Journal of New Theory
ISSN: 2149-1402

28 (2019) 74-83
Journal of New Theory
http://www.newtheory.org
Open Access

New Correlation Coefficients between Linguistic Neutrosophic Numbers and Their Group Decision Making Method

Yanfei Zhu[1], Wenhua Cui[2], Jun Ye[3]*

Article History
Received: 18.06.2019
Accepted: 09.09.2019
Published: 25.09.2019
Original Article

Abstract − Since linguistic neutrosophic numbers (LNNs) are depicted independently by the truth, indeterminacy, and falsity linguistic variables in indeterminate and inconsistent linguistic environment, they are very fit for human thinking and expressing habits to judgments of complex objects in real life world. Then the correlation coefficient is a critical mathematical tool in pattern recognition and decision making science, but the related research was rarely involved in LNN setting. Hence, this work first proposes two new correlation coefficients of LNNs based on the correlation and information energy of LNNs as the complement/extension of our previous work, and then develops a multiple criteria group decision making (MCGDM) method based on the proposed correlation coefficients in LNN setting. Lastly, a decision making example is provided to illustrate the applicability of the developed method. By comparison with the MCGDM methods regarding the existing correlation coefficients based on the maximum and minimum operations of LNNs, the decision results indicate the effectiveness of the developed MCGDM approach. Hence, the proposed approach provides another new way for linguistic neutrosophic decision making problems.

Keywords − *Linguistic neutrosophic number, correlation coefficient, multiple criteria group decision making*

1. Introduction

The decision making problems usually imply inconsistent, incomplete, and indeterminate information, along with truth, falsity, indeterminacy information in assessment process. Then, neutrosophic theory [1] is a powerful mathematical tool for expressing truth, falsity, indeterminacy information effectively. Hence, it has been used for various problems, such as medical image processing [2-4], medical diagnosis [5-7], fault diagnosis [8-10], and decision making [11-23]. However, when human thinking complicated objects usually contain subjectivity and vagueness, it is difficult to give accurate assessment values of complicated/ill-defined problems regarding the expression of qualitative information by numerical values, but linguistic variables/term values can effectively represent qualitative information and customarily accord with human thinking and expressing habits. Hence, some single-valued and interval neutrosophic linguistic numbers [24-26] and single-valued neutrosophic trapezoid linguistic numbers [27], and interval neutrosophic uncertain linguistic numbers [28] were proposed based on the combination of both linguistic variables and neutrosophic numbers and applied to decision making. On the one hand, there also exists the difficulty of qualitative information expressed by using the neutrosophic numbers. On the other hand, they cannot also express the truth, falsity, indeterminacy linguistic values in inconsistent and indeterminate linguistic setting.

[1]zhuyf00523@163.com; [2]wenhuacui@usx.edu.cn; [3]*yehjun@aliyun.com (Corresponding Author)
[1] Shaoxing People's Hospital, Shaoxing, P.R. China
[2,3]Department of Electrical and Information Engineering, Shaoxing University, Shaoxing, P.R. China

To solve these issues, linguistic neutrosophic numbers (LNNs) [29] were presented for describing the truth, falsity, indeterminacy linguistic information in inconsistent, incomplete, and indeterminate linguistic setting, and then some aggregation operators were introduced and applied in linguistic neutrosophic MCGDM problems [29, 30]. Furthermore, cosine measures based on the vector space and the distance of LNNs [31], correlation coefficients based on the minimum and minimum operations of LNNs [32], and bidirectional project measures based on the project models of LNNs [33] were presented respectively and applied to MCGDM problems in LNN setting.

However, the correlation coefficient is a critical mathematical tool in pattern recognition and decision making science, but the related research was rarely involved in LNN setting. Therefore, this study proposes two new correlation coefficients of LNNs as the complement/extension of our previous work [32], and then develops their MCGDM approach for solving the indeterminate and inconsistent linguistic decision making problems in LNN setting. To do so, this study is constructed as the following work framework. Section 2 introduces some preliminaries of LNNs. The correlation coefficients of LNNs are proposed based on the correlation and information energy of LNNs in Section 3. Section 4 presents a MCGDM approach based the proposed correlation coefficients in LNN setting. Section 5 presents a decision making example to show the applicability of the proposed MCGDM approach in LNN setting. Section 6 gives the comparison of the proposed approach with decision making approaches based on existing correlation coefficients of LNNs to indicate the effectiveness of the proposed approach. Section 7 contains conclusions and further research.

