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Öğretmen Adaylarının Değişkenlerin Kullanımı ile İlgili Bilgileri[a]

Nihat Boz[b]

Özet

Bu çalışmada öğretmen adaylarının değişkenler hakkındaki bilgileri incelenecektir. Üç farklı üniversiteden ve farklı sınıflardan 184 öğretmen adayından anketlerle veri toplanmıştır. Bu anketlerin kategorilere konularak analizinden sonra 10 öğretmen adayı seçilip röportaj yapılmıştır. Öğretmen adayları harfleri, alışkın olduğu durumlarda yani harflerin bilinmeyen rolü aldığı durumlarda kolayca kullanıyorlar. Diğer yandan harfleri genel sayı ya da fonksiyonlarda değişken olarak kullanmada zorlanıyorlar. Soruların seviyesine uygun ispat çeşitlerini kullanmıyorlar. Cevapları, rutin sorulara şipşak cevaplar vermeye hazır olduklarını gösteriyor. Bu çeşit bir anlayış 'eylem seviyesinde' bir anlayış olarak etiketlendirilebilir. Bu sorunun altında yatan ana sebep okullarımızda öğrencilerimizin en kısa sürede doğru cevabı bulmayı empoze edilmesiyle ilgili olabilir. Bu nedenle manayı bir kenara bırakıp, doğru cevabı bulmaya odaklanıyor olabilirler. Öğrencilerin manaya önem vermelerine yardımcı olmak için öğrencilerin öğrenmelerinin her aşamasında manayı aramalarına olanak sağlayacak sınıf ortamları oluşturmalıyız.

Anahtar kelimeler:Öğretmen adayları, bilgi, anlama, değişkenler, bilinmeyen, genel sayı

Prospective Teacher's Knowledge of Principal Uses of Variables

Abstract

In this study, prospective teachers' knowledge of principal uses of variables will be explored. 184 prospective mathematics teachers from three different universities and different year groups completed a questionnaire. After analysing the questionnaires by means of categorisation of responses, 10 students were selected and interviewed. The results indicate that prospective teachers are comfortable with using letters in familiar contexts where the variables assume the role of unknowns. On the other hand, students have difficulties in using letters as generalised numbers or arguments in functional relationships. They could not give appropriate kinds of proof according to the level of questions. Their responses show that they are accustomed to produce mechanistic

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[a] Bu makale yazarın doktora tezine dayandırılmıştır, hiç bir konferansta sunulmamıştır.

[b] Arş. Gör. Dr., Gazi Üniversitesi, Gazi Eğitim Fakültesi, Ankara.

answers to routine questions. This kind of understanding can be labelled as 'action level' of understanding. The main underlying reasons of such problems may be due to the fact that at schools students are imposed to get the right answers in a shortest possible time. Therefore, they put the meaning aside, and focus on getting answers right. In order to help students to give importance to meaning, educators have to create classroom contexts where students are searching for meaning at every stage of learning.
Key words: Prospective teachers, knowledge, understanding, variables, unknown, generalized number

1. INTRODUCTION

The word algebra has different meanings and interpretations, however within the context of school mathematics, algebra deals with expressions and equations, e.g. simplification of expressions, solutions of equations, substitution within expressions, and the use of these to construct and solve word problems. It is commonly agreed that the use of literal symbols is among the most important building blocks for understanding algebra. According to Eisenberg (1991) understanding the use of variables is the basis of all abstractions in mathematics. It is so important that not understanding it may block students' success in algebra (Leitzel, 1989).

In discussing the richness and multiplicity of meanings of the notion of variable, Schoenfeld and Arcavi (1988) point out that it is almost impossible to capture the meaning of the term variable by a single word, furthermore it is very difficult to give a full definition that captures the essence of the term variable. They assert that this difficulty may be partly lessened by considering how variables are used and work. Philipp (1992) puts variables into seven categories depending on their usages,

1. Labels f, y in $3f = 1y$ (3 feet in 1 yard); 2. Constants $\prod$, e, c; 3. Unknowns x in $5x$-9=91; 4. Generalised numbers a, b in a + b = b + a; 5. Varying quantities x, y in y= 9x-2; 6. Parameters m, b in y =m x + b; 7. Abstract symbols e, x in e*x= x.

In this study the word variable is used in a similar way, that is, by the word variable I will mean broadly 'letter standing for numbers'.

