

Opacity tables for using in Paczynski stellar modelling and their effects on stellar evolutions

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Abstract

We have constructed opacity tables for using in stellar modelling in a format particularly applicable to the Paczynski freeware program GOB. Our tables are derived from those of Allard and Weiss for lower metal content envelopes, using the Lagrange interpolation method. This allows accurate opacities to be given as numerical functions of density and temperature with given proportions of hydrogen (X), helium (Y), and ‘metals’ (Z). Inlek et al. showed that it is possible independently to check opacity effects from modern high-quality observational data. These tables will be useful for such purposes.

Keywords: Stars, general-structure, opacity tables.

Paczynski yıldız modellemesinde kullanılmak için opaklık tabloları ve onların yıldız evrimlerine etkileri

Özet

Yıldız modellemede Paczynski ’nin ücretsiz programı GOB’a uygulanabilir bir formatta kullanmak için opaklık tabloları oluşturduk. Tablolarımız, Lagrange interpolasyon yöntemi kullanılarak düşük metal içerikli zarflar için Allard ve Weiss ’in tablolarından elde edilmiştir. Bu, verilen hidrojen (X), helyum (Y) ve metallerin (Z) oranları ile yoğunluk ve sıcaklığın sayısal bir fonksiyonu olarak doğru opaklıkların elde edilmesini sağlar. Inlek ve grubu opaklık etkilerini yüksek kalitede modern gözlemlerle verilerden kontrol etmenin bağımsız olarak mümkün olduğunu göstermiştir. Bu tablolar bu tarz amaçlar için yararlı olacaktır.

Anahtar Kelimeler: Yıldızlar, genel-yapı, opaklık tabloları.

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1. Introduction

It is well known that radiative transport plays a major role in shaping the structure and evolution of stars; however, direct measurements of the opacity of matter to radiation in stellar interiors are not possible. Therefore, theoretical calculations are the main sources of opacity evaluation. Characterizing the radiative transport opacities are basic components for stellar structure and evolution modelling. Over the years, various opacity formulae and tables have been applied to structural and evolution models of stars. Schwarzschild [1] discussed in his seminal book the effects associated with the ionization of different atomic species, i.e. bound-free transitions, free-free absorption and electron scattering. Carson [2], Iglesias and Rogers [3] and Kurucz [4] have shown that the bound-bound (line) absorptions make an important contribution to opacity. Various theoretical methods have been developed to calculate opacities. Rogers and Iglesias [5] and Iglesias and Rogers [3] used the OPAL opacity code to calculate stellar opacities. The other code OP to calculate opacity tables has been used by Badnell et al. [6]. Blancard et al. [7] studied OP and OPAL codes compared by opacity model OPAS for the solar mixture. Mondet et al. [8] computed the opacities for various hydrogen and metallic element mass fractions according to the recent chemical composition of the solar photosphere. They discussed possible sources of uncertainty in their calculations and also compared Rosseland opacities to OPAL and OP data.

Our aim is examine the effect of varying opacities on the envelope structure with the aid of Paczynski's codes. In this study, we collected opacity data that were calculated using the OPAL code in the study of Allard [9] and Weiss [10]. Inlek et al [11] worked with the stellar model codes GOB (generates outer boundary conditions) and SCH (generates zero age main sequence star models) of Paczynski [12] and applied these programs to real observational data. They discussed the use of such sources as the OPAL code. It is well-known that opacity effects are strongest in the outer parts of the star, where the temperature gradient steepens as the major constituent elements start to recombine (cf. e.g. Young et al., 2001 [13]). In order to tailor recent detailed calculations of opacity values for low metallicity envelopes (e.g. those of Allard, [9]; Weiss, [10]) to the format required by GOB, we used four-point Lagrange interpolation [14]. Our results set out values of $\log \kappa$ for (envelope) \log density values in the range -12 to +3 and \log temperature in the range 3.25 to 7.00.

2. The tables

The element abundances included in the Allard [9] and Weiss [10] compositions are as follows: H, He, Li, Be, B, C, N, O, F, Ne, Na, Mg, Al, Si, P, S, Cl, Ar, K, Ca, Sc, Ti, V, Cr, Mn, Fe, Co and Ni. Relative element abundances are usually specified simply by X (hydrogen), Y (helium) and Z (total mass fraction of the other elements), where $X+Y+Z=1$. The relative proportions of the metals in the mixture for these calculations were given by Iglesias and Rogers [3]. The source data allow for accurate interpolation in temperature, density and composition. Those data were specified in terms of an additional parameter R, where $\log R = \log (\rho/T_6^3)$, ρ being the mass density in g/cm³, and $T_6=10^{-6}$ T, the temperature in kelvins [3]. The ranges of R and temperature seem to cover typical stellar conditions from the interior through the envelope extending towards outer regions. The new source data ranges are $3.75 \leq \log T \leq 8.7$ and $-8 \leq \log R \leq +1$ [3].