2. Some preliminaries of LNNs

Fang and Ye [29] proposed a LNN concept regarding the truth, indeterminacy, and falsity linguistic term variables v_a, v_b, v_c, and then the values of the linguistic term variables can be obtained from a given linguistic term set $V = \{v_0, v_1, \ldots, v_q\}$ with odd cardinality q+1. Thus, a LNN is expressed as $s = \langle v_a, v_b, v_c \rangle$ for $s \in V$ and $a, b, c \in [0, q]$.

For three LNNs $s = \langle v_a, v_b, v_c \rangle$, $s_1 = \langle v_{a_1}, v_{b_1}, v_{c_1} \rangle$, and $s_2 = \langle v_{a_2}, v_{b_2}, v_{c_2} \rangle$ in V, their operational laws are introduced as follows [29]:

(i) $s_1 \oplus s_2 = \langle v_{a_1}, v_{b_1}, v_{c_1} \rangle \oplus \langle v_{a_2}, v_{b_2}, v_{c_2} \rangle = \left\langle v_{a_1+a_2-\frac{a_1a_2}{q}}, v_{\frac{b_1b_2}{q}}, v_{\frac{c_1c_2}{q}} \right\rangle$;

(ii) $s_1 \otimes s_2 = \langle v_{a_1}, v_{b_1}, v_{c_1} \rangle \otimes \langle v_{a_2}, v_{b_2}, v_{c_2} \rangle = \left\langle v_{\frac{a_1a_2}{q}}, v_{b_1+b_2-\frac{b_1b_2}{q}}, v_{c_1+c_2-\frac{c_1c_2}{q}} \right\rangle$;

(iii) $ps = p\langle v_a, v_b, v_c \rangle = \left\langle v_{q-q\left(1-\frac{a}{q}\right)^p}, v_{q\left(\frac{b}{q}\right)^p}, v_{q\left(\frac{c}{q}\right)^p} \right\rangle$ for $p > 0$;

(iv) $s^p = \langle v_a, v_b, v_c \rangle^p = \left\langle v_{q\left(\frac{a}{q}\right)^p}, v_{q-q\left(1-\frac{b}{q}\right)^p}, v_{q-q\left(1-\frac{c}{q}\right)^p} \right\rangle$ for $p > 0$.

Let $s_k = \langle v_{a_k}, v_{b_k}, v_{c_k} \rangle$ (k = 1, 2, …, n) be a group of LNNs in V, then the LNN weighted arithmetic averaging operator is introduced as follows [29]:

$$LNNWAA(s_1, s_2, ..., s_n) = \sum_{k=1}^{n} \rho_k s_k = \left\langle v_{q-q\prod_{k=1}^{n}\left(1-\frac{a_k}{q}\right)^{\rho_k}}, v_{q\prod_{k=1}^{n}\left(\frac{b_k}{q}\right)^{\rho_k}}, v_{q\prod_{k=1}^{n}\left(\frac{c_k}{q}\right)^{\rho_k}} \right\rangle, \tag{1}$$

where $\rho_k \in [0, 1]$ is the weight of s_k (k =1, 2, …, n) with $\sum_{k=1}^{n} \rho_k = 1$.

Assume two linguistic neutrosophic sets (LNSs) are $S_1 = \{s_{11}, s_{12}, \ldots, s_{1n}\}$ and $S_2 = \{s_{21}, s_{22}, \ldots, s_{2n}\}$, where $s_{1k} = \langle v_{a_{1k}}, v_{b_{1k}}, v_{c_{1k}} \rangle$ and $s_{2k} = \langle v_{a_{2k}}, v_{b_{2k}}, v_{c_{2k}} \rangle$ (k = 1, 2, …, n) are two groups of LNNs in $V = \{v_0, v_1, \ldots, v_q\}$. Let $f(v_y) = y$ be a linguistic scale function. Then, based the minimum and maximum operations of LNNs, Shi and Ye [32] proposed three weighted correlation coefficients between S_1 and S_2:

$$M_1(S_1,S_2) = \sum_{k=1}^{n} \rho_k \frac{\min\left(f(v_{a_{1k}}), f(v_{a_{2k}})\right) + \min\left(f(v_{b_{1k}}), f(v_{b_{2k}})\right) + \min\left(f(v_{c_{1k}}), f(v_{c_{2k}})\right)}{\sqrt{f(v_{a_{1k}}) f(v_{a_{2k}})} + \sqrt{f(v_{b_{1k}}) f(v_{b_{2k}})} + \sqrt{f(v_{c_{1k}}) f(v_{c_{2k}})}}$$
$$= \sum_{k=1}^{n} \rho_k \frac{\min(a_{1k}, a_{2k}) + \min(b_{1k}, b_{2k}) + \min(c_{1k}, c_{2k})}{\sqrt{a_{1k}a_{2k}} + \sqrt{b_{1k}b_{2k}} + \sqrt{c_{1k}c_{2k}}}, \quad (2)$$