However, research conducted in many countries indicates that students experience difficulties on their journey to learning the concept of variable. Although it is so fundamental and so difficult to learn for some, we do not know enough about teachers' or prospective teachers' knowledge base for teaching this concept; in particular subject matter knowledge. In this study Turkish prospective teachers' knowledge of variables will be explored.

2. THEORETICAL BACKGROUND

Knowledge and understanding in mathematics education have varied definitions. As Sierpinska (1994) points out there are at least four kinds of models or theories of understanding in mathematics education. Different theories show that there is not a unified theory of knowledge and understanding of a topic. Each one has its own strong points in explaining relevant concepts. For example, Van Hiele's theory of understanding geometrical topics has been very widely adopted in mathematics education to explain the development of geometrical concepts in students' minds. However, theories that have foundations in 'seeing the process of understanding as a dialectic game' seem most appropriate for explaining the knowledge and understanding of variables. That is, it will be proposed that knowledge and understanding of variables appears in different forms, and it is a combination and integration of the different forms of knowledge into one concept that is empowering. Integrating procedural and conceptual understanding by the aid of symbolism, gives one power to move flexibly between them.

This interplay of different forms of understanding of symbols is explained by the notion of "procept". It is formulated by Gray and Tall (1994) to elucidate the phenomenon, in which the symbols function dually as both process and product, a notion that embraces the ambiguous use of *notation* to stand either for a *pro*cess or for the con*cept* produced by that process. The issues which are brought out by Gray and Tall (1994), and Sfard and Linchevski (1994) explain a very important phenomenon in the development of mathematical knowledge. Within this development, once expressions can be regarded as a whole, as objects, they can be manipulated at a higher plane of thought. In the context of variables these manipulations do not involve just operations with symbols, but also analysis of their roles, meanings, (and) referents. That is ‘procepts’ enable us to consider variables as objects whose *roles*, *meanings* and other relevant features can be analysed.

Knowledge and understanding variables, therefore, involves being able to ascribe different roles to symbols to represent problem situations and to manipulate them. It also involves, as Ursini and Trigerous (1997) point out, being able to “distinguish between the different roles and shift from one to another in a flexible way, integrating them as components of the same mathematical object.” (p. 254).

These views show that knowledge of variables is made up of different aspects. Making these distinctions between these aspects proves useful for describing students’ understanding of variables. Therefore, knowledge of variables can be discussed under three different headings: knowledge of variables in;

1. *The principal uses of letters*,
2. *Awareness of different roles of variables*,
3. *Flexibility, versatility and connectedness among different roles and uses*.

However, since this paper will deal with the principal uses of variables, this aspect of variables will be discussed.

2.1 Principal uses of variables

The understanding of the use of variables as *unknowns*, as *generalised numbers*, and as *arguments in functional relationships* will be regarded as principal uses of variables. The conception of variables as *unknowns* is inextricably related to regarding algebra as a study of *procedures* for *solving* certain kinds of problems (Usiskin, 1988). He asserts that the key instructions in this conception of algebra are *simplify* and *solve*. However, before simplifying one needs to recognise and to see that there is something unknown in the problem, and that this recognised unknown can be found by first constructing and then by simplifying the equation within the restrictions of the problem. This process is called ‘symbolising a problem by posing an equation’ (Ursini and Trigueros, 1997) and they add that understanding variables as unknown also involves being able to substitute to the variable the value or values that make the equation a true statement.

The literature on students' understanding of variables reports that beginning students have difficulties in using symbols as unknowns to solve equations (Filloy & Rojano, 1989; Herscovics & Linchevski, 1994). It is reported that students tend to use *reverse* operations instead of using *forward* operations that requires using symbols as unknowns. For instance, when it is asked to do this problem "When 3 is added to 5 times a certain number, the sum is 40. Find the number!" some students may solve this problem by undoing, using reverse operations: they first take away 3 from 40 and then divide the resulting number by 5 to find the number. This way of solving this problem is called the *arithmetical* method. However, to solve this problem using *algebra* it needs to be represented as $5x+3 = 40$ and forming this equation requires forward operations. The ability to form equations by a forward approach is called the 'algebraic method'. The lack of ability to use forward operations is termed a cognitive gap (or didactic cut) between arithmetic and algebra (Filloy & Rojano, Herscovics & Linchevski, 1994). However, solving this equation to find the unknown x does not require an ability to operate spontaneously with or on the unknown, since x appears on only one side of the equation. For those equations where x appears on both sides, then it is required that x is manipulated to find its value. It is suggested that this difference in solution methods is another place where the cognitive gap between arithmetic and algebra occurs (Hercovics &, Linchevski, 1994).

