JMR

BURSA ULUDAĞ UNIVERSITY JOURNAL OF MOSAIC RESEARCH

AIEMA - TÜRKİYE

OFFPRINT / AYRIBASIM

VOLUME 12 2019

JMR

Volume 12

2019

Bursa Uludağ University Press | Bursa Uludağ Üniversitesi Yayınları
Bursa Uludağ University Mosaic Research Center | Bursa Uludağ Üniversitesi Mozaik Araştırmaları Merkezi
Series - 3 | Serisi - 3

JMR - 12

AIEMA - Türkiye is a research center that aims to study, introduce and constitute a data bank of the mosaics from the prehistoric times till today. The best presentation of the mosaics of Turkey is the ultimate goal of this center functioning depending on AIEMA. A data bank of Turkey mosaics and a corpus including Turkey mosaics are some of the practices of the center. Additionally, this center also equips a periodical including the art of ancient mosaics and original studies namely JMR.

The JMR (Journal of Mosaic Research) is an international journal on mosaics, annually published by the Bursa Uludağ University Mosaic Research Centre. The aim of this journal is to serve as a forum for scientific studies with critical analysis, interpretation and synthesis of mosaics and related subjects. The main matter of the journal covers mosaics of Turkey and other mosaics related to Turkey mosaics. Besides, the journal also accommodates creative and original mosaic researches in general. Furthermore, together with articles about mosaics, the journal also includes book presentations and news about mosaics.

JMR is a refereed journal. The articles sent to our journal are scanned with the “Ithenticate” plagiarism program, and the referee evaluation process is initiated according to the report result received from the program.

The manuscripts can be written in English, German, French or Turkish. All authors are responsible for the content of their articles.

JMR is indexed as a full text by EBSCO since 2009; by TÜBİTAK - ULAKBİM Social Sciences Databases since 2014 and by Clarivate Analytics (Thomson Reuters) - Emerging Sources Citation Index (ESCI) since 2016. Articles are published with DOI number taken by Crossref.

JMR is published each year in November.

The abbreviations in this journal are based on German Archaeological Institute publication criterions, Bulletin de l’Association international pour l’Etude de la Mosaique antique, AIEMA - AOROC 24.2016, La Mosaique Gréco-Romaine IX and Der Kleine Pauly.

AIEMA - Türkiye, prehistorik dönemden günümüze kadar uzanan zaman süreci içerisindeki mozaikler hakkında bilimsel çalışmalar yapmayı, bu mozaikleri tanıtmayı ve söz konusu mozaikler hakkında bir mozaik veri bankası oluşturmayı amaçlayan bir araştırma merkezidir. AIEMA’ya bağlı olarak, Türkiye mozaiklerinin en iyi şekilde sunumu, bu merkezin işleyişinin nihai hedefidir. Türkiye mozaik veri bankası ve Türkiye mozaiklerini de içeren bir korpus hazırlanması çalışmaları, merkezin faaliyetlerinden bazılarıdır. Ayrıca, merkezin, antik mozaikler hakkında özgün çalışmaları içeren JMR (Journal of Mosaic Research) adında bir süreli yayını vardır.

JMR (Journal of Mosaic Research) Dergisi, her yıl Bursa Uludağ Üniversitesi Mozaik Araştırmaları Merkezi tarafından, mozaikler konusunda yayınlanan uluslararası bir dergidir. Bu derginin amacı, mozaikler hakkında eleştirel bir analiz, yorumlama, mozaik ve onunla ilgili konuların sentezi ile bilimsel çalışmalar için bir platform oluşturmaktır. Derginin temel konusu, Türkiye mozaikleri ve Türkiye mozaikleriyle ilişkili mozaiklerdir. Bunun yanında, dergi yaratıcı ve özgün mozaik araştırmaları içeren diğer mozaiklerle ilgili makaleleri de kabul etmektedir. Ayrıca dergide, mozaikler hakkındaki makalelerle birlikte, kitap tanıtımları ve haberler de bulunmaktadır.

JMR hakemli bir dergidir. Dergimize gönderilen makaleler, “Ithenticate” intihal programı ile taranmakta olup, programdan alınan rapor sonucuna göre hakem değerlendirme süreci başlatılmaktadır.

Makaleler İngilizce, Almanca, Fransızca ve Türkçe dillerinde yazılabilir. Dergide yayınlanan makalelerin sorumluluğu makale sahiplerine aittir.

JMR, 2009 yılından itibaren EBSCO tarafından tam metin olarak, 2014 yılından itibaren TÜBİTAK - ULAKBİM Sosyal Bilimler veri tabanları tarafından ve 2016 yılından itibaren ise Clarivate Analytics (Thomson Reuters) - Emerging Sources Citation Index (ESCI) tarafından taranmaktadır. Makaleler, Crossref'ten alınan DOI numarası ile yayınlanmaktadır.

JMR, her yıl Kasım ayında yayınlanmaktadır.

Bu dergideki makalelerde kullanılacak olan kısaltmalar Alman Arkeoloji Enstitüsü yayın kuralları, Bulletin de l’Association international pour l’Etude de la Mosaique antique, AIEMA - AOROC 24.2016, La Mosaique Greco Romaine IX ve Der Kleine Pauly dikkate alınarak yapılmalıdır.

Journal of Mosaic Research
ISSN 1309-047X
E-ISSN 2619-9165

Printed by / Baskı
12.matbaa
İbrahim Karaoğlanoğlu Cad. No: 35 Kat: 1
34418 Seyrantepe / Kağıthane-İstanbul
Tel: +90 212 281 25 80; www.onikincimatbaa.com
Certificate No: 33094

For detailed information please visit website / Detaylı bilgi için lütfen web sitesini ziyaret ediniz:

http://arkeoloji.uludag.edu.tr/JMRe/

Address / Adres:
Bursa Uludağ University / Bursa Uludağ Üniversitesi
Faculty of Art and Sciences / Fen Edebiyat Fakültesi
Department of Archaeology / Arkeoloji Bölümü
16059 - Nilüfer / BURSA – TÜRKİYE

Tel & Fax: + 90 224 2941892
E.mail: mosaicsjournal@gmail.com
aiematurkey@uludag.edu.tr
Facebook: @journalofmosaicresearch
Instagram: @journalofmosaicresearch
Twitter: @mosaicsresearch

CONTENTS

JOURNAL OF MOSAIC RESEARCH

Archaeology / Arkeoloji

DOI: 10.26658/jmr.614865 JMR 12, 2019 107-118

The Third Dimension of the Magdouh Mosaic in Antioch

Antakya Magdouh Mozaiği'nin Üçüncü Boyutu

Hakan HİSARLIGİL * - Beyhan BOLAK HİSARLIGİL**

(Received 06 July 2019, accepted after revision 30 September 2019)

Abstract

This study investigates the complex geometry of the pattern of circles in the Magdouh Mosaic, which dates to between the fifth and sixth centuries, found in Antioch-on-the-Orontes (modern Antakya, Turkey). The mosaic has a complex pattern that is composed of tangent, overlapping and intersecting circles suggesting that it was worthwhile analyzing in 3D. Through such an analysis, it was observed that the extension of the pattern of circles into a third dimension looks like spheres packed together that intersect in three directions at right angles. This can also be demonstrated by packing octahemioctahedra, which are a cluster of eight regular packed tetrahedra contained within the spheres. In addition, the packing of octahemioctahedra in this way also suggests the packing of stella octangula contained within hemispheres that intersect each other. Furthermore, tracing the line segments that are emphasized in each circle in the Magdouh Mosaic in such a 3D structure turns them into space-filling rhombohedra that can be dissected into two tetrahedra and an octahedron. Thus, the result of this study shows that the Magdouh Mosaic in Antioch is the top view of a square-based pyramidal cluster of intersecting hemispheres that hold space-filling rhombohedra inside them.