$$M_2(S_1,S_2) = \sum_{k=1}^{n} \rho_k \frac{f(v_{a_{1k}}) f(v_{a_{2k}}) + f(v_{b_{1k}}) f(v_{b_{2k}}) + f(v_{c_{1k}}) f(v_{c_{2k}})}{\left(\max(f(v_{a_{1k}}), f(v_{a_{2k}}))\right)^2 + \left(\max(f(v_{b_{1k}}), f(v_{b_{2k}}))\right)^2 + \left(\max(f(v_{c_{1k}}), f(v_{c_{2k}}))\right)^2}$$
$$= \sum_{k=1}^{n} \rho_k \frac{a_{1k}a_{2k} + b_{1k}b_{2k} + c_{1k}c_{2k}}{\left(\max(a_{1k}, a_{2k})\right)^2 + \left(\max(b_{1k}, b_{2k})\right)^2 + \left(\max(c_{1k}, c_{2k})\right)^2}, \quad (3)$$

$$M_3(S_1,S_2) = \sum_{k=1}^{n} \rho_k \frac{\min\left(f(v_{a_{1k}}), f(v_{a_{2k}})\right) + \min\left(f(v_{b_{1k}}), f(v_{b_{2k}})\right) + \min\left(f(v_{c_{1k}}), f(v_{c_{2k}})\right)}{\max\left(f(v_{a_{1k}}), f(v_{a_{2k}})\right) + \max\left(f(v_{b_{1k}}), f(v_{b_{2k}})\right) + \max\left(f(v_{c_{1k}}), f(v_{c_{2k}})\right)}$$
$$= \sum_{k=1}^{n} \rho_k \frac{\min(a_{1k}, a_{2k}) + \min(b_{1k}, b_{2k}) + \min(c_{1k}, c_{2k})}{\max(a_{1k}, a_{2k}) + \max(b_{1k}, b_{2k}) + \max(b_{1k}, a_{2k})}, \quad (4)$$

where $\rho_k \in [0, 1]$ is the weight of s_{jk} (j =1, 2; k =1, 2, …, n) with $\sum_{k=1}^{n} \rho_k = 1$.

3. Correlation coefficients between LNNs

As the complement/extension of existing correlation coefficients of LNNs [32], this section proposes two new correlation coefficients between two LNNs based on the correlation and information energy of LNNs.

Definition 1. Set two linguistic neutrosophic sets (LNSs) as $S_1 = \{s_{11}, s_{12}, \ldots, s_{1n}\}$ and $S_2 = \{s_{21}, s_{22}, \ldots, s_{2n}\}$, where $s_{1k} = \langle v_{a_{1k}}, v_{b_{1k}}, v_{c_{1k}} \rangle$ and $s_{2k} = \langle v_{a_{2k}}, v_{b_{2k}}, v_{c_{2k}} \rangle$ (k = 1, 2, …, n) are two groups of LNNs in $V = \{v_0, v_1, \ldots, v_q\}$. Let $f(v_y) = y$ be a linguistic scale function. Then we can define the correlation of LNSs S_1 and S_2 as follows:

$$L(S_1,S_2) = \sum_{k=1}^{n} (f(v_{a_{1k}}) f(v_{a_{2k}}) + f(v_{b_{1k}}) f(v_{b_{2k}}) + f(v_{c_{1k}}) f(v_{c_{2k}}) = \sum_{k=1}^{n} (a_{1k}a_{2k} + b_{1k}b_{2k} + c_{1k}c_{2k}). \quad (5)$$

Based on Eq. (5), it is obvious that the correlations between S_1 and S_1 and between S_2 and S_2 yield the following forms:

$$L(S_1,S_1) = \sum_{k=1}^{n} (f(v_{a_{1k}}) f(v_{a_{1k}}) + f(v_{b_{1k}}) f(v_{b_{1k}}) + f(v_{c_{1k}}) f(v_{c_{1k}}) = \sum_{k=1}^{n} (a_{1k}^2 + b_{1k}^2 + c_{1k}^2), \quad (6)$$

$$L(S_2,S_2) = \sum_{k=1}^{n} (f(v_{a_{2k}}) f(v_{a_{2k}}) + f(v_{b_{2k}}) f(v_{b_{2k}}) + f(v_{c_{2k}}) f(v_{c_{2k}}) = \sum_{k=1}^{n} (a_{2k}^2 + b_{2k}^2 + c_{2k}^2), \quad (7)$$

which are also named the information energy of LNSs S_1 and S_2.