We present suitably formatted tables for $X = 0.7$ and $Z = 0.01$ from Allard [9] and $X = 0.7$; metal mass fractions $Z = 0.001, 0.002, 0.004$ from Weiss'[10] calculations, respectively. To create the full two-dimensional tables needed for the Paczynski [12] programs, a two-step procedure is required: firstly, we interpolated density values from the source to the required table format. These opacities were then interpolated to the required temperatures at the new densities. We thus had the logarithm of the opacity, $\kappa(\text{cm}^2/\text{g})$, as a function of $\log T$ for columns of constant $\log \rho$. $\log \kappa$ is given for 31 values of $\log \rho$ in the range of -12 to +3 in steps of 0.5, and 51 values of $\log T$ from 3.25 in steps of 0.075.

3. Remarks

The main opacity changes in more recent calculations are due to additional metals in the mixture; for this reason, opacity enhancements are smaller for the lower metallicity mixture [3]. We studied the effects of lower metallicity mixture opacity tables in some stellar modelling examples for intermediate-low mass stars. Inlek et al. [11] showed that increases of the radius are associated with increasing envelope opacity. In their study, changes from the earlier LAOL opacity tables [15] to later ones that included more detailed line absorption effects changed with the radii up to $\sim 5\text{-}10\%$. In our examples, we have found an increase in the radius (1 - 3%) for the opacities with different Z value. This can be seen in Table 1.

Table 1: GOB results for radii using opacity tables with different Z value.

Parameter	Value				
	0.6	1	2.5	5	10
M (Sun)	0.6	1	2.5	5	10
R (Weiss Z=0.001)	0.3434	0.5195	1.0118	1.7219	2.3571
R (Weiss Z=0.002)	0.3480	0.5220	1.0127	1.7500	2.3584
R (Weiss Z=0.004)	0.3491	0.5272	1.0194	1.7552	2.3601
R (Allard Z=0.01)	0.3532	0.5300	1.0228	1.7656	2.3781
log L	-0.8276	0.0409	1.6068	2.8296	3.6749
log T _e	3.6520	3.7692	4.0115	4.2085	4.3545

We anticipate that our low metallicity opacity tables may be useful in the tests of luminosity values for Population II stars, in the studies on globular clusters, and on the Bulge or nearby galaxies. Our results are very usefull for testing eclipsing binary data and in the near future observational data for the new planetary systems.

Since the early twenties many opacity calculations have been published. The opacity theory has not been static. For example stellar structure coefficient k_2 has been calculated as theoretical and has been derived by observationally. For the eclipsing binary stars components it is impossible derived by observationally at the same time. The benefical of the theoretical calculations is the possibility for this purpose.

4. Conclusion

The present study presents the opacity tables for different metallicity for Paczynski [12] codes GOB and SCH in Tables 2-5. These results are very beneficial for astrophysical modelling programs. Our next step is conducting a study with the new tables from Colgan et al. [16] and checking the results with observational data on GOB and SCH.

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Table 2. (Continued).