Keywords: *Ancient mosaics, Antioch, geometry, cubic close packing, rhombohedron.*

Öz

Bu çalışma, Antakya'da bulunan, beşinci ve altıncı yüzyıllara ait Magdouh Mozaiğinin karmaşık geometrik düzeninin üç boyutlu içeriğini araştırmaktadır. Bunun nedeni, mozaik kompozisyonunu oluşturan geometrik düzenin yüzeyde oldukça güçlü bir derinlik etkisi oluşturmasıdır. Bu amaçla çalışma kapsamında bu mozaiğin geometrik dokusunu oluşturan elemanlar ile bunların arasındaki ilişkilerin üçüncü boyuttaki geometrik karşılıkları belirlenmeye çalışılmıştır. Bu kapsamda öncelikle mozaik bütününe bakıldığında görsel kompozisyon düzlemsel olarak dört katmandan oluşan daire kümelerinden oluşan kare tabanlı tepeler olarak görülmektedir. Bu tepelerde simetrik olarak dört farklı doğrultuda birbirine teğet olan, birbirleri üstüne binen ve birbirleri ile kesişen daireler bir arada görülmektedir. Üçüncü boyuta taşındığında da bu kümelerin dokusunu oluşturan dairelerin, dik koordinat sisteminde birbirleriyle kesişen kürelere karşılık geldiği belirlenmiştir. Kürelerin dik koordinat sisteminde bu oranda kesişmesi ile oluşan üç boyutlu ağ içerisinde her bir küre, bir küp sekizyüzlünün kenarlarını oluşturacak şekilde paketlenen sekiz adet düzgün dörtyüzlü bir küme içerirken, kesişen herhangi iki kürenin kesişen yarıları ise yıldızlaşmış bir düzgün sekizyüzlü içermektedir. Her bir yıldızlaşmış sekizyüzlü ise birbirleriyle kesişen dört adet rombik altı yüzlü birime ayrıştırılabilmektedir. Bu gometrik içerik bir bütün olarak değerlendirildiğinde, Magdouh Mozaik yüzeyi üçüncü boyutta uzayı boşluksuz doldurabilen eş altı yüzlü birimlerin paketlenmesi olarak karşılık bulmaktadır.

Anahtar Kelimeler: *Antik mozaikler, Antakya, geometri, sıkışık kübik paketlenme, rombohedron.*

* Hakan Hisarlıgil, İstanbul Rumeli University, Department of Architecture, İstanbul, Türkiye.
ORCID ID: https://orcid.org/0000-0003-0616-9539. E-mail: hhisarligil@gmail.com

** Beyhan Bolak Hisarlıgil, İstanbul Rumeli University, Department of Architecture, İstanbul, Türkiye.
ORCID ID: https://orcid.org/0000-0002-3343-0440. E-mail: bolakb@gmail.com

Introduction

Throughout history, the Ancient Greek and Roman mosaic pattern designs vary from lively scenes of everyday life or mythology to abstract geometric patterns. Compared to research into mythology and everyday life, studies on abstract geometrical patterns in ancient mosaics seem relatively limited (Parzysz 2009). Among geometrical mosaic patterns, the use of a circle – as a basic geometrical figure – is called a scale-pattern (imbrication), peltae or intersecting (interlacing or overlapping) circles in mosaic art found in various places. Among these, the Magdouh Mosaic is one that is especially investigated because it is a complex pattern that represents various patterns of circles as one, and also because it is a part of a rich collection of mosaics with geometrical patterns found in the same era and within the same geography.

For Levi (1947) – who catalogued the Antioch Mosaics chronologically – no other site has been so widely representative of a series with a chronology determined by stratigraphic evidence (Haynes 1951: 111). The Antioch mosaics demonstrate a remarkable continuity with the Hellenistic group artistic tradition (Cimok 2000: 15). The single city of Antioch – through a rich collection of material – is one of the few sites anywhere in the ancient world that have provided so long and continuous a series of mosaics from the eastern Roman Empire. Dating from the late first century or early second century AD to the mid-sixth century AD, the mosaics represent the crucial period spanning the classical to the medieval (Dunbabin 1989: 313). It is only Antioch that has a continuous series of mosaics of particular importance for those interested in early Christian art (Hopkins 1948: 91).

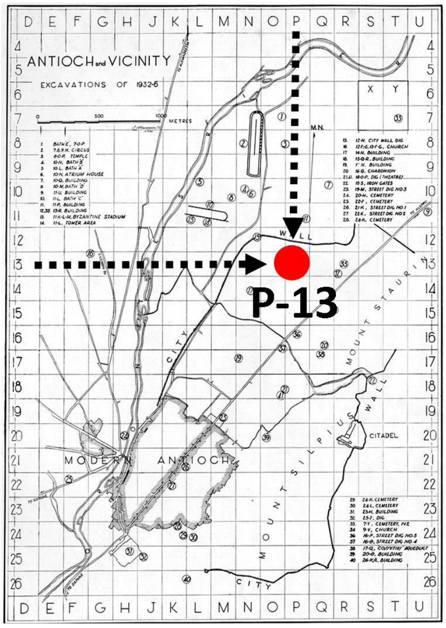
Within such a context, this study, by introducing the descriptions of the geometrical content of the patterns observed by some researchers, explains the geometrical construction of the circular patterns in the Magdouh Mosaic. Secondly, it looks at the 3D content of the planar composition that is a result of a comprehensive geometrical analysis identifying not just a number of polyhedrons, but also types of lattice coordinate axes that would explain the ultimate structure of the design.

The Magdouh Mosaic in Antioch

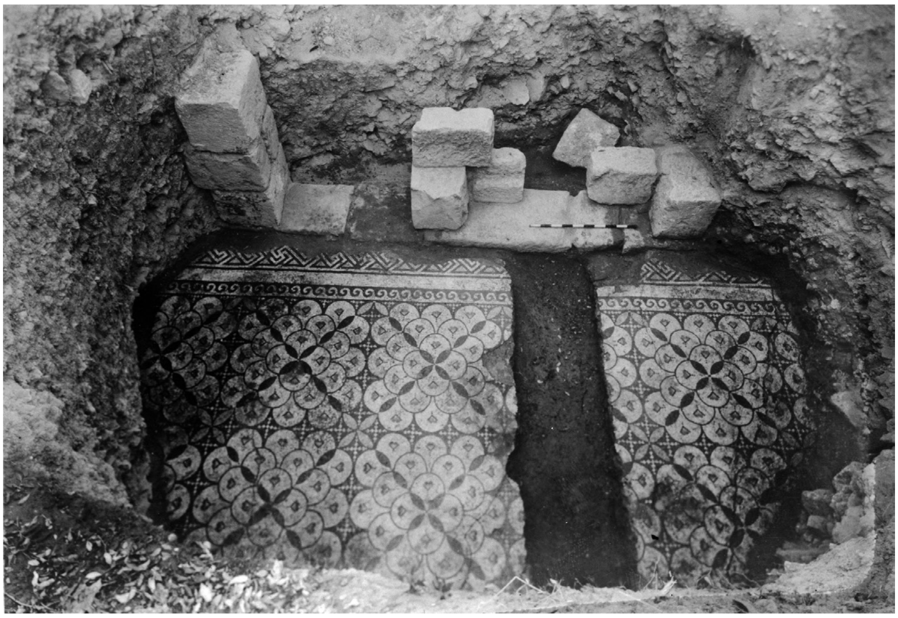
The excavation of Antioch-on-the-Orontes (modern Antakya, Turkey) from 1932– 1939[1] led to the landmark discovery of over three hundred mosaics dating from the first decades of the second century to the sixth century. Antioch-on-the-Orontes was founded in 300 BC as the capital of the Hellenistic Seleucid Empire, and later, the third most important city of the Roman Empire. The floor mosaics were uncovered from the houses of Antioch and its surrounding area, the nearby garden suburb of Daphne, and the port city of Seleucia Pieria. The mosaics found are kept in the Hatay Archaeological Museum in Antioch and scattered across the Louvre Museum in Paris and in various US collections (Campbell 1988; Barsanti 2012: 25). Doro Levi's (1947) "Antioch Mosaic Pavements" still serves as the most comprehensive study and analysis of the mosaics of Antioch uncovered by these excavations. Levi discusses the compositions of the figured scenes and the decorative pattern of the mosaics with the plans of the buildings

1 The history of the excavations began in 1932 and continued until 1939. With the start of the war in Europe it became impossible to carry out any further work. The mosaics were first published as three volume of reports in Elderkin 1934, Stillwell 1938, and Stillwell 1941.

they paved in chronological order[2]. The Magdouh Mosaic or Magdoue Mosaic is a floor mosaic dating to the sixth century (Levi 1947: 626) uncovered in 1937 from the land of Magdouh in Sector 13-P of the excavation area, which is a short distance from the inner face of the north wing of the town walls (Levi 1947: 357) (Fig. 1a[3], Fig. 1b[4]).



a



b

Figure 1
(a) Site plan of "Antioch and Vicinity" showing excavations and the location of Magdouh Mosaic.
(b) Excavation view of the mosaic floor, Magdouh Mosaic, View to North.