Thus, the two correlation coefficients of LNSs S_1 and S_2 are given by

$$Q_1(S_1,S_2)=\frac{L(S_1,S_2)}{\sqrt{L(S_1,S_1)}\sqrt{L(S_2,S_2)}}$$
$$=\frac{\sum_{k=1}^{n}\left(f(v_{a_{1k}})f(v_{a_{2k}})+f(v_{b_{1k}})f(v_{b_{2k}})+f(v_{c_{1k}})f(v_{c_{2k}})\right)}{\sqrt{\sum_{k=1}^{n}\left(f^2(v_{a_{1k}})+f^2(v_{b_{1k}})+f^2(v_{c_{1k}})\right)}\sqrt{\sum_{k=1}^{n}\left(f^2(v_{a_{2k}})+f^2(v_{b_{2k}})+f^2(v_{c_{2k}})\right)}} \quad (8)$$
$$=\frac{\sum_{k=1}^{n}(a_{1k}a_{2k}+b_{1k}b_{2k}+c_{1k}c_{2k})}{\sqrt{\sum_{k=1}^{n}(a_{1k}^2+b_{1k}^2+c_{1k}^2)}\sqrt{\sum_{k=1}^{n}(a_{2k}^2+b_{2k}^2+c_{2k}^2)}},$$

$$Q_2(S_1,S_2)=\frac{L(S_1,S_2)}{\max\{L(S_1,S_1),L(S_2,S_2)\}}$$
$$=\frac{\sum_{k=1}^{n}\left(f(v_{a_{1k}})f(v_{a_{2k}})+f(v_{b_{1k}})f(v_{b_{2k}})+f(v_{c_{1k}})f(v_{c_{2k}})\right)}{\max\left\{\sum_{k=1}^{n}\left(f^2(v_{a_{1k}})+f^2(v_{b_{1k}})+f^2(v_{c_{1k}})\right),\sum_{k=1}^{n}\left(f^2(v_{a_{2k}})+f^2(v_{b_{2k}})+f^2(v_{c_{2k}})\right)\right\}} \quad (9)$$
$$=\frac{\sum_{k=1}^{n}(a_{1k}a_{2k}+b_{1k}b_{2k}+c_{1k}c_{2k})}{\max\left\{\sum_{k=1}^{n}(a_{1k}^2+b_{1k}^2+c_{1k}^2),\sum_{k=1}^{n}(a_{2k}^2+b_{2k}^2+c_{2k}^2)\right\}}.$$

Then, it is obvious that Eqs. (8) and (9) satisfies the following conditions:

(a) $Q_1(S_1, S_2) = Q_1(S_2, S_1)$ and $Q_2(S_1, S_2) = Q_2(S_2, S_1)$;

(b) $Q_1(S_1, S_2) = Q_2(S_1, S_2) = 1$ for $S_1 = S_2$;

(c) $Q_1(S_1, S_2)$, $Q_2(S_1, S_2) \in [0, 1]$.

PROOF.

It is clear that the conditions (a) and (b) are true. Hence, we only verify the condition (c) below.

For the proof of $Q_1(S_1, S_2)$, if $k = 1$, Eq. (8) is reduced to the following cosine measure of LNNs [31]:

$$Q_1(S_1,S_2)=Cos(S_1,S_2)=\frac{f(v_{a_{1k}})f(v_{a_{2k}})+f(v_{b_{1k}})f(v_{b_{2k}})+f(v_{c_{1k}})f(v_{c_{2k}})}{\sqrt{f^2(v_{a_{1k}})+f^2(v_{b_{1k}})+f^2(v_{c_{1k}})}\sqrt{(f^2(v_{a_{2k}})+f^2(v_{b_{2k}})+f^2(v_{c_{2k}})}} \quad (10)$$
$$=\frac{a_{1k}a_{2k}+b_{1k}b_{2k}+c_{1k}c_{2k}}{\sqrt{a_{1k}^2+b_{1k}^2+c_{1k}^2}\sqrt{a_{2k}^2+b_{2k}^2+c_{2k}^2}}.$$

Obviously, the cosine measure of LNNs introduced by Shi and Ye [31] is a special case of the correlation coefficient $Q_1(S_1, S_2)$ when $k = 1$.