log T	log ρ=																																		
	-12.0	-11.5	-11	-10.5	-10.0	-9.5	-9.0	-8.5	-8.0	-7.5	-7.0	-6.5	-6.0	-5.5	-5.0	-4.5	-4.0	-3.5	-3.0	-2.5	-2.0	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5	2.0	2.5	3.0				
5.500	.47	.47	.47	.47	.49	.48	.46	.46	.44	.42	.37	.29	.15	.06	.35	.71	1.10	1.49	1.87	2.20	2.47	2.72	2.99	3.24	3.47	.79	.79	.79	.79	.79	.79				
5.575	.47	.47	.47	.47	.48	.47	.44	.47	.46	.45	.43	.40	.32	.17	.09	.43	.83	1.22	1.59	2.49	2.20	2.45	2.70	2.96	3.21	.97	.97	.97	.97	.97	.97				
5.650	.47	.47	.47	.48	.47	.46	.47	.47	.46	.45	.44	.40	.33	.17	.12	.51	.94	1.33	1.68	1.96	2.20	2.44	2.70	2.96	1.15	1.15	1.15	1.15	1.15	1.15					
5.725	.47	.47	.47	.47	.47	.47	.44	.47	.46	.45	.43	.40	.32	.14	.18	.61	1.05	1.43	1.74	1.99	2.22	2.46	2.71	1.32	1.32	1.32	1.32	1.32	1.32						
5.800	.47	.47	.47	.47	.47	.49	.47	.47	.46	.46	.45	.43	.38	.28	.08	.27	.72	1.16	1.51	1.78	2.01	2.25	2.49	1.50	1.20	.71	-.27	-1.20	-2.29	-3.39					
5.875	.47	.47	.47	.47	.47	.48	.49	.46	.47	.47	.46	.45	.44	.41	.35	.23	.02	.40	.85	1.25	1.56	1.82	2.06	2.30	2.55	1.75	1.19	.24	-.70	-1.77	-2.86				
5.950	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.45	.43	.39	.31	.15	.14	.55	.98	1.34	1.63	1.89	2.13	2.37	2.27	1.67	.74	-.19	-1.25	-2.33					
6.025	.47	.47	.47	.47	.47	.47	.47	.47	.48	.46	.46	.45	.44	.42	.36	.25	.04	.30	.73	1.13	1.45	1.73	1.98	2.21	2.43	2.68	1.25	.32	-.74	-1.81					
6.100	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.45	.44	.42	.39	.32	.18	.09	.47	.90	1.27	1.58	1.84	2.05	2.25	2.48	1.74	.81	-.24	-1.29					
6.175	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.44	.43	.40	.35	.25	.06	.25	.65	1.06	1.41	1.68	1.88	2.06	2.27	2.12	1.29	.26	-.79					
6.250	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.45	.44	.42	.37	.29	.15	.09	.44	.84	1.21	1.49	1.70	1.86	2.06	2.28	1.70	.75	-.30					
6.325	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.45	.41	.34	.23	.03	.26	.62	.99	1.27	1.49	1.66	1.84	2.08	1.84	1.18	.19					
6.400	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.45	.41	.32	.15	.10	.42	.75	1.03	1.25	1.43	1.62	1.86	2.14	1.41	.65					
6.475	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.47	.47	.47	.47	.47	.47	.45	.40	.29	.08	.21	.52	.79	1.01	1.20	1.40	1.64	1.92	1.15	.97				
6.550	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.47	.47	.47	.47	.47	.46	.44	.38	.24	-.01	.27	.54	.77	.96	1.16	1.40	1.69	2.00	.86					
6.625	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.47	.47	.47	.46	.44	.38	-.20	.04	.30	.53	.73	.93	1.17	1.46	1.77	.43				
6.700	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.48	.48	.48	.48	.47	.47	.45	.42	.33	.15	.08	.30	.51	.71	.94	1.22	1.54	1.84		
6.775	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.48	.48	.48	.48	.48	.48	.47	.47	.45	.40	-.29	-.12	.09	.29	.49	.71	.99	1.31	1.62	
6.850	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.48	.48	.48	.48	.48	.48	.47	.46	-.44	-.38	-.26	-.09	.09	.29	.50	.77	1.08	1.39		
6.925	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.48	.48	.48	.48	.48	.48	.47	-.46	-.43	-.35	-.23	-.07	.10	.31	.56	.86	1.17		
7.000	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.48	.48	.48	.48	.48	.48	.47	-.45	-.41	-.33	-.21	-.05	.14	.37	.64	.95

Table 3. OPAL opacity table interpolated for X=0.7 and Z=0.001 with metal distribution from Weiss (1995).