It has bisected panels; one half is displayed in the Hatay Archaeological Museum in Antakya, Turkey (Fig. 2a)[5] and the other half is in Princeton University Archaeological Archives in the USA. It is a type of pattern formed from intersecting circles and triangular parts of scale patterns. Although the circular pattern is common among mosaics worldwide, this particular pattern is unique to the Roman region of Syria, which partly includes the Mediterranean coastline from north to south (Fig. 2b). Almost all examples in this region can be found in different structures, such as synagogues, churches, baths and pavements dated to between the fifth and seventh centuries[6]. Although the patterns show differences in terms of color, size and structure, it keeps its unique circular design.

Similar specific patterns and simpler versions were also used over several decades. This pattern was used during the Hellenistic period and was a common design throughout the Roman period (Nassar 2010: 193). Compared to other circular patterns, the Magdouh Mosaic is more complex and has more visual

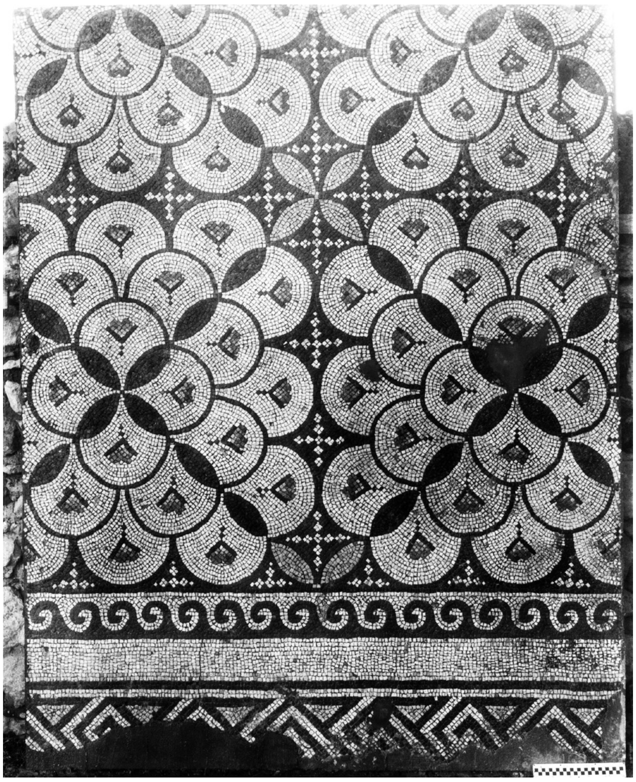
2 Campbell (1988), catalogued the whole Antioch Mosaics with some revised dating after Levi (1947), and Cimok (2000) presented mosaic pavements displayed in the Hatay Archaeological Museum, Turkey. Antioch mosaics were also studied by Kondoleon 2000; Becker - Kondoleon 2005, in general. Unless stated otherwise, the dates given in this study for Antioch mosaics are those of Levi 1947.

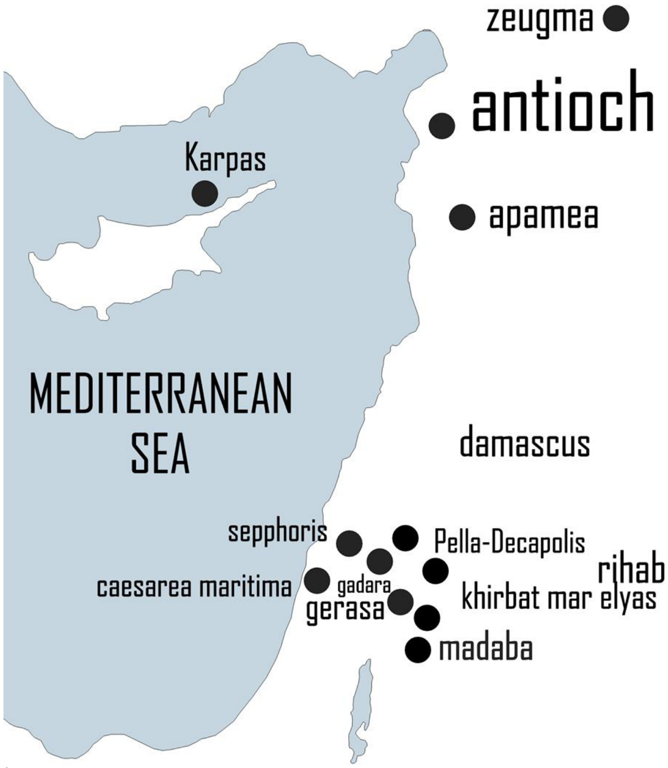
3 Princeton University Archaeological Archives, Antioch II, plan 1, accessed July 15, 2019, http://vrc.princeton.edu/archives/items/show/14543.

4 Levi II, plate CXXXVIIa, September 3 1937, Princeton University Archaeological Archives, accessed May 22, 2018, http://vrc.princeton.edu/archives/items/show/15522.

5 Antioch III, plate 47, September 22 1937, Antioch Museum Archives, accessed May 22, 2018, http://vrc.princeton.edu/archives/items/show/15849.

6 The same mosaic pattern is found in Zeugma, Apamea Synagogue, Hisham's Palace, House of Aion (500), Magdouh Mosaic (500–525), Diocaesarea or Sepphoris (Zippori), Caesarea Maritima, Basilica of Agias Trias, Karpas Peninsula, Cyprus 6th century, Khirbat Mar Elyas, Church of Saint George in Mount Nebo (535/6), Church of Dayr (557/8), Rihab in the Church of Saint Menas (635), Decapolis church at Pella, Baths of Herakleides at Gadara.


a

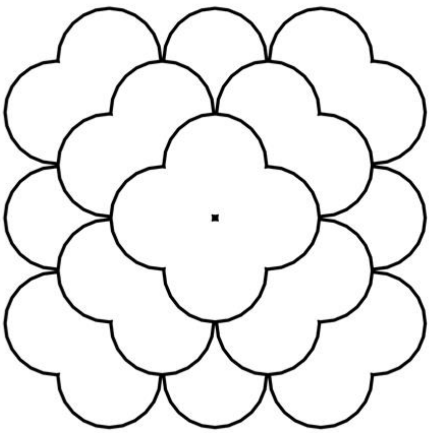


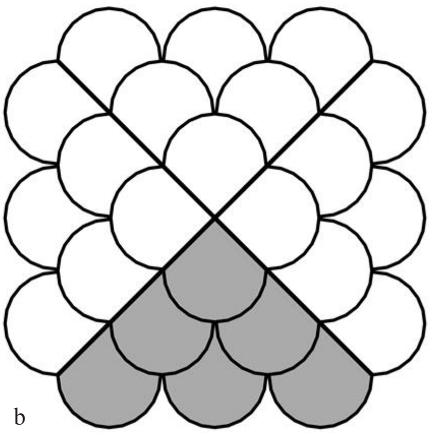
b

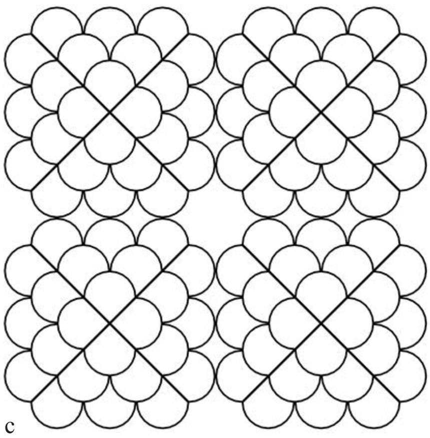
content that make it hard to describe easily. At first glance, the clusters of circles in pyramid form come to the forefront and afterwards the whole pattern becomes an array of these clusters. For this reason, some researchers describe this as a single cluster, while others describe the pattern as a whole. For instance, borrowing Biebel's term "scalloped square" (Biebel 1938: 335), Levi describes the Magdouh Mosaic as a square of scales (Levi 1947: 444). Hachlili describes the Magdouh Mosaic as both a "scalloped square with a dot in its center" (Fig. 3a) and as "multi-lobed scales" (Fig. 3b) (Hachlili 1998: 202-203). Nassar and Sabbagh describe the overall pattern as "tangent multi-lobed scales" with spindles radiating in four directions from a central quadrilobe (Nassar - Sabbagh 2016: 548) (Fig. 3c). The pattern is also identified as "the scalloped square with floret pattern" (Balderstone 2009: 96).

Figure 2
(a) Raised mosaic panel A, Magdouh Mosaic.
(b) Map of locations that has the examples of the Magdouh Mosaic in the region.