Since there exists $Cos(S_1, S_2) \in [0, 1]$ regarding the property of the cosine measure between LNNs [31], there is also $Q_1(S_1, S_2) \in [0, 1]$ if $k = 1$. Thus, it is obvious that $Q_1(S_1, S_2) \in [0, 1]$ is true if $k = n$.

For the proof of $Q_2(S_1, S_2)$, since $\max\left\{\sum_{k=1}^{n}(a_{1k}^2+b_{1k}^2+c_{1k}^2),\sum_{k=1}^{n}(a_{2k}^2+b_{2k}^2+c_{2k}^2)\right\} \geq$ $a_{1k}a_{2k}+b_{1k}b_{2k}+c_{1k}c_{2k}$ can holds for $a_{jk}, b_{jk}, c_{jk} \in [0, q]$ ($j = 1, 2$; $k = 1, 2, \ldots, n$) in $V = \{v_0, v_1, \ldots, v_q\}$, it is clear that there exists $Q_2(S_1, S_2) \in [0, 1]$.

Hence, this proof is finished. □

If the importance of each LNN s_{jk} (j = 1, 2; k = 1, 2, …, n) in S_1 and S_2 is indicated by the weight value ρ_k for $\rho_k \in [0, 1]$ and $\sum_{k=1}^{n}\rho_k = 1$, the weighted correlation coefficients of LNSs S_1 and S_2 can be expressed by

$$W_1(S_1,S_2)=\frac{\sum_{k=1}^{n}\rho_k\left(f(v_{a_{1k}})f(v_{a_{2k}})+f(v_{b_{1k}})f(v_{b_{2k}})+f(v_{c_{1k}})f(v_{c_{2k}})\right)}{\sqrt{\sum_{k=1}^{n}\rho_k\left(f^2(v_{a_{1k}})+f^2(v_{b_{1k}})+f^2(v_{c_{1k}})\right)}\sqrt{\sum_{k=1}^{n}\rho_k\left(f^2(v_{a_{2k}})+f^2(v_{b_{2k}})+f^2(v_{c_{2k}})\right)}}$$
$$=\frac{\sum_{k=1}^{n}\rho_k(a_{1k}a_{2k}+b_{1k}b_{2k}+c_{1k}c_{2k})}{\sqrt{\sum_{k=1}^{n}\rho_k(a_{1k}^2+b_{1k}^2+c_{1k}^2)}\sqrt{\sum_{k=1}^{n}\rho_k(a_{2k}^2+b_{2k}^2+c_{2k}^2)}}, \tag{11}$$

$$W_2(S_1,S_2)=\frac{\sum_{k=1}^{n}\rho_k\left(f(v_{a_{1k}})f(v_{a_{2k}})+f(v_{b_{1k}})f(v_{b_{2k}})+f(v_{c_{1k}})f(v_{c_{2k}})\right)}{\max\left\{\sum_{k=1}^{n}\rho_k\left(f^2(v_{a_{1k}})+f^2(v_{b_{1k}})+f^2(v_{c_{1k}})\right),\sum_{k=1}^{n}\rho_k\left(f^2(v_{a_{2k}})+f^2(v_{b_{2k}})+f^2(v_{c_{2k}})\right)\right\}}$$
$$=\frac{\sum_{k=1}^{n}\rho_k(a_{1k}a_{2k}+b_{1k}b_{2k}+c_{1k}c_{2k})}{\max\left\{\sum_{k=1}^{n}\rho_k(a_{1k}^2+b_{1k}^2+c_{1k}^2),\sum_{k=1}^{n}\rho_k(a_{2k}^2+b_{2k}^2+c_{2k}^2)\right\}}. \tag{12}$$

Obviously, the weighted correlation coefficients of Eqs. (11) and (12) also satisfy these conditions:

(a) $W_1(S_1, S_2) = W_1(S_2, S_1)$ and $W_2(S_1, S_2) = W_2(S_2, S_1)$;

(b) $W_1(S_1, S_2) = W_2(S_1, S_2) = 1$ for $S_1 = S_2$;

(c) $W_1(S_1, S_2), W_2(S_1, S_2) \in [0, 1]$.