Sfard and Linchevski (1994) view using a letter as an unknown as a stage in algebraic development which is termed as '*algebra of a fixed value*' as opposed to *'functional algebra'*, where letters represent changing rather than constant magnitudes. Using symbols in a context in which any number may be substituted for the symbol marks a shift in algebra from a science of constant quantities into a science of changing magnitudes (Sfard and Linchevski, 1994).

Sfard and Linchevski (1994) suggest that this second stage of algebraic development involves using letters as variables but they don't mention explicitly using letters as generalised numbers. This may be due to the fact that both conceptions of variables require conceiving symbols as any number not a fixed number, thus symbols may assume a range or series of values. However, 'letter as variable' implies more than just a range of values. Understanding letters as variables implies conceiving how one set of numbers changes in relation to another. Therefore, it should be noted that using letters as generalised numbers is one of the aspects of understanding variables which should be dealt with in a separate category.

The conceptions of variables as *generalised* numbers involve using letters to generalise already recognised numerical or geometrical patterns which requires being aware that a symbol can stand for a general indeterminate object (Ursini and Trigueros, 1997). According to Usiskin (1988) this conception of algebra is algebra as *generalised* arithmetic, and the key instructions within this conception are *translate* and *generalise.*

Generalizing is the process of exploring a given situation for patterns and relationships, organizing data systematically, recognizing the relations and expressing them verbally and symbolically, and seeking explanation and appropriate kinds of justification or proof according to level (Bell, 1995, p.50).

For instance, divisibility properties, such as 'the difference between the third power of a whole number and the number itself is always divisible by 6', provide an example of using letters as general numbers to prove general properties.

The research on children's understanding of variables show that difficulties arise in the students' journey to conceiving letters as generalised numbers. (Lee & Wheeler, 1989; Kuchemann, 1981; Booth, 1984). For example, Kuchemann (1981) reports that 25% of third-year students gave a correct answer to the question whether l+m+n=l+p+n is always, sometimes, never true. This shows that many children have difficulty in allowing two different letters to have the same value. After a follow up study, Booth (1984) reports that there is a strong resistance to the conception of letter as generalised number, even within the context of a teaching program specifically designed to address this aspect of algebra.

Goulding and Suggate (2001) report that 61% of 201 PGCE (Postgraduate Certificate in Education) students have difficulties using letters as generalised numbers to prove that the addition of two odd numbers results in an even number. 19% of these students use only specific numbers or diagrams, 16% of them use two equal odd numbers, and 32% of them cannot complete an algebraic proof.

The conceptions of variables *as arguments in functional relationships* involves recognising how one set of numbers changes in relation to another irrespective of the representation used (table, graph, formula, etc.), finding out the values of one variable given the value of the other one (independent or dependent), and being able to symbolise a relation based on the analysis of the data of a problem. (Ursini and Trigueros, 1997). According to Usiskin (1988) the fundamental distinction between this and previous conceptions of letters is that, here, variables *vary,* and two key instructions under this conception of algebra are *relate* and *graph*. However, not only graph but also other representations table, formula are also involved in understanding letters as variables.

Kuchemann (1981) reports that only 3% of 11 year old students tested gave a correct answer to the question "Which is larger, 2n or n+2? Explain", a question which tests the existence of the understanding of letters as *arguments in functional relationships*.

Ursini and Trigueros (1997) reports that even after several algebra courses, starting university students still have difficulties in understanding the principal uses of variables. Their understanding of each use of the variable concept remains at an action level where they produce mechanistic answers to routine questions.

In order to recap what constitutes this aspect of subject matter knowledge of variables, I would like to mention that this aspect of subject matter includes an ability to interpret, symbolise and manipulate symbols as numbers, whether it is a fixed, or varying. However, each of the usages has its own related way of interpreting, manipulating and symbolising.

3. METHODOLOGY

It was decided that the most appropriate research strategy for this study would be *cross-sectional survey study*, since the *purpose* of this study was to examine the *knowledge of variables.* Conducting a cross-sectional survey served my intention to elicit information from a single period of time, rather than to study changes over time.

In this study knowledge of variables was defined in an appropriate way. By an appropriate way I mean regarding knowledge of variables not as the number of courses taken. The theoretical framework, which was used to find suitable questions for knowledge of variables, was based on several bodies of work; the research on students' understanding of variables, the research on teachers' professional knowledge. A free response (open-ended) questionnaire was administered to 184 prospective teachers. After analysing the questionnaire data, I conducted interviews to validate the responses given to questionnaire questions.