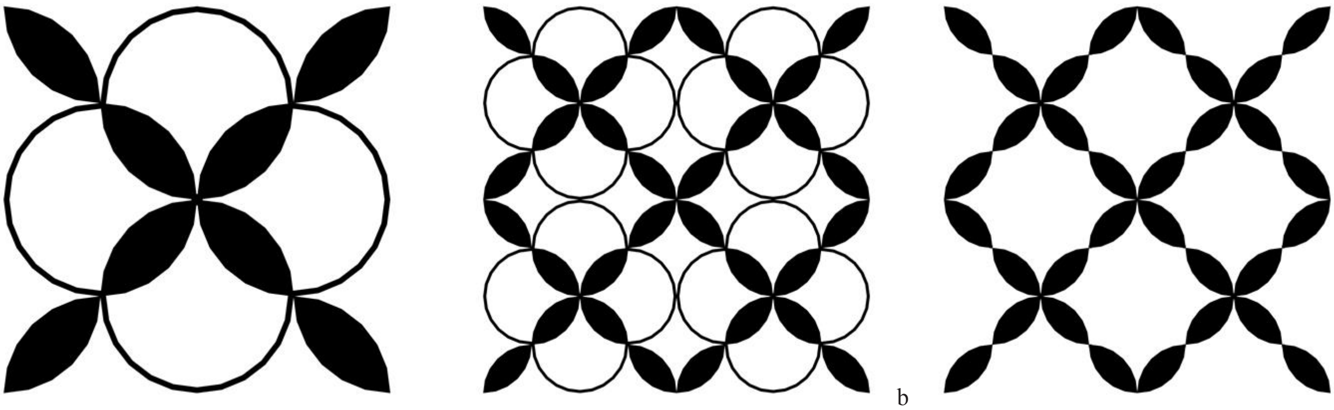
Figure 3
(a) Magdouh Mosaic as a "scalloped square with a dot in its center".
(b) Pattern of "multi-lobed scales".
(c) Pattern of "tangent multi-lobed scales".


a


b


c

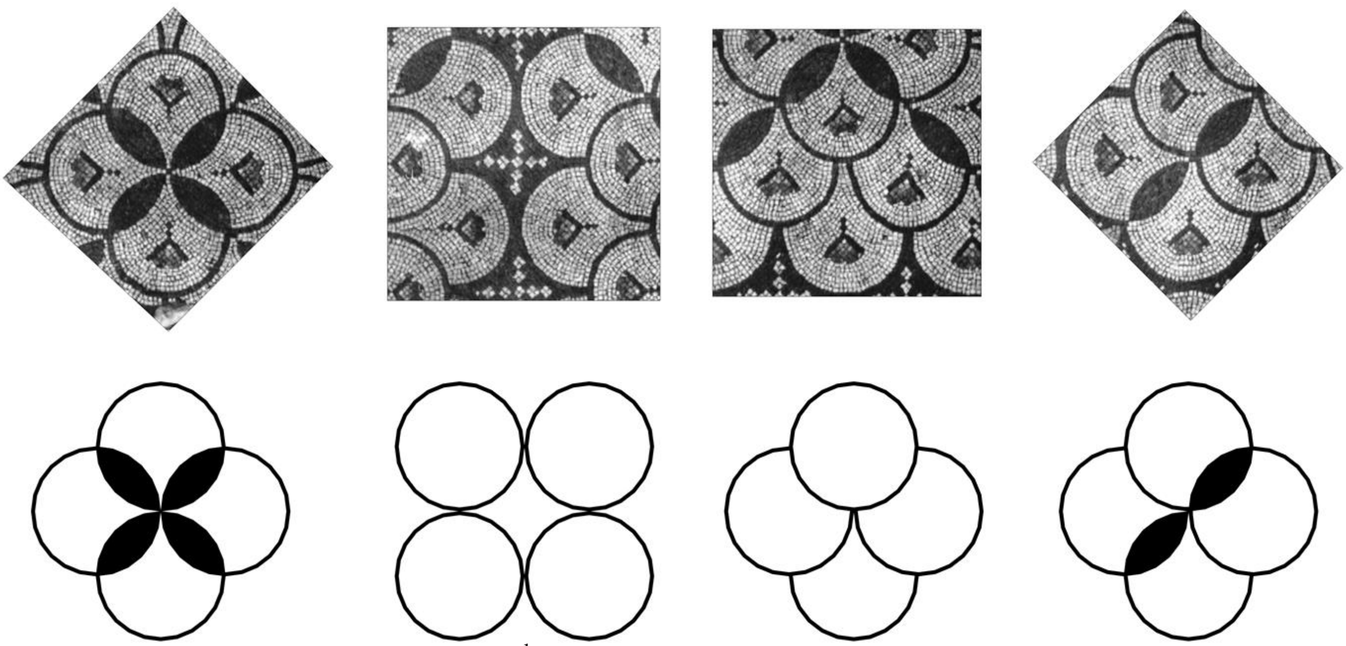
The simplest version of this pattern is also called "quatrefoil petals" with four intersecting circles forming a quadrilobe scale, where the small crosslets are placed in the spaces between the four petals (Nassar - Sabbagh 2016: 191) (Fig. 4a). Such a quatrefoil pattern is known to have been used over several decades. Moreover, there are type of mosaic patterns that are derived from the "cruciferous flower" using only the area of the four lens-shaped regions (Fig. 4b) as in Terrace House 2 at Ephesus in Turkey, which is dated to the late third / early fourth century AD (al-Muheisen - Nassar 2014: 98).



a b

Figure 4
(a) "Quatrefoil petals" of four intersecting circles with the four lens-shaped regions.
(b) Pattern generated by the four lens-shaped regions.

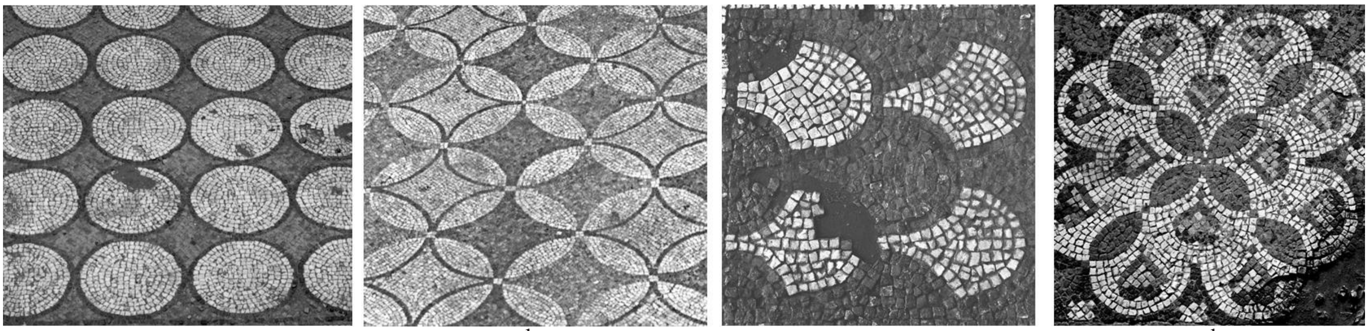
When the Magdouh Mosaic is observed in this context, it can be said that the greater the number of layers, the more the pattern becomes legible. The increasing number of layers makes the fourfold relationship of circles come to the fore in the pattern. While the first four intersect each other at the top layer (Fig. 5a), the other four become a tangent at the bottom layer (Fig. 5b). Furthermore, two tangent circles at the middle layer are tangent to the circles at the top and bottom layers (Fig. 5c) as well. In addition, the two intersecting circles at each layer are tangent to the other pairs of intersecting circles and likewise extend in four directions (Fig. 5d).



