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Çarpık ve Parçalı Mıknatıs Geometrileri Kullanarak Vuruntu Torku Minimizasyonu

Araştırma Makalesi / Research Article

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ÖZ

Çalışmada, 2,5 kW gücünde 14 kutuplu, 84 oluklu sabit mıknatıslı senkron generatörün (SMSG) analitik tasarımı, analizi ve optimizasyonu gerçekleştirilmiştir. Bu tasarıma ait verim, tork, vuruntu torku ve manyetik akı yoğunluğu gibi performans özellikleri değerlendirilmiştir. Ardından mıknatıs geometrisinin vuruntu torkuna etkisini incelemek amacıyla ilgili generatörün 3 boyutlu modeli çıkarılmıştır. Bu bağlamda bölünmüş ve kaykık/çarpıklık (skew) verilmiş mıknatıs yapılarının etkileri araştırılmıştır. İlk tasarımda mıknatıs tek parça modellenmiş ve vuruntu torkunun etkin değeri 436,75 mNm olarak bulunmuştur. İkinci tasarımda mıknatıs yüzeyi boyunca belli ölçüde çarpık bir yarık açılmış ve bu işlem vuruntu torkunu 434,58 mNm değerine düşürmüştür. Diğer tasarımda, birinci tasarımdaki mıknatıs iki parçaya bölünmüş ve bölünen mıknatıslar çarpık şekilde tekrar birleştirilmiştir. Böylece vuruntu torkunun değeri 159,60 mNm olarak bulunmuştur. Son olarak, son modeldeki mıknatıs geometrisine belli ölçülerde iki tane yarık açılarak vuruntu torku 89,95 mNm'e kadar düşürülmüştür. Elde edilen sonuçlardan, son tasarımın, ilk tasarıma kıyasla vuruntu torku değerinde %80'e varan bir iyileşme sağladığı sonucuna varılmıştır.

Anahtar Kelimeler: Vuruntu torku, kaykık/çarpıklık, SMSG, sonlu elemanlar analizi.

Cogging Torque Minimization Using Skewed and Separated Magnet Geometries

ABSTRACT

In the study, analytical design, analysis and optimization of a 2.5 kW 14-pole, 84-slot permanent magnet synchronous generator (PMSG) have been performed. The performance characteristics of this PMSG such as efficiency, torque, cogging torque and magnetic flux density are assessed. Then, 3D model of the respective generator is acquired to examine the effect of magnet geometry on the cogging torque produced. In that context, the effects of splitted and skewed magnet structures are examined. In the first design, the magnet is modelled with one piece and the rms value of the cogging torque is found as 436.75 mNm. In the second case, a certain skewed slit is made alongside the magnet and that yields a slightly reduced cogging torque of 434.58 mNm. In the other design, the magnet of the first design is divided into two sub-parts, which are then combined together in a skewed fashion. Thus, the value of cogging torque is found as 159.60 mNm. Eventually, by making two certain slits on the last model, cogging torque is further depressed down to 89.95mNm. It is concluded from the obtained results that the last design contributes an improvement in the value of cogging torque up to 80% compared to the initial design.

1. INTRODUCTION

Nowadays, the importance of renewable energy sources increases due to the increasing awareness on the environmental issues and the conflicts in the Middle East area producing high percentage of fossil fuels. In addition to being an environmentally friendly and local solution, intense efforts are underway to utilize the renewables to diversify energy sources [1,2]. When considering renewable energy sources, Turkey is a rich country specifically in terms wind, solar and hydro energy. It ranks first amongst the UE countries in the subject of wind energy potential. Despite all this potential, a large amount of its energy is supplied by fossil fuels, which are

also being imported [3]. By the end of 2017, the installed capacity based on renewable energy sources is 38.743 MW in Turkey. When the distribution of renewable energy resources within the installed power is known as 27.273 MW hydraulic, 6.516 MW wind, 1.028 MW geothermal and 2.653 MW solar energy in this respect. The ministry of energy aims to increase the share of renewable energy in the energy supply and expects to increase, for instance, wind power to 10.000 MW [4-8].