log T	log ρ=																												
-12.0 -11.5 -11 -10.5 -10.0 -9.5 -9.0 -8.5 -8.0 -7.5 -7.0 -6.5 -6.0 -5.5 -5.0 -4.5 -4.0 -3.5 -3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0																													
3.250 -4.48 -3.80 -3.20 -2.86 -2.72 -2.67 -2.64 -2.63 -2.62 -2.62 -2.61 -2.60 -2.60 -2.59 -2.58 -2.56 -2.54 -2.52 -2.49 -2.46 -2.42 -5.54 -4.59 -4.59 -4.59 -4.59 -4.59	3.325 -4.73 -4.71 -4.59 -4.14 -3.52 -3.04 -2.80 -2.69 -2.65 -2.63 -2.61 -2.60 -2.59 -2.57 -2.55 -2.52 -2.49 -2.44 -2.39 -2.33 -2.26 -2.18 -2.09 -5.32 -4.41 -4.41 -4.41 -4.41 -4.41	3.400 -4.53 -4.51 -4.48 -4.41 -4.29 -4.05 -3.60 -3.12 -2.83 -2.69 -2.62 -2.57 -2.53 -2.48 -2.43 -2.36 -2.29 -2.20 -2.09 -1.98 -1.85 -1.72 -1.58 -5.09 -4.23 -4.23 -4.23 -4.23 -4.23	3.475 -4.44 -4.41 -4.38 -4.31 -4.21 -4.08 -3.90 -3.64 -3.28 -2.90 -2.60 -2.41 -2.25 -2.13 -2.01 -1.88 -1.73 -1.59 -1.43 -1.28 -1.12 -0.96 -0.80 -4.87 -4.05 -4.05 -4.05 -4.05 -4.05	3.550 -4.31 -4.29 -4.32 -4.24 -4.10 -3.92 -3.68 -3.39 -3.06 -2.72 -2.38 -2.07 -1.82 -1.62 -1.45 -1.27 -1.08 -0.90 -0.71 -0.52 -0.33 -0.14 -0.00 -4.65 -3.87 -3.87 -3.87 -3.87 -3.87	3.625 -4.13 -4.14 -4.23 -4.24 -4.12 -3.94 -3.68 -3.37 -3.04 -2.69 -2.35 -2.03 -1.72 -1.44 -1.20 -0.98 -0.76 -0.53 -0.30 -0.08 -0.15 -0.38 -0.61 -0.66 -4.43 -3.70 -3.70 -3.70 -3.70 -3.70	3.700 -3.96 -3.99 -4.03 -3.92 -3.71 -3.39 -3.03 -2.69 -2.35 -2.04 -1.73 -1.43 -1.13 -0.86 -0.60 -0.33 -0.06 -0.21 -0.17 -0.13 -0.09 -0.05 -0.02 -0.01 -0.01 -0.01 -0.01 -0.01 -0.01 -0.01 -0.01	3.775 -1.50 -1.68 -1.83 -1.91 -1.90 -1.80 -1.65 -1.46 -1.25 -1.04 -0.81 -0.58 -0.51 -0.24 -0.05 -0.33 -0.61 -0.89 -1.17 -1.46 -1.72 -1.25 -3.99 -3.34 -3.34 -3.34 -3.34 -3.34	3.850 -.60 -.65 -.72 -.76 -.75 -.70 -.60 -.47 -.31 -.14 -.04 -.24 -.45 -.67 -.08 -.37 -.66 -.96 1.25 1.54 1.83 2.08 1.39 -.77 -.16 -.16 -.16 -.16 -.16	3.925 -.37 -.23 -.05 .12 .27 .38 .47 .57 .66 .77 .90 1.05 1.20 1.38 .36 .67 .98 1.29 1.60 1.91 2.22 2.45 1.51 -.355 -.298 -.298 -.298 -.298 -.298	4.000 -.45 -.32 -.09 .23 .60 .95 1.22 1.41 1.54 1.64 1.74 1.82 1.93 2.05 2.20 .97 1.28 1.58 1.88 2.18 2.47 2.66 1.61 -.333 -.280 -.280 -.280 -.280 -.280	4.075 -.48 -.40 -.25 .00 .36 .80 1.25 1.65 1.98 2.21 2.36 2.47 2.57 2.67 2.78 1.32 1.59 1.85 2.12 2.38 2.64 2.78 1.71 -.311 -.262 -.262 -.262 -.262 -.262	4.150 -.47 -.43 -.33 -.16 .10 .46 .90 1.39 1.87 2.30 2.64 2.89 3.06 3.19 3.32 1.76 2.01 2.25 2.49 2.73 2.96 3.01 1.82 -.289 -.244 -.244 -.244 -.244 -.244	4.225 -.47 -.45 -.39 -.27 -.07 .23 .61 1.06 1.55 2.05 2.53 2.96 