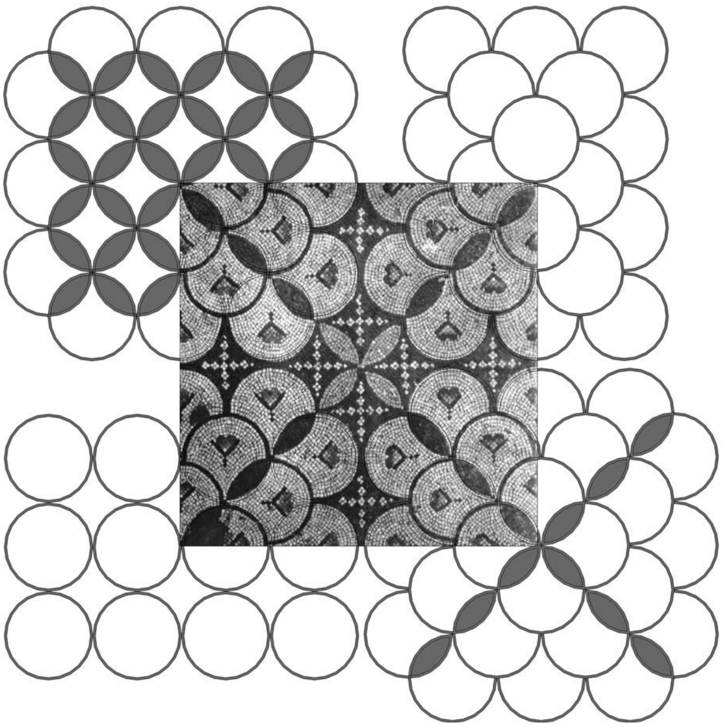
a b c d

Figure 5
(a) Four intersecting circles at the top layer of the pattern. (b) Four tangent circles at the bottom layer. (c) Two tangent scales at the middle layer are tangent to the circles at the top and bottom layers. (d) Pairs of intersecting circles are tangent at different layers.

Such relationships suggested by layers in the Magdouh Mosaic can be distinguished by the broken and continuous curves of the circles. Mosaic examples using such effects can also be observed in other types of patterns of overlapping circles found separately in Antioch. The geometric pattern from the floor of Room 4 of the Barracks (Fig. 6a)[7] is an array featuring square packing of tangent circles, while the other one in Room 2 of the House of the Bird Rinceau (Fig. 6b)[8] gives the effect of intersecting circles. The pattern that is composed of fan-shaped motifs in the central field panel at the west edge of Bath C, Room 51 (Fig. 6c)[9] gives an effect of stacking circles, which is also known as imbricated scales in literature. To conclude, the complex pattern of overlapping circles in the Magdouh Mosaic is also a four-layer version of the one found on the floor of Room 2 of the House of the Bird Rinceau in Antioch (Fig. 6d)[10] suggesting a combination of each of the three patterns in one (Fig. 6e).



a b c d



e

Figure 6
(a) Detail of geometric mosaic in Room 4, Barracks. (b) Mosaic floor of Room 2 with geometric overlapping circles, House of the Bird-Rinceau. (c) Central field panel at west edge, Bath C, Room 51. (d)Detail of geometric mosaic squares in Room 2, House of the Bird-Rinceau. (e) Magdouh Mosaic as a combination of each of the three patterns in one example.

7 Levi II, plate CXXIXc, November 27 1934, Princeton University Archaeological Archives, accessed August 2, 2018, http://vrc.princeton.edu/archives/items/show/14087

8 October 17 1934, Princeton University Archaeological Archives, accessed May 22, 2018, http://vrc.princeton.edu/archives/items/show/13963

9 Levi II, plate CXIXe, Princeton University Archaeological Archives, accessed May 22, 2018, http://vrc.princeton.edu/archives/items/show/13245

10 Levi II, plate CXXXVIIIa, September 28 1934, Princeton University Archaeological Archives, accessed August 2, 2018, http://vrc.princeton.edu/archives/items/show/13886

The plane geometry of the Magdouh Mosaic can be simply defined as the intersection of a multilayered, rectangular array of square packed overlapping circles where each one is centered right above the gap between the adjacent circles beneath. In the case of the Magdouh Mosaic, the layer at the bottom is an array of 4×3 tangent circles, while the middle one is a 3×2 array. The layer at the top is the intersection of two pairs of tangent circles at 90° at the tangent point (Fig. 7).

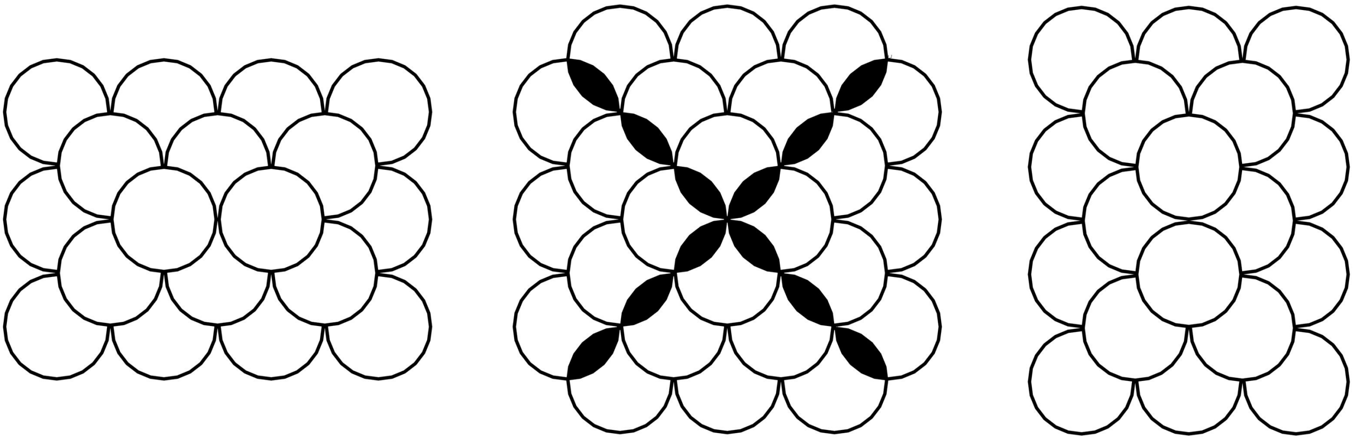


Figure 7
Intersection of a cluster of a three-layered rectangular array of square packed circles at 90 degrees.

The 3D Geometry of the Magdouh Mosaic

When both the content and configuration is extended into three dimensions, the intersection of both the packing of spheres or its lattice at 90 degrees gives the overall 3D structure of the Magdouh Mosaic. Here, the packing of spheres gives a cubic close packed structure, also known as face-centered cubic packing, in which each layer fits right into the gaps of the other layers both above and below it. Therefore, the coordination number of each sphere is 12 since it is tangent with 12 other spheres around it (Fig. 8). When the centers of the spheres are joined by lines, they constitute an array of equilateral triangles in four directions that would generate either a tetrahedron completely surrounded by four octahedra sharing their faces, or an octahedron completely surrounded by eight tetrahedra sharing their faces (Kappraff 2002: 350).