The sample of this study comprised three different year groups of students from three different Universities. The students ranged from the second year to the fourth year of courses in Mathematics Education Departments. One hundred and eighty-four students (184) completed the questionnaire. All universities in the sample draw students from all over Turkey. However, they accept students who achieve a higher score on the University Entrance Exam than other Turkish universities do. In this respect all three universities in my sample are in the top ten.

The interview sample was chosen to establish the range of responses given by students from all universities who completed the questionnaire. For this reason, ten students chosen from a university were interviewed.

The selection of interviewee students was done by mainly considering the relationships found from the analysis of questionnaire responses. That is, I chose such students who gave particular responses to particular questions.

While designing the questionnaire, in view of the fact that I did not have enough elicited information to write appropriate response categories, I preferred to use free-response questions. Furthermore, since I was interested in subjects' solution methods rather than whether or not they solved questions correctly, I chose to use free-response questions.

Questionnaires were distributed and completed over a period of three weeks. To prevent administration differences and biases, certain directions were given in the classroom by me and I observed each session of questionnaire administration. Beside this, detailed directions and explanations were written on the questionnaires. Prospective teachers' instructors/mentors were present at the time of the administration.

To sum up, in this study, the main data was collected by means of a questionnaire which contains free-response, open-ended questions. 184 prospective mathematics teachers from three different Turkish Universities completed the questionnaire. 10 of these prospective teachers were later chosen to take part in semi-structured interviews.

3.1 Data Analysis

There were many different kinds of responses to each question. The categorisation of these responses for each question was strictly based on the entire range of given responses considering the similarities among them. In order to form categories, all responses from all year groups for each question were collated. Firstly, general categories were formed by putting similar answers together. Deciding on the similarity of responses is based on words, phrases or solution methods that seem similar. These categories were gradually modified or replaced during the subsequent stages of analysis. Therefore, this allowed me to combine categories in a flexible way by identifying subcategories under these general categories. For those responses which could not be put into already formed categories, a new category called "other" or "miscellaneous" was formed. Regrouping or linking of categories is accomplished by use of the literature review.

The validity of this study was improved by triangulating the data types; interview data was incorporated to the findings of questionnaire data. Semi-structured interviews were audio-taped and then transcribed. The students who were interviewed were selected as a subset of the students who were completed the questionnaires. Random selection procedures were used after putting students into different groups according to questionnaire data. This improved the validity of the study by preventing systematic bias. The subjects were told that their responses would not be marked and their responses would have no effect on their course marks. They were asked to write down what they really thought, and to give as much detail in their answers as possible. The validity of this study was also improved by drawing from published literature. Both contradicting and substantiating studies will be considered while presenting the findings.

Since the data obtained from questionnaires is in the form of nominal categories, the *kappa coefficient* was used to measure the reliability of the findings. Each of these questions together with the categories formed and their descriptions were given to my colleagues and wanted them to match responses to given categories. I compared the degree of agreement between my categories and the categories my colleagues put by finding the kappa coefficient. Kappa coefficient was higher than 0, 90 for all the three questions.

To summarise, analysis of raw questionnaire data is accomplished by means of categorisation. The validity of this study is established by a theoretical framework which is developed through extensive investigation of the relevant literature.

4. FINDINGS

In this section, the findings related to the analysis of the following three problems will be presented. These problems are for eliciting prospective teachers' knowledge of using variables as *unknowns (problem 1)*, as *generalized numbers (problem 2)* and as *arguments in functional relations (problem 3)*.

Problem 1*: Five years ago, the sum of the ages of a mother and her daughter was 53. Now, if mother is 27 years older than her daughter, how old was the daughter 3 years ago?*

Problem 2*: Prove that the sum of n consecutive even integers is divisible by n.*

Problem 3*: Two mobile phone companies rent line. Both of them take some money for line rental per month and some money for calls per minute. The following table (Table 1, Table 2) give some information about their pricing for some particular number of call minutes and for some number of months.*

For what number of call minutes per month would the price be the same? If you are a big user of mobile phones which company is preferable?