3.30 3.54 3.73 3.89 2.40 2.62 2.83 3.05 3.25 3.19 1.92 -.266 -.226 -.226 -.226 -.226 -.226	4.300 -.48 -.45 -.41 -.34 -.20 .04 .38 .81 1.29 1.78 2.29 2.80 3.28 3.68 3.99 4.22 2.78 2.97 3.17 3.36 3.52 3.35 2.02 -.244 -.08 -.08 -.08 -.08 -.08	4.375 -.48 -.45 -.42 -.36 -.26 -.08 .20 .59 1.07 1.57 2.10 2.62 3.15 3.65 4.08 4.42 4.69 3.31 3.49 3.67 3.80 3.49 2.12 -.222 -.190 -.190 -.190 -.190 -.190	4.450 -.45 -.44 -.41 -.35 -.26 -.13 .10 .44 .89 1.39 1.92 2.46 3.01 3.56 4.06 4.49 4.84 3.60 3.78 3.97 4.07 3.62 2.23 -.200 -.172 -.172 -.172 -.172 -.172	4.525 -.45 -.44 -.42 -.36 -.26 -.11 .09 .38 .78 1.25 1.78 2.32 2.88 3.45 3.99 4.48 4.89 5.22 4.04 4.23 4.30 3.73 2.33 -.178 -.154 -.154 -.154 -.154 -.154	4.600 -.46 -.45 -.43 -.40 -.32 -.17 .07 .38 .76 1.19 1.68 2.20 2.74 3.30 3.86 4.38 4.83 5.20 4.28 4.43 4.43 3.80 2.43 -.156 -.136 -.136 -.136 -.136 -.136	4.675 -.47 -.45 -.44 -.42 -.38 -.28 -.09 .21 .61 1.08 1.57 2.08 2.59 3.11 3.64 4.16 4.63 5.02 4.50 4.57 4.49 3.86 2.52 -.134 -.119 -.119 -.119 -.119 -.119	4.750 -.47 -.45 -.44 -.43 -.41 -.35 -.23 -.01 .31 .73 1.23 1.76 2.31 2.83 3.35 3.85 4.32 4.73 5.07 4.70 4.53 3.92 2.62 -.112 -.010 -.010 -.010 -.010 -.010	4.825 -.47 -.47 -.45 -.44 -.42 -.39 -.31 -.16 .08 .42 .85 1.34 1.88 2.43 2.98 3.50 3.98 4.42 4.79 4.94 4.72 4.02 2.72 -.90 -.83 -.83 -.83 -.83 -.83	4.900 -.47 -.47 -.45 -.44 -.43 -.40 -.35 -.25 -.08 .19 .57 1.02 1.52 2.06 2.61 3.15 3.68 4.15 4.54 4.86 4.87 4.11 2.82 -.68 -.65 -.65 -.65 -.65 -.65	4.975 -.47 -.47 -.45 -.45 -.44 -.42 -.38 -.30 -.17 .04 .36 .77 1.25 1.77 2.31 2.86 3.40 3.90 4.33 4.65 5.00 4.19 2.93 -.46 -.47 -.47 -.47 -.47 -.47	5.050 -.47 -.47 -.47 -.43 -.43 -.42 -.40 -.34 -.23 -.06 .21 .58 1.04 1.54 2.07 2.61 3.15 3.66 4.10 4.43 4.67 4.27 3.03 -.24 -.29 -.29 -.29 -.29 -.29	5.125 -.47 -.47 -.47 -.42 -.41 -.40 -.39 -.36 -.29 -.15 .08 .42 .85 1.33 1.84 2.36 2.88 3.38 3.82 4.15 4.40 4.33 3.12 -.01 -.11 -.11 -.11 -.11 -.11	5.200 -.47 -.47 -.47 -.46 -.40 -.39 -.38 -.36 -.31 -.21 -.04 .25 .63 1.09 1.57 2.05 2.53 3.00 3.43 3.79 4.06 4.28 3.22 .21 .07 .07 .07 .07 .07	5.275 -.47 -.47 -.47 -.47 -.43 -.41 -.40 -.38 -.34 -.28 -.15 .06 .37 .78 1.22 1.68 2.12 2.56 2.98 3.36 3.68 3.96 3.32 .43 .25 .25 .25 .25 .25	5.350 -.47 -.47 -.47 -.49 -.47 -.44 -.43 -.41 -.38 -.34 -.25 -.11 .14 .47 .86 1.28 1.70 2.12 2.53 2.92 3.28 3.57 3.41 .65 .43 .43 .43 .43 .43	5.425 -.47 -.47 -.47 -.49 -.48 -.46 -.46 -.45 -.43 -.40 -.34 -.24 -.06 .20 .53 .91 1.31 1.70 2.10 2.49 2.87 3.21 3.51 .87 .61 .61 .61 .61 .61