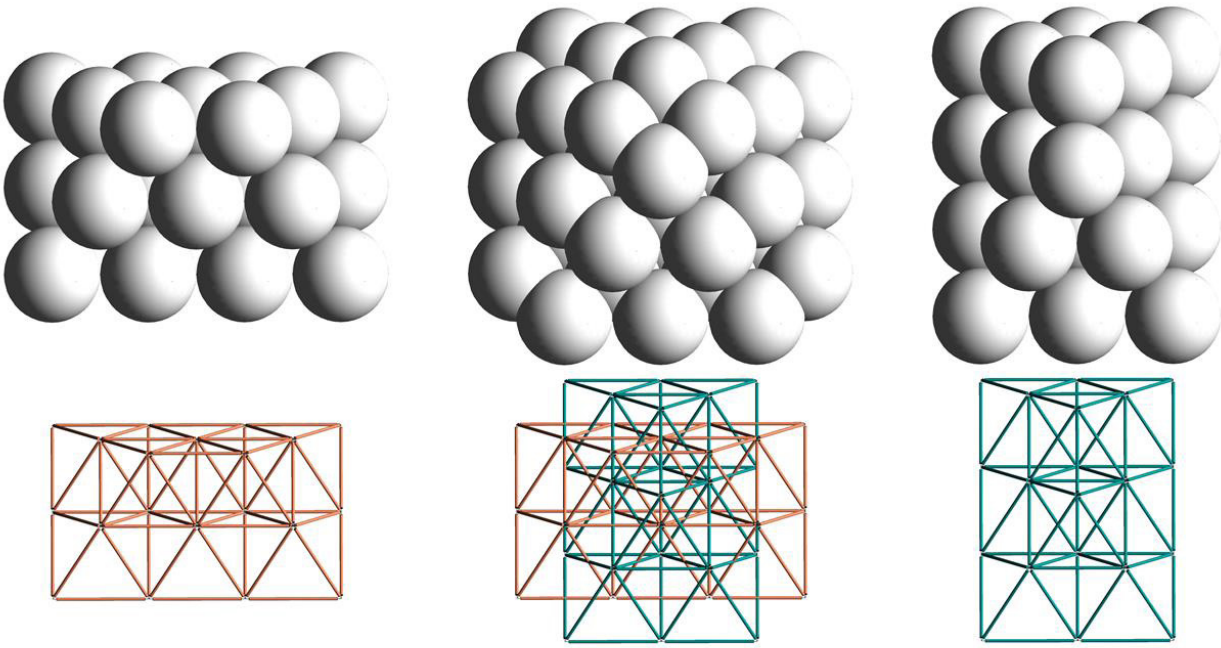
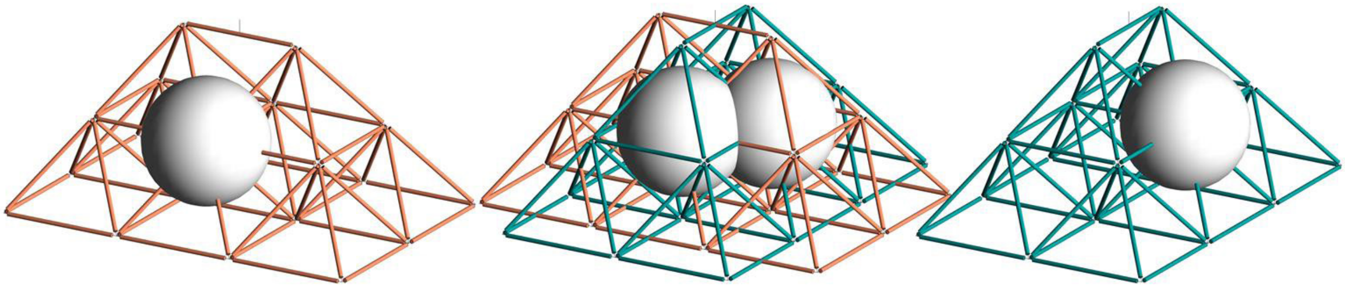
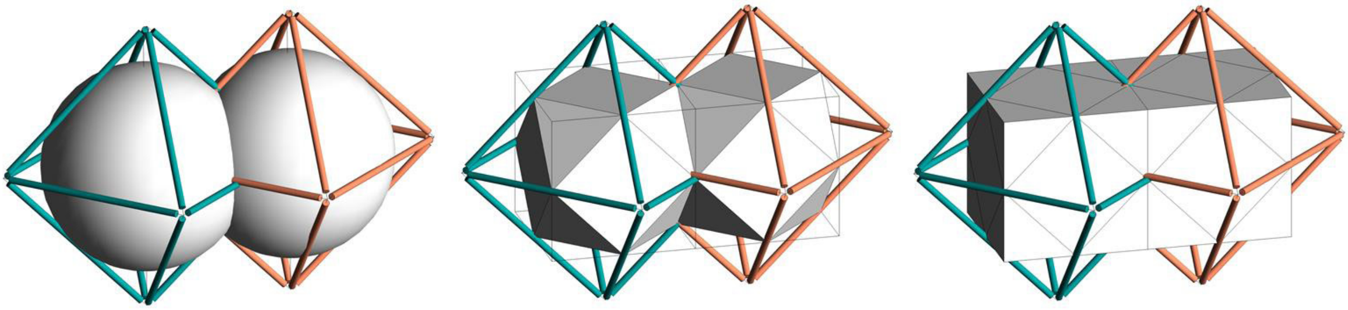


Figure 8
3D geometry of the Magdouh Mosaic as the intersection of identical clusters of spheres at 90 degrees and their space frame.

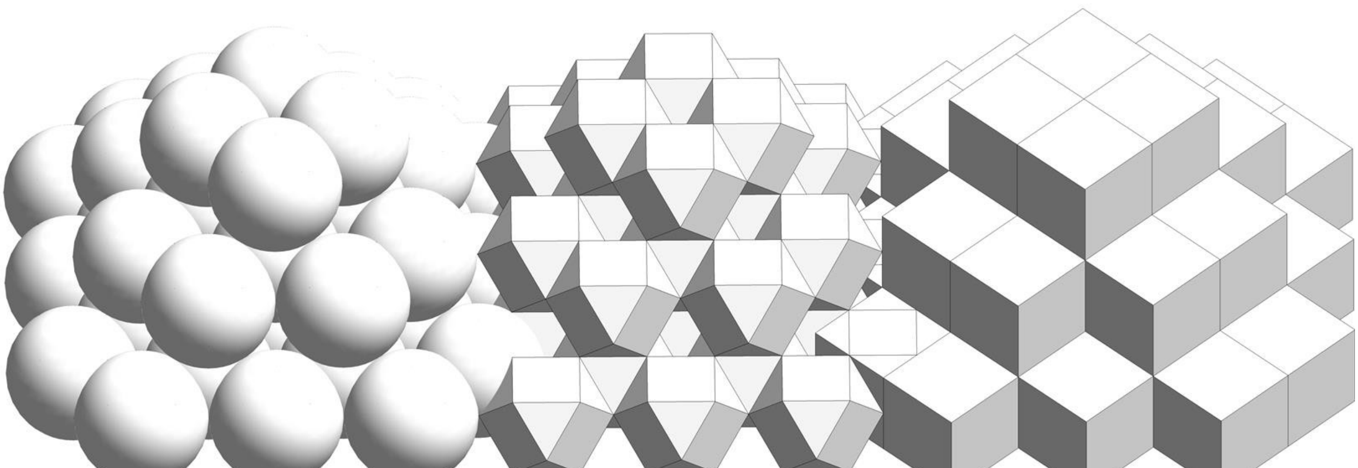
In close cubic packing of spheres, the spheres not only generate the lattice of a tetrahedron and an octahedron, but also bisect the edges of either, defining another cuboctahedron and octahedron lattice. In addition, the intersection of close cubic packing of spheres at right angles also gives the vertices of further cuboctahedra and octahedra. Therefore, either case suggests the packing of cuboctahedra with octahedral voids (Fig. 9a). Moreover, if the cuboctahedron is replaced by a sphere inside the cube, it becomes a union of a cube and a sphere. The intersection of both gives circles tangent at the midpoint of the edges of the cube that defines the vertices of a cuboctahedron (Fig. 9b). Thus, such a packing of intersecting spheres suggests not only the packing of cuboctahedra but packing of cubes as well. Thus, the cuboctahedron in the geometry of the Magdouh Mosaic can also be explained as a polyhedron that mediates between the sphere and the cube (Fig. 9c).



a



b



c

Figure 9
(a) Spheres bisecting the lattice and the pairs of spheres intersecting at 90 degrees in the intersecting lattice. (b) Packing of intersecting spheres that resembles the packing of both cuboctahedra and cubes. (c) Cluster of spheres that shows packing of both cuboctahedra and cubes in the Magdouh Mosaic.

The geometrical relationship among these solids can be explained best by the unique property of the cuboctahedron, which is the congruency of its edge length to its radius length. Since the radius of the cuboctahedron is also identical to the radius of the sphere circumscribed by it, the square root (√2) of the length of the diagonal gives the edge length of the cube in which it is imbedded. Moreover, if a cuboctahedron is replaced by a sphere inside the cube, the edge of the cube is the chord length of the intersecting spheres. In that case, the sagitta of a chord[11], which is perpendicular from the midpoint of the arc's chord to the arc itself – as the height of a minor arc or segment – is (2–√2)/2. The multiplication of the length of the sagitta by two gives the length of the intersection of the spheres which is 2–√2 (Fig. 10).