For the wind turbine systems, various kinds of generators are used. The generators can have advantages and disadvantages relative to each other depending on the type and classification. For instance, the permanent magnet synchronous generators (PMSGs) with high power densities are used in increasing scales in the commercial sector. Since PMSGs can be connected to

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wind turbines without any gear system, they have been often preferred in wind power plants with low and medium power range [9]. By the discovery of high-performance Neodymium-Iron-Boron magnets and developments in power electronic, permanent magnet (PM) synchronous machines have shown rapid development. Indeed, PM machines are preferred for their low speed and variable speed applications. Since the parameters such as efficiency and power coefficient are independent of rotation speed in the performance of PM machine, the machines can suit for low cycle applications [10-13]. In addition to its advantages, one of the major disadvantages of PM machines is the generation of cogging torque. The cogging torque on electric machines is the torque that is generated by the interaction between the stator slots and the permanent magnets which make up the rotor magnetic field when the phase currents are zero. The value of the cogging torque is independent of the stator current and varies depending on the rotor position and that is undesirable and causes vibration and acoustic noise on the machine.

Many methods exist in the machine literature in order to reduce the cogging torque. In fact, those methods for cogging torque reduction can be classified into two main groups: Machine-based and control-based. Optimization of cogging torque can be accomplished by skewing stator slots or rotor magnets, injection the harmonic current, changing the embrace and offset of the magnet, designing a fractional slot, and adding additional slots or teeth [11,14-17]. In terms of control method, the torque harmonic spectrum of a 12/10 pole machine is investigated for the optimization of the cogging torque. To compensate for the harmonic components, an additional torque component is applied to the machine beside the current harmonics. Both simulations and experiments proved the effectiveness of those methods [18]. However, the effectiveness of these methods is often poor, practically [19].

Machine - based work can be divided into two groups: Stator and rotor magnets. In the literature, the authors studied the cogging torque of 5 different PM machines with different groove counts due to the rotor position. It was found that the motor with the fractional - slot winding structure leads to low cogging torque, however the average torque was decreased [20]. For the study made on the stator slots, three different anti-notch geometries were added and an optimization was ascertained. In other finite element studies, the cogging torque value was decreased by examining the effect of the anti-notch form with a semi-cylindrical structure [21]. In PMSGs, the cogging torque can be optimized by using the finite element method (FEM) to skew the stator slots. For instance, Tseng et al. [22] found that cogging torque is reduced by 89% compared to the straight slot structure by using the Taguchi method.

When the studies on the magnet part are examined, the decrease in the cogging torque by obtaining the overlaid form of the opposite electromotive force harmonic components was ascertained by changing the magnet

shape similar to the study [18]. It was found that, the cogging torque was reduced by 90% in the third harmonic superimposed structure and it also reduced the fluctuations in the output torque [23]. However, the difficulty of applying the desired form to the magnet structure should not be ignored in this study. In a similar study, aimed the exploration of the effect of the structures with the rounded edge magnet on the cogging torque of the PMSG, an optimization on the air gap was applied. With the proposed method, the change in the reluctance was minimized and the cogging torque value was reduced by 40% [9]. It was determined that the magnetic pole eccentric reduced the torque by reducing the air-gap harmonic components. In addition, the magnetic pole structure reduced the fluctuation in the cogging torque [24].

In the present study, a 2.5 kW, 14 pole surface mounted PM synchronous generator has been designed and the cogging torque calculations have been performed for various structures of PM magnets, which can be produced easily by rare-earth elements. After an initial design stage, a 3D model of the generator has been created for the finite element analysis and the rotor magnets are designed with multi-step magnet-skew and a skewed slit to test and optimize the most suitable cogging torque value.