Table 1: First Company

	10 Mins	20 Mins	40 Mins	60 Mins	70 Mins	100Mins	200mins
1 Month	35	50	80	110	125	170	320
2 Months	55	70	100	130	145	190	340
3 Months	75	90	120	150	165	210	360
4 Months	95	110	140	170	185	230	380
5 Months	115	130	160	190	205	250	400

Table 2: Second Company

	10 Mins	20 Mins	40 Mins	60 Mins	70 Mins	100Mins	200mins
1 Month	38	58	98	138	158	218	418
2 Months	53	73	113	153	173	233	433
3 Months	68	88	128	168	188	248	448
4 Months	83	103	143	183	203	263	463
5 Months	98	118	158	198	218	278	478

In the categorisation of the responses to these problems the focus was on whether or not letters are used in the solution. The Table 3 shows the results of this categorisation. The reasons for such a focus were two fold: firstly I was not interested in whether or not the respondent got the correct answer; rather I was interested in the solution methods. Secondly, using letters for these solutions give clues about respondents' level of understanding of variables. For example, in problem 1, using symbols to solve word problems which ask for the unknown(s) requires forward operations. By forward operations, I mean opposite of using inverse of the operations which are given in the problem. Kieran (1992) and Usiskin (1988) consider symbolizing a word problem by using forward operations as a transition from arithmetic methods to algebraic methods. After symbolizing a word problem, to find the unknown, it is required to operate on or with the unknowns, *e.g.* grouping like terms, decomposition of a term, cancellation of like terms etc. Being able to carrying out such operations is considered as a sign of having crossed the "cognitive gap" between algebra and arithmetic by Herscovics and Linchevski (1994). As can be seen in the

Table 3, all but 5 respondents (No Letter category) could confer this sign of being crossed the cognitive gap. 178 respondents used letters in their solutions. However, second problem which is for probing participants' knowledge of using letters as generalised numbers tells a different story.

The categorisation of responses to this question is on subjects' preferences for representing the first number in the sequence. The reason for concentrating on the representation of the first even number is two fold: firstly, representing the first number by a letter shows one's desire to solve this problem for all sequences of the given form not for any particular one. This desire is considered as a sign of using letters as generalised numbers.

Secondly, after one decides to use a letter for representing the first number, he/she may encounter some problems in choosing this letter; he/she must produce a letter different from *n*, since *n* is already given in the question as the number of even integers and also the number which divides the sum.

The Table 3 shows the frequencies of the categorisation. 75 respondents used a letter to represent the first even number in the sequence, 94 students did not. The solutions of the respondents who are in the "No Letter" category is similar to Diophantine's methods of using letters whereas the solutions in the letter category is similar to Vietan's methods. In the history of algebraic development there are two eras which were dominated by Diophantus (c. 250 A. D.) who used letters only to denote unknowns and Viète (1540- 1563) who came after Diophantus and who is believed to be "the first to replace numerical givens with symbols" (Sfard & Linchevski, 1994).

Table 3: Results of problem 1, 2 and 3

	Letter	No Letter
Problem 1	178	5
Problem 2	75	94
Problem 3	39	145

The difference of usage of symbols between Diophantus and Viète marks a line between two different stages of algebra according to Sfard and Linchevski (1994). They call it these stages as "algebra of a fixed value" and "functional algebra". Harper (1987) also holds this view that these two stages in algebraic development exist. In order to see students' algebraic development, he asks a question which was solved differently through out the historical development of algebra. This question asks to prove that one can always find what the two numbers are if he/she knows the difference and the sum of those two numbers. The interesting thing about the solution of this question is that it is solved by Diophantus by using letters as only unknowns to find those two numbers by supposing a particular sum and difference. For example, assuming the sum is 50 and the difference is 20 then letting these numbers be x and y and finding the values of x and y. This solution method does not make use of letters to prove that this assertion is true for any case, namely it does not give a general solution by making use of letters.

However, Viète solves this problem very differently, he does not use any specific numbers, and instead he uses to represent also the sum and the difference. This implies that the usage of letters takes on a very different role. Viète is able to use letters to represent also generalised numbers.

In interview I asked two students the problem which was solved by Diophantus and Viète in the history, to confirm my hypothesis that there is a parallelism between different solutions of this question and the solutions of Diophantus and Viète. For example the following student was in the "Letter" category:

> **Researcher**: For example let's say we know the difference and the sum of two numbers. Can we always find what those numbers are?
> **Student**: Yes we can find them
> **Researcher**: How?
> **Student**: Let these two numbers be a and b, their sum be c and difference be d. (She writes these on the paper: $a+b=c$, $a-b=d$, $2a=c+d$, $a=(c+d)/2$) Then if I add both sides, bs are cancelled and we get 2a equals c+d, we divide this by 2 to get a equals c+d divided by 2. Yes also b can be found from the first or second equation.