Table 3. (Continued).

log T	log ρ=																															
	-12.0	-11.5	-11	-10.5	-10.0	-9.5	-9.0	-8.5	-8.0	-7.5	-7.0	-6.5	-6.0	-5.5	-5.0	-4.5	-4.0	-3.5	-3.0	-2.5	-2.0	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5	2.0	2.5	3.0	
5.500	-.47	-.47	-.47	-.47	-.49	-.48	-.47	-.47	-.46	-.46	-.44	-.41	-.34	-.22	-.02	.26	.60	.96	1.33	1.71	2.09	2.48	2.84	3.16	3.42	.79	.79	.79	.79	.79	.79	.79
5.575	-.47	-.47	-.47	-.47	-.48	-.47	-.44	-.47	-.47	-.46	-.45	-.41	-.34	-.20	.02	.32	.66	1.01	1.37	1.73	2.11	2.55	2.83	3.12	.97	.97	.97	.97	.97	.97	.97	
5.650	-.47	-.47	-.47	-.47	-.48	-.47	-.46	-.47	-.47	-.46	-.44	-.41	-.33	-.18	.06	.38	.72	1.06	1.41	1.77	1.15	2.52	2.83	1.15	1.15	1.15	1.15	1.15	1.15	1.15		
5.725	-.47	-.47	-.47	-.47	-.47	-.44	-.47	-.47	-.47	-.46	-.46	-.44	-.40	-.31	-.15	.12	.45	.79	1.12	1.47	1.90	2.21	2.55	1.32	1.32	1.32	1.32	1.32	1.32	1.32		
5.800	-.47	-.47	-.47	-.47	-.47	-.49	-.47	-.47	-.47	-.47	-.46	-.45	-.43	-.38	-.28	-.09	.19	.52	.86	1.19	1.55	1.92	2.27	1.50	1.20	.71	-.27	-1.20	-2.29	-3.39		
5.875	-.47	-.47	-.47	-.47	-.47	-.48	-.49	-.46	-.47	-.47	-.47	-.46	-.45	-.42	-.36	-.23	-.02	.28	.61	.95	1.30	1.66	2.01	2.34	1.75	1.19	.24	-.70	-1.77	-2.86		
5.950	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.46	-.46	-.44	-.40	-.32	-.16	.08	.39	.74	1.08	1.43	1.77	2.09	2.27	1.67	.74	-.19	-1.25	-2.33			
6.025	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.46	-.46	-.45	-.43	-.37	-.27	-.07	.21	.54	.89	1.23	1.54	1.85	2.18	2.53	1.25	.32	-.74	-1.81			
6.100	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.46	-.46	-.45	-.44	-.41	-.34	-.20	.04	.36	.71	1.03	1.31	1.60	1.93	2.28	1.74	.81	-.24	-1.29			
6.175	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.46	-.46	-.45	-.42	-.38	-.28	-.10	.17	.50	.81	1.09	1.36	1.68	2.03	2.12	1.29	.26	-.79			
6.250	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.46	-.44	-.41	-.34	-.21	.00	.29	.58	.85	1.12	1.43	1.78	2.14	1.70	.75	-.30			
6.325	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.46	-.43	-.39	-.29	-.14	.09	.35	.61	.87	1.18	1.53	1.91	1.84	1.18	.19			
6.400	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.47	-.46	-.43	-.37	-.25	-.08	.14	.38	.64	.93	1.28	1.67	2.05	1.41	.65				
6.475	-.48	-.48	-.48	-.48	-.48	-.48	-.48	-.48	-.48	-.48	-.47	-.47	-.47	-.47	-.46	-.45	-.42	-.34	-.22	-.04	.17	.41	.70	1.03	1.42	1.81	1.15	.97				
6.550	-.48	-.48	-.48	-.48	-.48	-.48	-.48	-.48	-.48	-.48	-.48	-.47	-.47	-.47	-.47	-.47	-.46	-.45	-.40	-.32	-.18	.00	.21	.47	.79	1.17	1.56	1.93	.86			
6.625	-.49	-.49	-.49	-.49	-.49	-.49	-.49	-.49	-.49	-.49	-.49	-.49	-.49	-.49	-.47	-.47	-.47	-.47	-.46	-.44	-.38	-.29	-.14	.04	.27	.57	.93	1.32	1.69	.43		
6.700	-.50	-.50	-.50	-.50	-.50	-.50	-.50	-.50	-.50	-.50	-.50	-.50	-.50	-.50	-.48	-.48	-.48	-.48	-.48	-.47	-.47	-.45	-.42	-.36	-.25	-.10	.10	.36	.70	1.08	1.45	1.79
6.775	-.51	-.51	-.51	-.51	-.51	-.51	-.51	-.51	-.51	-.51	-.51	-.51	-.51	-.51	-.48	-.48	-.48	-.48	-.48	-.48	-.47	-.47	-.45	-.41	-.33	-.21	-.05	.18	.48	.84	1.22	1.57
6.850	-.53	-.53	-.53	-.53	-.53	-.53	-.53	-.53	-.53	-.53	-.53	-.53	-.53	-.53	-.48	-.48	-.48	-.48	-.48	-.48	-.47	-.46	-.44	-.39	-.30	-.17	.03	.29	.62	.99	1.34	
6.925	-.56	-.56	-.56	-.56	-.56	-.56	-.56	-.56	-.56	-.56	-.56	-.56	-.56	-.56	-.48	-.48	-.48	-.48	-.48	-.48	-.47	-.45	-.42	-.36	-.26	-.11	.12	.41	.76	1.11		
7.000	-.61	-.61	-.61	-.61	-.61	-.61	-.61	-.61	-.61	-.61	-.61	-.61	-.61	-.61	-.48	-.48	-.48	-.48	-.48	-.48	-.48	-.47	-.44	-.40	-.33	-.21	-.03	.23	.55	.89		