In that frame, this paper is structured as follows: Initially, the design parameters of the generator, the material information, the 3D model, the winding structure and the mesh structure are discussed in the second section. In addition, the definition of the cogging torque in a PM machine, reduction methods and mathematical model are presented. In Section 3, new models of generator and cogging torque values of these structures are obtained. Finally, the conclusion is given in section IV.

2. SPECIFICATION OF THE DESIGNED PM GENERATOR

PMSG design starts with the generator sizing equations. The output equation of the generator is given by Equation 1.

$$S = 11.K_{w1} \cdot \bar{B} \cdot ac \cdot \left(\frac{D}{1000}\right)^2 \cdot \left(\frac{L}{1000}\right) \cdot n \quad (1)$$

In the equation, S is the power, K_{w1} the winding factor, \bar{B} the specific magnetic loading, D the stator outer diameter, L the stack length of the generator and ac the specific electrical loading. ac depends on variables such as nominal power, frequency and nominal voltage, and is expressed by Equation 2.

$$ac = \frac{(\delta a_z) Z}{\pi D} \quad (2)$$

During the design phase, specific magnetic loading and specific electric loading are selected by the designer. The

hysteresis and eddy current losses, saturation point, stray losses, cooling type and load profile of the materials used are important variables while selecting magnetic loading. On the other hand, copper loss, cooling type, insulator and load profile are taken into consideration for selecting electrical loading [25].

The generator used in the current study is a direct drive, inner rotor, radial flux and PM structure. PMSGs are machines with high efficiency, easy production, low torque fluctuation and high-power density. PM generators can be directly connected to the wind turbines without any gear system and that superiority makes the PMSGs frequently used in wind turbines [26]. The design parameters of the generator are summarized in Table 1.

Table 1. Design parameters of PMSG

Parameter	Unit	Value
Rated Power	kW	2.5
Rated Speed	rpm	428.5
Rated Voltage	V	120
Magnet Material	-	N35
Stator-Rotor Material	-	M19
Stator Outer Diameter	mm	280
Air-gap	mm	0.8
Length of Stator	mm	54
Pole Number	-	14
Number of Slots	-	84

3D model for an initial design, the stator winding connection and the mesh structure used in the analysis of the finite element method are given in Figure 1.

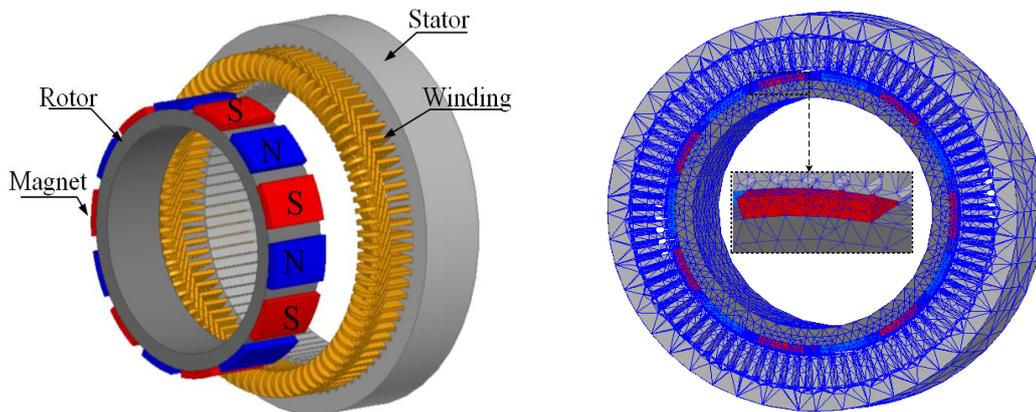


Figure 1. The designed generator with Model 1 magnet type

3. COGGING TORQUE OF THE PM GENERATOR

Cogging torque in the PM machines is caused by the magnetic interaction between the stator and the magnets placed on the rotors. This effect is undesirable as it produces noise and vibration in PM machines. The high value of the cogging torque prevents rotor rotation and thereby it exhibits resistant to generate electricity at low wind speeds [27]. In the literature, there are many methods for the minimization of cogging torque, such as lot skewing, magnet skewing, closed slot, unequal pole width, pole arc optimization and auxiliary slots. Cogging torque can be written as a function of the reluctance, the air-gap flux and the rotor position and expressed by Equation 3 [28].

