
OBJ.. theta3 =E= SUM (r, u(r) * Yo(r));
* subprocess3 efficiency score
*___
model subprocess3 /
EQA
EQB
EQC
EQD
EQE
EQF
EQG
EQH
OBJ
/;
ALIAS (j,o);
LOOP (o,
           LOOP (i, Xo(i) = X(o, i));
           LOOP (r, Yo(r) = Y(o, r));
           LOOP (1, To(1) = T(0, 1));
           LOOP (p, Zo(p) = Z(o, p));
           LOOP (m, thetaallo(m) = thetaall(o, m));
           SOLVE subprocess2 USING LP maximizing theta3;
);
```