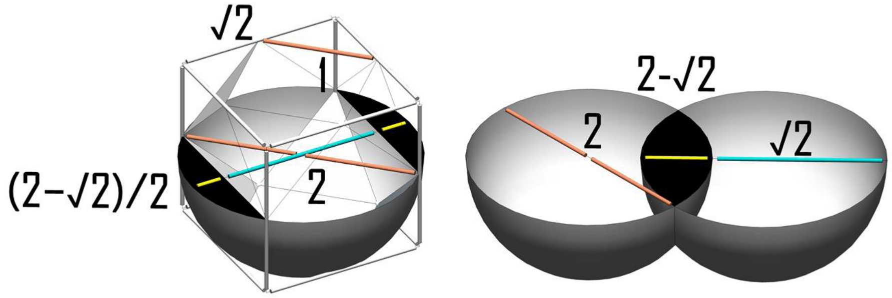
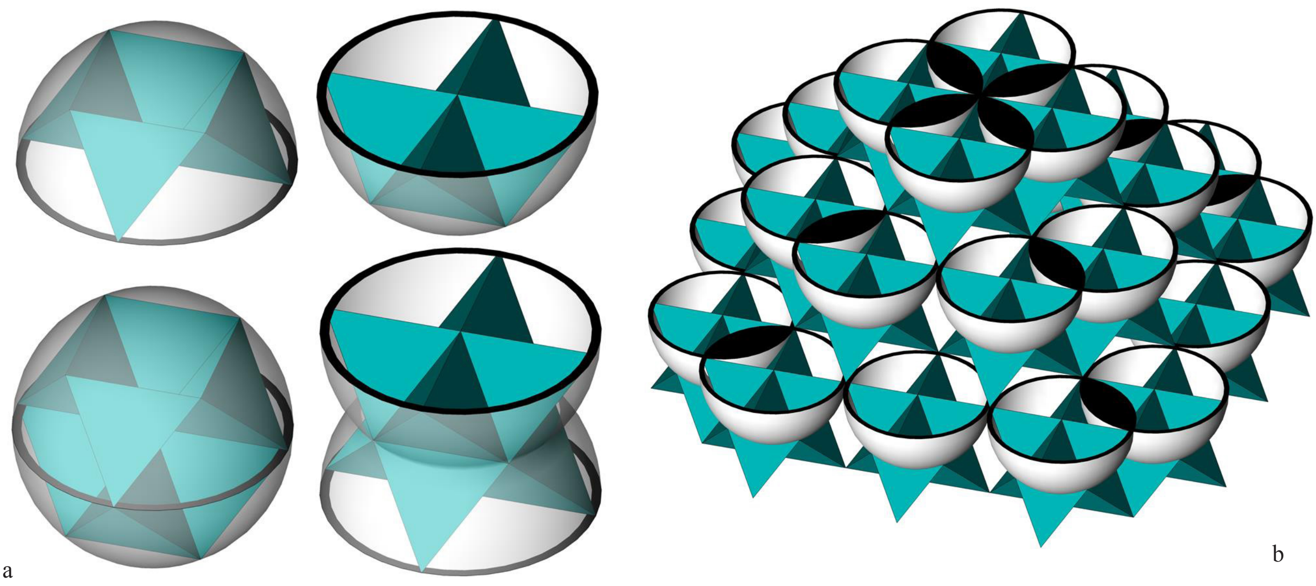


Figure 10
Dimensions showing the relationship between the sphere, cube and cuboctahedron.

The pattern of the Magdouh Mosaic is not just composed of overlapping circles, but also of the line segments that radiate from the centers of the circles through the layout of the pattern. The angle and direction of these lines match the orthogonal projection of the lattice generated by packing of tetrahedra (Fig. 11a). In other words, the hemisphere with four tetrahedra inside it represents a half of not only the octahemioctahedron, but the stella octangula as well. Therefore, the geometric content of the Magdouh Mosaic can also be explained as packing of stella octangula produced by the intersection of each pair of spheres in such a packing (Fig. 11b).



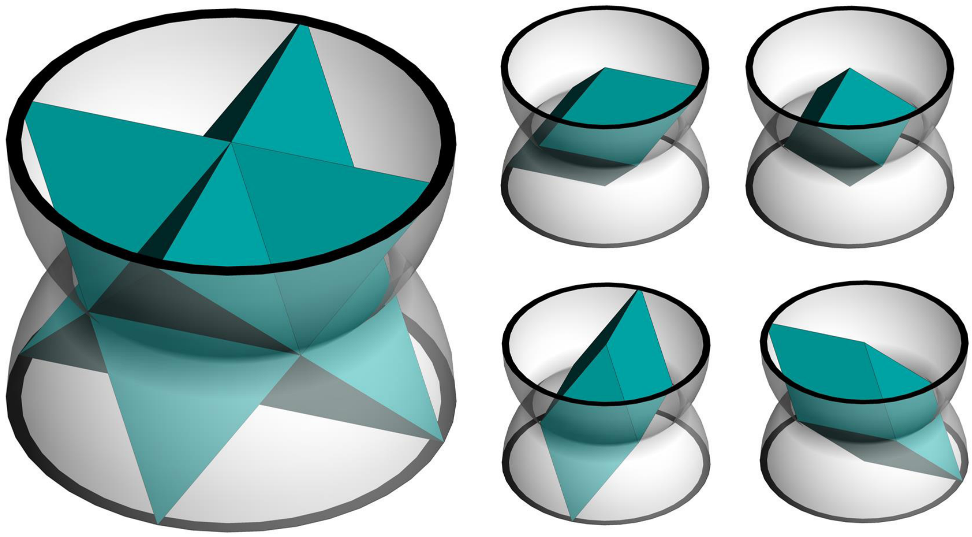
a b

Figure 11
(a) Hemisphere with four tetrahedra inside it that represents a half of both the octahemioctahedron and stella octangula.
(b) Packing of stella octangula inside the packing of hemispheres.

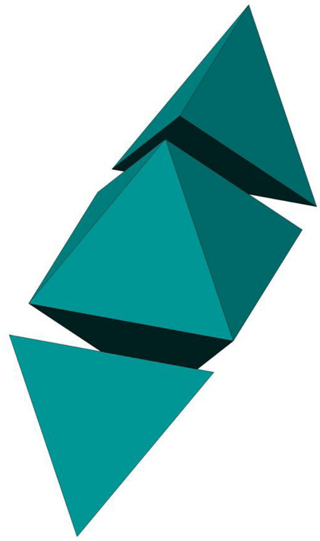
11 The sagitta is a line segment drawn perpendicular to a chord between the midpoint of that chord and the arc of the circle (Concise Dictionary of Mathematics 2013: 81).

In such a geometrical context, the Magdouh Mosaic also suggests the packing of rhombohedra, since the stella octangula can be dissected into four rhombohedra where an octahedron represents the region of intersection with eight regular tetrahedra attached on opposite faces (Fig. 12a). In that case, such a rhombohedron can be decomposed into two equal regular tetrahedra and a regular octahedron (Fig. 12 b).

Figure 12
(a) Dissection of stellaoctangula into four rhombohedra four rhombohedra.
(b) Dissection of rhombohedron into an octahedron and two tetrahedra.



a



b

This rhombohedron, which is a special kind of parallelepiped, can fill a space represented by a tetrahedral–octahedral honeycomb, since the dihedral angles of both (70°32' and 109°28') are supplementary. In other words, it is a regular octahedron augmented by two regular tetrahedra to give a trigonal trapezohedron, which is formed by the six congruent 60-degree rhombic faces (Fig. 13a). Being the smallest possible basic three-dimensional unit cell, it can fill the entire space only by translation on three axes of the cubic closed packed spheres (Williams 1979: 133). Likewise, packing of 60-degree rhombohedra defines the three rotational symmetrical axes of the cube, which are the edge, face-diagonal, and body-diagonal directions (Fig. 13a). Here, the rhombohedron is the primitive-cell, which fills only one fourth of the conventional cubic cell that has a volume a^3 (Blakemore 1985: 27) (Fig. 13a). The geometric relation between the primitive cell and the conventional cell can be displayed by a rhombohedron inserted in a cubic frame. The intersection of this combination with the shell, which is generated by the union of two hemispheres, reveals the unit cell that would explain the geometry Magdouh Mosaic in third dimension further (Fig. 13b). In that case, each circle becomes a hemisphere containing a tetrahedron and a pyramid that is half octahedron. Furthermore, this geometric form appears as an oblique equilateral triangular prism inserted in a hemisphere. In addition, the "Y" figure at the center of each circle in Magdough Mosaic comes into view as the edges of this unit cell that meets at the vertex that sits right on the body center of the sphere (Fig. 13b). In this context, the cluster of packing of circles in the mosaic corresponds to the square-based pyramidal ball packing in which the top halves of the balls are removed (Fig. 13c). Corollary, the pattern of circles in the Magdouh Mosaic becomes the top view of this 3D structure (Fig. 13d)[12].