$$T_{cog} = \frac{1}{2} \phi_g^2 \frac{d\mathfrak{R}}{d\theta} \quad (3)$$

In Equation 3, ϕ_g expresses the amount of air-gap flux, \mathfrak{R} air-gap reluctance and θ rotor position. As seen from

the equation, the cogging torque is the interaction of the magnets (i.e. air-gap flux) and the stator teeth (i.e. the source of the variable air - gap reluctance). Periodically change in the reluctance of the air gap also causes a cogging torque which changes periodically. Due to this periodic variation, the value of the cogging torque can be calculated from Equation 4 with the Fourier series,

$$T_{cog} = \sum_{k=1}^{\infty} T_{mk} \sin(mk\theta) \quad (4)$$

In the equation, m , k and T_{mk} denote the number of rotor poles and the smallest number of stator slot numbers, an integer and a Fourier coefficient. The finite element analysis is applied, while there is no current in the stator of the generator of the 3D model. In those analyzes, the torque variation is obtained according to the rotor position of the initial designed generator (Figure 2).

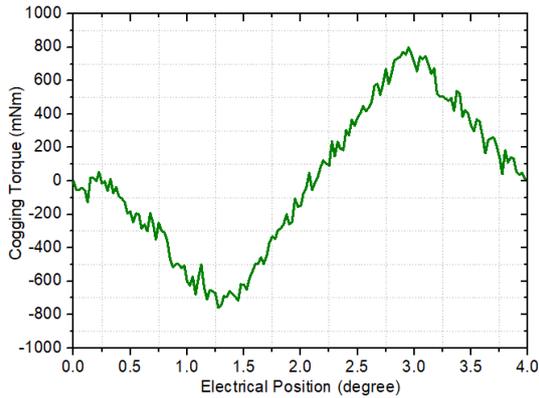


Figure 2. Cogging torque waveform of the initial design

In the first design (i.e. Model 1) the magnet is designed as a single piece and the rms value of the cogging torque is obtained according to the rotor position as 436.75 mNm.

4. FEM ANALYSIS OF PM GENERATOR

The analysis of electric machines includes a complicated solution and an interrelated problem. It is possible to create the desired design by using an approximation method and a computer software working parallel with the finite difference and finite element methods. By applying a FEM to machine design, it is possible to determine the electromagnetic parameters at a very high accuracy in that manner [29-31]. Finite element analysis was developed to find approximate numerical solutions of the magnitudes that are continuous in a particular region such as the fields and their variations on the

studied region. Then the solution can be expressed by partial differential equations in accordance with the Maxwell's equations. In this method, the region to be solved is divided into small regions with a finite number to read the data. For each node in the region, a cumulative solution value is calculated within a certain error limit. By operating the method, an iteration to reduce that error is applied to the nodes. In many packages, that can be adjustable to ascertain the desired numerical accuracy [32].

Both 2D and 3D models of the machine are designed and finite element analysis is performed with computer aided design program. The use of FEM provides the designer with time and economic benefits. The application of the FEM to machine design ensures that critical design parameters such as cogging torque, winding voltages, and induced torque of the machine are determined with very high accuracy. In order to optimize the cogging torque of the generator, which passed the initial design, rotor magnets were designed with two pairs of pieces and a skewed slit.

In permanent magnet machines cogging torque can be suppressed by partitioning the magnets off in the axial axis. In the present study, the rotor pertaining to models 3 and 4 is designed in two-stage structure. The mechanical magnitude of skew angle of the magnet piece is calculated by Eq. 5 [33].

$$\theta = \frac{360}{2 \times Z} \quad (5)$$

Where Z stands for the number of stator slots. In view of this, the skew angle of our design is computed as 2.14° . The structures formed are given in Figure 3.