Table 4. (Continued).

log T	log ρ=																														
	-12.0	-11.5	-11	-10.5	-10.0	-9.5	-9.0	-8.5	-8.0	-7.5	-7.0	-6.5	-6.0	-5.5	-5.0	-4.5	-4.0	-3.5	-3.0	-2.5	-2.0	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5	2.0	2.5	3.0
5.500	.47	.47	.47	.49	.48	.47	.46	.46	.45	.43	.39	.31	.17	.06	.36	.71	1.08	1.45	1.80	2.15	2.51	2.86	3.17	3.43	.79	.79	.79	.79	.79	.79	.79
5.575	.47	.47	.47	.48	.47	.44	.47	.47	.46	.45	.44	.40	.31	.15	.11	.43	.79	1.14	1.48	1.81	2.16	2.52	2.85	3.13	.97	.97	.97	.97	.97	.97	.97
5.650	.47	.47	.47	.48	.47	.46	.47	.47	.46	.46	.44	.40	.30	.13	.15	.50	.86	1.20	1.52	1.84	2.19	2.54	2.85	1.15	1.15	1.15	1.15	1.15	1.15	1.15	
5.725	.47	.47	.47	.47	.47	.44	.44	.47	.47	.47	.46	.45	.43	.39	.29	.09	.22	.58	.93	1.25	1.56	1.90	2.25	2.57	1.32	1.32	1.32	1.32	1.32	1.32	1.32
5.800	.47	.47	.47	.47	.49	.47	.47	.47	.47	.47	.46	.45	.42	.37	.25	.02	.30	.66	.99	1.31	1.63	1.98	2.31	1.50	1.20	.71	-.27	-1.20	-2.29	-3.39	
5.875	.47	.47	.47	.47	.48	.49	.46	.47	.47	.46	.46	.44	.41	.34	.19	.06	.40	.75	1.07	1.40	1.74	2.07	2.38	1.75	1.19	.24	-.70	-1.77	-2.86		
5.950	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.45	.43	.38	.29	.11	.17	.51	.86	1.19	1.52	1.84	2.15	2.27	1.67	.74	-.19	-1.25	-2.33		
6.025	.47	.47	.47	.47	.47	.47	.47	.48	.48	.47	.47	.46	.46	.44	.41	.35	.23	.00	.31	.67	1.01	1.34	1.64	1.92	2.22	2.55	1.25	.32	-.74	-1.81	
6.100	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.46	.45	.43	.39	.31	.14	.13	.48	.84	1.16	1.45	1.70	1.99	2.31	1.74	.81	-.24	-1.29	
6.175	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.46	.45	.44	.41	.35	.24	.03	.28	.64	.97	1.24	1.48	1.75	2.07	2.12	1.29	.26	-.79	
6.250	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.45	.43	.38	.30	.15	.11	.43	.75	1.01	1.25	1.51	1.83	2.16	1.70	.75	-.30	
6.325	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.45	.42	.36	.24	.05	.22	.51	.77	1.01	1.27	1.58	1.93	1.84	1.18	.19	
6.400	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.45	.41	.33	.18	.03	.28	.52	.76	1.03	1.34	1.69	2.06	1.41	.65	
6.475	.48	.48	.48	.48	.48	.48	.48	.48	.48	.47	.47	.47	.47	.47	.47	.47	.46	.45	.40	.30	.14	.07	.30	.53	.79	1.09	1.45	1.82	1.15	.97	
6.550	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.47	.47	.47	.47	.47	.46	.44	.38	.27	-.10	.10	.31	.56	.85	1.20	1.58	1.93	.86		
6.625	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.47	.47	.47	.47	.47	.46	.43	.36	-.24	-.07	.12	.34	.62	.96	1.34	1.70	.43	
6.700	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.48	.48	.48	.48	.47	.47	.45	.41	-.33	-.21	-.04	.16	.41	.73	1.10	1.46	1.80	
6.775	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.48	.48	.48	.48	.48	.48	.48	.47	.46	.44	-.39	-.30	-.18	.00	.22	.51	.86	1.22	1.56
6.850	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.48	.48	.48	.48	.48	.48	.48	.47	.47	.46	-.42	-.36	-.25	-.10	.09	.35	.68	1.04	1.39
6.925	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.48	.48	.48	.48	.48	.47	.46	-.44	-.39	-.31	-.18	-.01	.22	.51	.86	1.23
7.000	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.48	.48	.48	.48	.48	.47	.46	-.43	-.37	-.28	-.15	.05	.31	.63	.99	