12 Raised mosaic panel B, Levi II, plate CXXXVIIb, September 22 1937, Princeton University Archaeological Archives, accessed July 18, 2019, http://vrc.princeton.edu/archives/items/show/15848

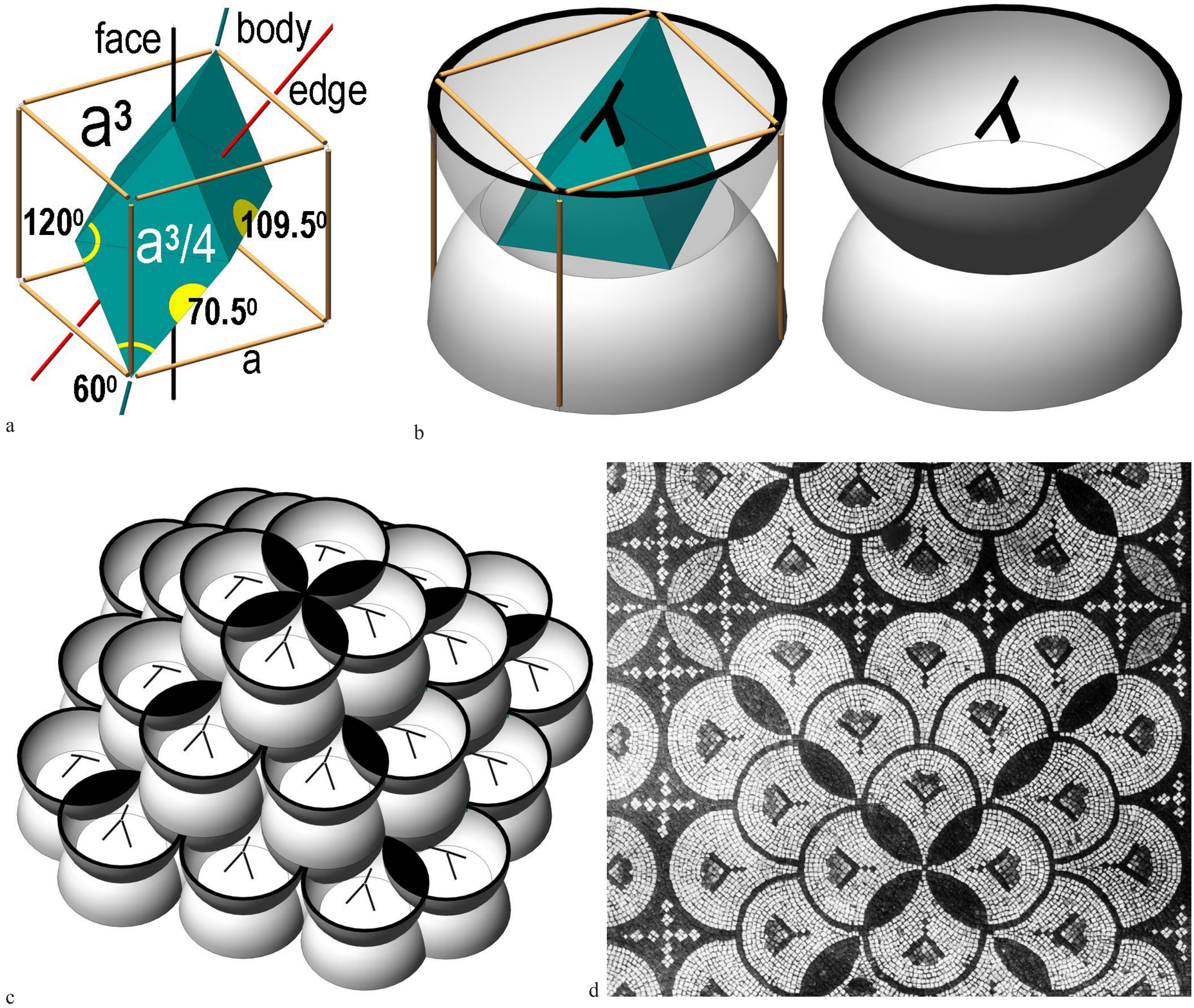


Figure 13
(a) Geometrical properties of a rhombohedron. (b) Unit cell of the mosaic pattern. (c) Square-based pyramidal unit cell packing. (d) Magdouh Mosaic as the top view of the unit cell packing.

Conclusion

In this study, the complex relations of overlapping circles in the Magdouh Mosaic are analyzed by extending them to three dimensions. In that case, the plane geometry of the Magdouh Mosaic produces a square-based pyramidal cluster of spheres generated by the intersection of the close packing of spheres at 90 degrees that defines space filling tetrahedral–octahedral honeycomb structure. In this structure, both octahemioctahedron and stella octangula appears as primary polyhedra that mediate between the sphere and the cube. The intersection spheres at 90 degrees in three directions, in which the length of the intersection is $2-\sqrt{2}$, produces hemispheres that contains four tetrahedra that appears as half of either octahemioctahedron or the stella octangula. Accordingly, the sphere contains an octahemioctahedron, while intersecting hemispheres cover the stella octangula in this structure. The latter configuration also suggests space-filling rhombohedra, since the stella octangula can be dissected into four rhombohedra. Moreover, bisection of rhombohedron gives an oblique equilateral triangle prism, which is a union of tetrahedron and half octahedron, inserted in each hemisphere. This figure outlines the primary building block of the space filling structure that is represented by a square-based pyramidal cluster of intersecting spheres. The overall outcome of this study shows that examples of geometric mosaics are essential materials that could help to investigate the mathematical content of ancient mosaics further.

Acknowledgement

All photographs and images are by the authors except for the photos of the mosaics from Antioch published with the reprint permission kindly granted by Antioch Expedition Archives, Department of Art and Archaeology, Princeton University.

Bibliography – Kaynaklar

al-Muheisen - Nassar 2014 — Z. al-Muheisen - M. Nassar. "Geometric Mosaic Pavements at Ras ed-Deir, Jordan", GrRomByzSt 54, 1, 87–104.

Balderstone 2009 — S. Balderstone, "A New View of the Pella Churches in Terms of Dating and Analysis: The Typological Context", Levant 41, 1, 93-106.

Barsanti 2012 — C. Barsanti, "The Fate of the Antioch Mosaic Pavements: Some Reflections", JMR 5, 25-42.

Becker - Kondoleon 2005 — L. Becker - C. Kondoleon, The Arts of Antioch: Art Historical and Scientific Approaches to Roman Mosaics and a Catalogue of the Worcester Art Museum Antioch Collection, Worcester Art Museum, Worcester, MA.

Biebel 1938 — F. M. Biebel, "Mosaics", C.H. Kraeling (ed.), Gerasa, City of the Decapolis, American Schools of Oriental Research, New Haven (CT), 297-352.

Blakemore 1985 — J. S. Blakemore, Solid State Physics, Cambridge.

Campbell 1988 — S. Campbell, The Mosaics of Antioch, Pontifical Institute of Mediaeval Studies, Toronto.

Cimok 2000 — F. Cimok, Antioch Mosaics, İstanbul.

Concise Dictionary of Mathematics 2013 — Concise Dictionary of Mathematics, New Delhi.

Dunbabin 1989 — K. M. D. Dunbabin, "Roman and Byzantine Mosaics in the Eastern Mediterranean", JRA 2, 313-318.

Elderkin 1934 — G. W. Elderkin, Antioch on the Orontes I: The Excavations of 1932, Princeton.

Hachlili 1998 — R. Hachlili, Ancient Jewish Art and Archeology in the Diaspora, Leiden, Boston, Köln.

Haynes 1951 — I. E. Haynes, "Antioch Mosaic Pavements by Doro Levi", Antiquity 25, 98, 111-112.

Hopkins 1948 — C. Hopkins, "Antioch Mosaic Pavements", JNES 7, 2, 91-97.

Kappraff 2002 — J. Kappraff, Connections: The Geometric Bridge Between Art and Science 2nd Edition, World Scientific, Singapore.

Kondoleon 2000 — C. Kondoleon, "Mosaics of Antioch", C. Kondoleon (ed.), Antioch: The Lost Ancient City, Princeton, 63–77.

Levi 1947 — D. Levi, Antioch Mosaic Pavements, Princeton.

Nassar 2010 — M. Nassar, "Geometric Mosaic Pavements of Yasileh in Jordan", PEQ 142, 3, 182–198.

Nassar - Sabbagh 2016 — M. Nassar - A. Sabbagh, "The Geometric Mosaics at Khirbat Mar Elyas: A Comparative Study", GrRomByzSt 56, 3, 528–555.

Parzysz 2009 — B. Parzysz, "Using Key Diagrams to Design and Construct Roman Geometric Mosaics?", NNJ 11, 2, 273-288.

Stillwell 1938 — R. Stillwell, Antioch on the Orontes II: The Excavations of 1933-1936, Princeton.

Stillwell 1941 — R. Stillwell, Antioch on the Orontes III: The Excavations of 1937-1939, Princeton.

Williams 1979 — R. Williams, The Geometrical Foundation of Natural Structure a Source Book of Design, New York.