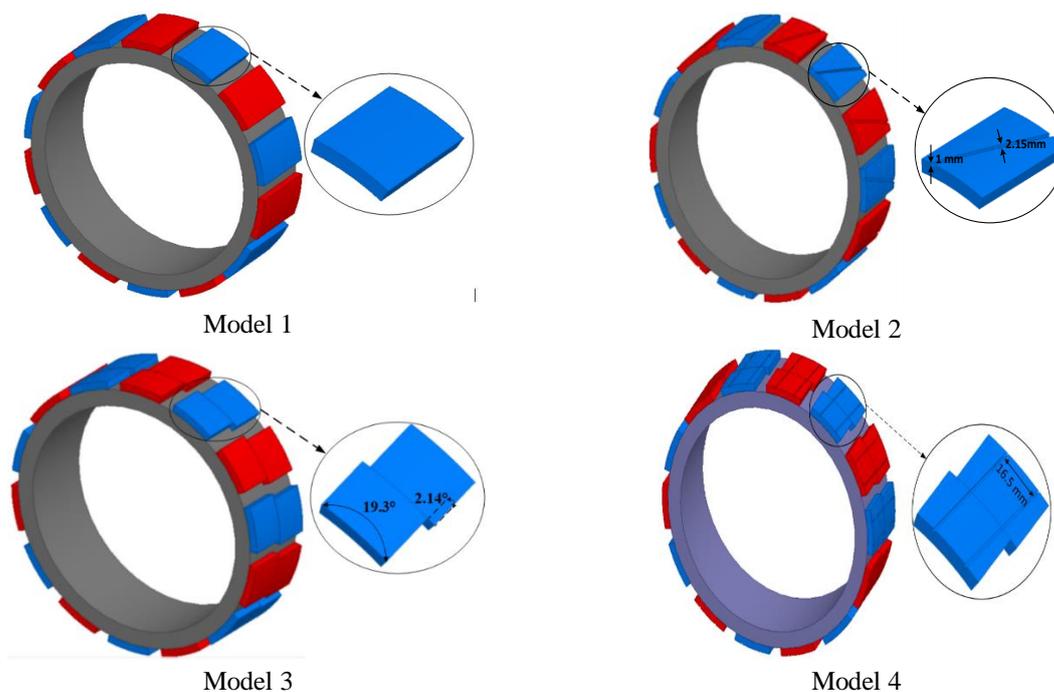


Figure 3. The designed magnet structures

When the designed magnets are examined, Model 2 is formed by opening a notch in the first designed magnet. Model 3 is formed by dividing the magnet into two equal parts in axial direction and half of the magnet part is shifted about half-slot in angular direction in accordance

with cylindrical coordinates. Finally, the magnets configured in Model 4 are derived from Model 3 by having additional 2 notches. The simulation results for the cogging torque are given in Figure 4.