This student uses letters to represent the sum and the difference as in problem 2 she used "2k" to represent the first even number in the sequence of *n* consecutive even integers. She writes in the questionnaire followings to prove the sum of *n* consecutive even integers is divisible by *n*:

> "The sum of n consecutive even integers= $(2k)+(2k+2)+\ldots\ldots..+(2k+2n)$, there are n of $2k$, therefore sum $=2kn(0+2+4+\ldots\ldots+2n)$ since it has factor n, it is divisible by n."

Although she does not complete the proof, she has a willingness to prove it for general cases by representing the first letter by "2k".

As Harper (1987), Sfard and Linchevski (1994) point out these two types of solutions require different cognitive demand and belong to different levels of understanding algebra. The superiority of Vietan type solution over that of Diophantine type solution can be explained by considering the different interpretations of the role of letters, that is, representing unknown numbers and representing general numbers.

Since understanding of the variable as arguments in functional relations involves, as Ursini and Trigerous (2001) point out, recognising the relation between quantities irrespective of the *representation* used, determining the values of one variable given the value of the other one, symbolising the relation based on the given data of a problem, the solutions of problem 3 are categorised also according to the employment of letters. As can be seen from the Table 3 the number of students who use letters to represent the problem is relatively smaller than the numbers in previous problems. Only 39 students used a letter to solve this problem, whereas 145 students did not employ letters.

Those students who did not use a letter write in the questionnaire similar to the following:

> "I checked the table and decided to choose the first company. Because, it is cheaper."

On the other hand, students who use letters try to generate functions from tables of data. For example they write similar to the following:

> "y=ax+cd where y is the cost for the first company, x is the total minute, c is the month, a is the cost per minute, d is the line rental per month. If we put the

> given numbers into this equation we can find the constants a and d..."

These students try to create expressions and equations to model the problem situation. In the interviews when one of them is asked how she solved this question, she replied:

> "I tried to find the function that gives the total cost at a given month for the given number of minutes. This way I could plot the graph of functions and see where they are intersecting and where one of them is bigger than the other."

This reply shows that this student is aware of the power of symbols. She models the problem situation by means of symbols and then operates with these to arrive at the solution. This shows that she is able to regard variables *as arguments in functional relationships* that involve recognising how one set of numbers changes in relation to another irrespective of the representation used (table, graph, formula, etc.).

5. DISCUSSION AND IMPLICATIONS

The Table 3 shows that as the question gets more complex the number of students who use letters gets smaller. This may be due to fact that solutions of these questions require different cognitive demands. As a matter of fact, as Harper (1987), Sfard and Linchevski (1994) point out that using letters to represent general numbers to give a general solution require different cognitive demand and belong to different levels of understanding algebra. The superiority of Vietan type solution over that of Diophantine type solution can be explained by considering the different interpretations of the role of letters, that is, representing unknown numbers and representing general numbers. According to Harper (1987) using letters as generalised numbers is more demanding since "the mental activity here operates with concepts which are not required if one is interested only in the determination of numerical values [as in the solution of Diophantus]…" (Harper, 1987).

Therefore, we can say that the students who give Vietan type solution could be able to operate with different kinds of concepts, *letters as givens,* which requires higher level of mental processes. Sfard (1995) explains this situation as follows.

Employing letters as givens, together with the subsequent symbolism for operations and relations, condensed and reified the whole of existing algebraic knowledge in a way that made it possible to handle it almost effortlessly, and thus to use it as a convenient basis for entirely new layers of mathematics. In algebra itself, symbolically represented equations soon turned into objects of investigation in their own right and the purely operational method of solving problems by reverse calculations was replaced by formal manipulations on prepositional formulas (p.24).

Consequently, those students who can employ letters as givens or as arguments in functional relationships may be in a higher layer of algebra understanding. The results show that very few students confer the sign of such understanding. Hence it can be claimed that the students' understanding of each use of the variable concept remains at an action level where they produce mechanistic answers to routine questions. This result is similar to the result of Ursini and Trigueros (1997) in which they work with starting university students in a different country.

It can be suggested that the type of problems found in this study are strongly related to the way in which mathematics in particular algebra is taught. Based on my experiences as a learner and my classroom observations I can claim that the focus in classes was to get correct solutions quickly. As a result students are addicted to the automatic symbolic manipulations. Therefore, in classes we have to motivate students to search for meaning at every stage of the learning.

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