Table 5. (Continued)

log T	log ρ=																															
	-12.0	-11.5	-11	-10.5	-10.0	-9.5	-9.0	-8.5	-8.0	-7.5	-7.0	-6.5	-6.0	-5.5	-5.0	-4.5	-4.0	-3.5	-3.0	-2.5	-2.0	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5	2.0	2.5	3.0	
5.500	.47	.47	.47	.47	.49	.48	.47	.46	.45	.44	.41	.36	.26	.10	.15	.48	.85	1.23	1.60	1.93	2.25	2.57	2.89	3.19	3.44	.79	.79	.79	.79	.79	.79	.79
5.575	.47	.47	.47	.47	.48	.47	.44	.47	.47	.46	.45	.43	.37	.27	.08	.22	.57	.94	1.30	1.63	1.93	2.24	2.56	2.87	3.15	.97	.97	.97	.97	.97	.97	.97
5.650	.47	.47	.47	.47	.48	.47	.46	.47	.47	.46	.45	.43	.38	.27	.05	.27	.65	1.02	1.36	1.66	1.95	2.26	2.58	2.87	1.15	1.15	1.15	1.15	1.15	1.15	1.15	
5.725	.47	.47	.47	.47	.47	.47	.44	.47	.47	.46	.46	.45	.42	.37	.25	.01	.34	.73	1.10	1.41	1.69	1.99	2.30	2.61	1.32	1.32	1.32	1.32	1.32	1.32	1.32	
5.800	.47	.47	.47	.47	.47	.49	.47	.47	.47	.47	.46	.46	.44	.41	.34	.20	.06	.43	.82	1.16	1.46	1.75	2.05	2.36	1.50	1.20	.71	-.27	-1.20	-2.29	-3.39	
5.875	.47	.47	.47	.47	.47	.48	.49	.46	.47	.47	.46	.45	.43	.39	.31	.13	.17	.54	.91	1.23	1.53	1.83	2.14	2.43	1.75	1.19	.24	-.70	-1.77	-2.86		
5.950	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.46	.45	.42	.36	.25	.04	.29	.66	1.02	1.33	1.64	1.94	2.23	2.27	1.67	.74	-.19	-1.25	-2.33		
6.025	.47	.47	.47	.47	.47	.47	.47	.47	.48	.48	.47	.47	.46	.46	.45	.43	.40	.32	.17	.09	.44	.82	1.16	1.48	1.77	2.03	2.29	2.59	1.25	.32	-.74	-1.81
6.100	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.46	.45	.44	.42	.37	.27	.07	.24	.62	.99	1.32	1.60	1.83	2.08	2.37	1.74	.81	-.24	-1.29	
6.175	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.46	.44	.42	.39	.32	.18	.06	.41	.80	1.14	1.41	1.63	1.86	2.14	2.12	1.29	.26	-.79			
6.250	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.46	.45	.42	.36	.25	.04	.29	.66	1.02	1.33	1.64	1.94	2.23	2.27	1.67	.74	-.19	-1.25	-2.33	
6.325	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.44	.40	.32	.17	.09	.44	.82	1.16	1.48	1.77	2.03	2.29	2.59	1.25	.32	-.74	-1.81
6.400	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.47	.46	.46	.45	.44	.42	.37	.27	.07	.24	.62	.99	1.32	1.60	1.83	2.08	2.37	1.74	.81	-.24	-1.29	
6.475	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.47	.47	.46	.44	.42	.39	.32	.18	.06	.41	.80	1.14	1.41	1.63	1.86	2.14	2.12	1.29	.26	-.79		
6.550	.48	.48	.48	.48	.48	.48	.48	.48	.48	.48	.47	.47	.46	.44	.41	.36	.25	.07	.23	.59	.93	1.20	1.42	1.63	1.90	2.19	1.70	.75	-.30			
6.625	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.49	.47	.47	.47	.47	.47	.45	.41	.32	.16	.04	.24	.46	.71	1.02	1.37	1.71	.43	
6.700	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.48	.48	.48	.48	.47	.47	.46	.44	.39	.29	.13	.06	.26	.49	.78	1.13	1.48	1.81
6.775	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.51	.48	.48	.48	.48	.48	.47	.46	.43	.37	.26	.10	.08	.29	.56	.89	1.25	1.58	
6.850	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53	.48	.48	.48	.48	.48	.48	.47	.45	.42	.34	.23	.08	.12	.36	.67	1.02	1.36	
6.925	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.56	.48	.48	.48	.48	.48	.48	.47	.45	.40	.32	.20	-.04	.18	.46	.79	1.13		
7.000	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.61	.48	.48	.48	.48	.48	.48	.47	.46	.43	.38	.29	-.16	.03	.27	.58	.91		