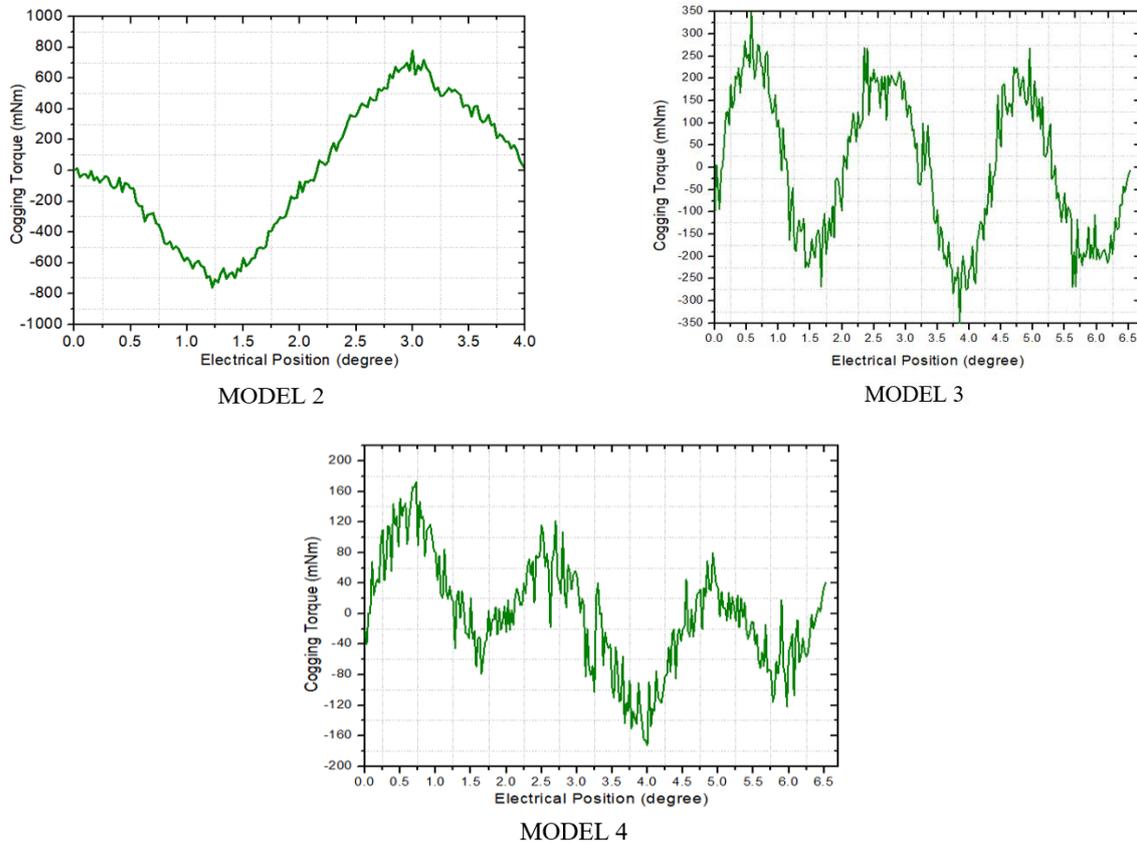


Figure 4. Cogging torque waveforms versus rotor electrical position for three different configurations

The rms values of the cogging torque from the simulations are summarized in Table 2. By designing the magnet in a piece structure leads to a significant reduction in cogging torque. The final design has improved by up to 80% according to the first design.

Table 2. RMS value of cogging torque

Model	Rms value
Model 1(initial design)	436.75 mNm
Model 2	434.58 mNm
Model 3	159.60 mNm
Model 3	89.95 mNm

It is obvious that the proposed new cogging torque reduction method is very effective for the radial flux PM machines and that yields to better power generation mechanism for the wind turbines even for lower wind rates and increases the productivity of the turbines. Taking benefits of the detailed transient analyses, output powers of the designs are tabulated in Table 3 from which

it is seen that the output power of the proposed model is decreased by 2.76% with regard to classical Model 1.

Table 3. Power values of models

Model	Value
Model 1(initial design)	2500 W
Model 2	2462 W
Model 3	2455 W
Model 3	2431 W

The magnetic field can be expressed by Maxwell's equations in a permanent magnet synchronous generator;

$$\begin{aligned} \nabla \times \vec{H} &= \vec{J} \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \end{aligned} \tag{6}$$

In the equation, \vec{J} is current density, \vec{H} magnetic field strength, \vec{B} magnetic flux density, \vec{E} electric field strength. The basic formulation of the vector potential for the magnetic field is expressed by Equation 7

$$\nabla \times (\nu \nabla \times \vec{A}) = \vec{J} \quad (7)$$

Here in ν , which is given by $\nu = \partial B / \partial H$ exhibits variable permeability owing to the nonlinearity of $B = f(H)$. The flux density distribution of the generators in Figure 5 and 6 can be expressed by Equations 8 and 9.

$$\frac{\partial}{\partial x} \left(\nu \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu \frac{\partial A}{\partial y} \right) + \frac{\partial}{\partial z} \left(\nu \frac{\partial A}{\partial z} \right) = \vec{J} \quad (8)$$

The magnetic flux density value is calculated from Equation 8 in 3D analyses [34].

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} \quad (9)$$

The flux density distributions obtained from the 3D electromagnetic analyses of the designed generators are given in Figure 5 for overall angular and radial geometry. Indeed, four plots indicate similar flux structures especially for the vicinities of cores and magnets. Thus, that result proves the reliability of the studied models. Especially at the vicinity of cores, $B=1.5$ T flux densities are achieved and indicated by a yellow colour in the coloured version of the paper. The main difference among the models is that the maximal fluxes are obtained with skewed forms in Model 3 and 4 as it is expected due to the geometries of magnets in Models 3 and 4. On the outer surface of the machine nearly 1.2 T flux density is available in the present models and it is already satisfactory for the use of the designs.

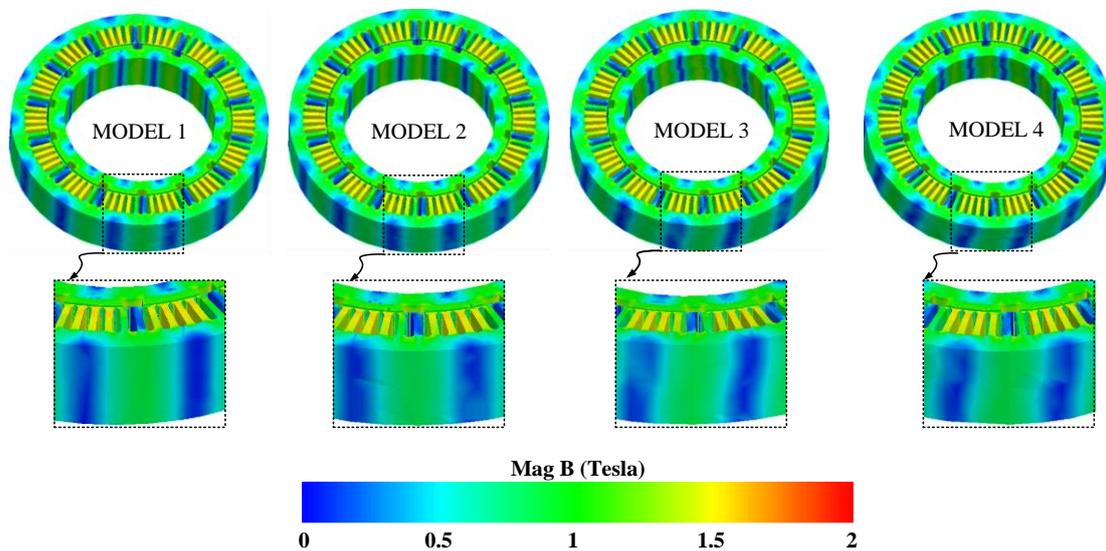


Figure 5. Magnetic flux densities of four models

In order to give a comprehensive idea on the air gap flux density along the angular direction, a series of analyses have been performed and the detailed plots have been drawn in Figure 6. According to figures, especially Figure 6(a) gives important results on the studied designs. For instance, for the same mesh structures, while Model 1, 2 and 3 give slightly higher flux densities around 0.705 T, the last model gives $B=0.690$ T. Indeed, the flux density fluctuations with respect to the angular direction give different characteristics for each model.

Especially for Model 2, which is denoted by blue colour in coloured version, the fluctuations over the angular position is strong. However, the fluctuations for Model 4 is lower compared to the other models. In Figure 6(b), the flux densities at the vicinities of a magnet and the cores are shown on the axial plane for all models. All models show the flux density values of 1.5 T and 1 T on the cores and on the magnet, respectively. These flux densities are sufficiently high for the cross-sectional area of the cores.

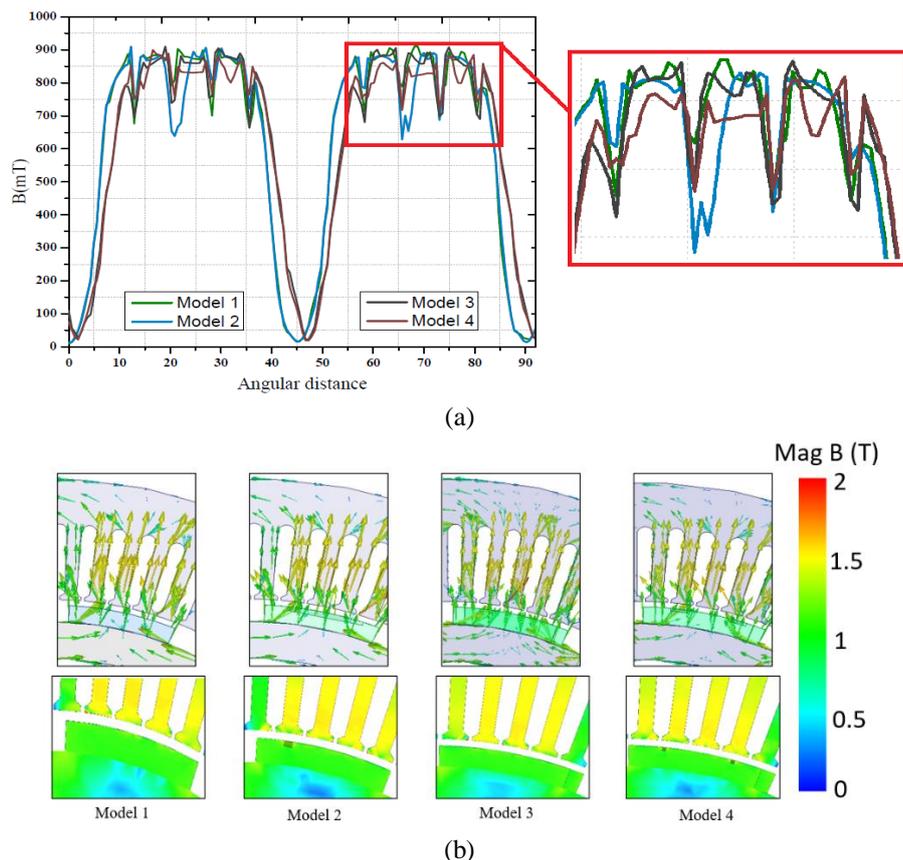


Figure 6. Flux densities (a) along with the angular position inside the air gap and (b) on the axial plane

6. CONCLUSION

New rotor designs with different magnet configurations are performed in order to lower the undesired cogging torque value. The machine (i.e. PMSG) has the power of 2.5 kW and 14-pole as appropriate for house-hold wind turbines. The study has focused on the optimization of the cogging torque, which causes noise and vibration in PM machines and prevents generators from generating electricity at low wind speeds. The rotor magnet has been designed by forming 4 different models such as pieced and notched structures. In the initial design, when the magnet is one piece and without skew and stator current is zero, the rotor position is changed and the rms value of the cogging torque has been obtained as 436.75 mNm. With reference to Model 1, one notch has been opened for each magnet and Model 2 has been created. In Model 2, with a slight change the rms value of cogging torque has been found 434.58 mNm. In Model 3, where the PM is formed by sliding two equal parts and half slot, the cogging torque has been reduced to 159.60 mNm with a rate of 64%. Finally, the Model 4 is designed by producing 2 notches from the magnets in the partitioned structures. In the latter design, the rms value of the cogging torque is calculated as 89.95 mNm, which yields to 80% improvement to the preliminary configuration.

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