4. MCGDM approach based on weighted correlation coefficients of LNNs

This section proposes a MCGDM approach based on the weighted correlation coefficients of LNNs.

Regarding a MCGDM problem in LNN setting, there are the set of m alternatives represented by $S = \{S_1, S_2, \ldots, S_m\}$ and the set of n criteria represented by $E = \{E_1, E_2, \ldots, E_n\}$. Then, the set of d decision makers is denoted by $D = \{D_1, D_2, \ldots, D_d\}$. Thus, when the j-th decision maker D_j give the fit evaluations of each alternative S_i (i = 1, 2, …, m) over criteria E_k (k = 1, 2, …, n), his/her evaluation values are expressed by a LNS $S_i^j = \{s_{i1}^j, s_{i2}^j, \ldots, s_{in}^j\}$, where $s_{ik}^j = <v_{a_{ik}}^j, v_{b_{ik}}^j, v_{c_{ik}}^j>$ is a LNN obtained from the given linguistic term set $V = \{v_0, v_1, \ldots, v_q\}$ for $v_{a_{ik}}^j, v_{b_{ik}}^j, v_{c_{ik}}^j \in [v_0, v_q]$ (i = 1, 2, …, m; j = 1, 2, …, d; k = 1, 2, …, n). Thus, the j-th decision matrix of LNNs $R^j = \left(s_{ik}^j\right)_{m\times n}$ (j = 1, 2, …, d) can be constructed in LNN setting.

Suppose the weight vector of criteria is $\rho = (\rho_1, \rho_2, \ldots, \rho_n)$ for $\rho_k \in [0, 1]$ and $\sum_{k=1}^{n}\rho_k = 1$, and then the weight vector of decision makers is $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_d)$ for $\lambda_j \in [0, 1]$ and $\sum_{j=1}^{d}\lambda_j = 1$. In this decision making problem, we can propose a MCGDM approach based on the weighted correlation coefficients in LNN setting, which is depicted by the following steps:

Step 1: Based on Eq. (1), the aggregated LNN $v_{ik} = <v_{a_{ik}}, v_{b_{ik}}, v_{c_{ik}}>$ is obtained by the following weighted aggregation operator:

$$s_{ik} = LNNWAA(s_{ik}^1, s_{ik}^2, ..., s_{ik}^d) = \sum_{j=1}^{d} \lambda_j s_{ik}^j = \left\langle v_{q-q\prod_{j=1}^{d}\left(1-\frac{a_{ik}^j}{q}\right)^{\lambda_j}}, v_{q\prod_{j=1}^{d}\left(\frac{b_{ik}^j}{q}\right)^{\lambda_j}}, v_{q\prod_{j=1}^{d}\left(\frac{c_{ik}^j}{q}\right)^{\lambda_j}} \right\rangle. \tag{13}$$

Then, the aggregated matrix of LNNs is constructed as follows:

$$R = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_{mn} \end{bmatrix}.$$

Step2: Regarding the concept of the ideal solution (alternative), we can determine the ideal solution $S^* = \{s_1^*, s_2^*, ..., s_n^*\}$ from the aggregated matrix R, where $s_k^* = < v_{a_k^*}, v_{b_k^*}, v_{c_k^*} > = < \max_i(v_{a_{ik}}), \min_i(v_{b_{ik}}), \min_i(v_{c_{ik}}) >$ is the ideal LNN (k = 1, 2, …, n; i = 1, 2, …, m).

Step 3: Based on Eq. (11) or Eq. (12), the weighted correlation coefficient between S_i (i = 1, 2, …, m) and S^* is given by

$$W_1(S_i, S^*) = \frac{\sum_{k=1}^{n} \rho_k (a_{ik}a_k^* + b_{ik}b_k^* + c_{ik}c_k^*)}{\sqrt{\sum_{k=1}^{n} \rho_k (a_{ik}^2 + b_{ik}^2 + c_{ik}^2)} \sqrt{\sum_{k=1}^{n} \rho_k \left((a_k^*)^2 + (b_k^*)^2 + (c_k^*)^2\right)}}, \tag{14}$$

$$\text{or } W_2(S_i, S^*) = \frac{\sum_{k=1}^{n} \rho_k (a_{ik}a_k^* + b_{ik}b_k^* + c_{ik}c_k^*)}{\max\left\{\sum_{k=1}^{n} \rho_k (a_{ik}^2 + b_{ik}^2 + c_{ik}^2), \sum_{k=1}^{n} \rho_k \left((a_k^*)^2 + (b_k^*)^2 + (c_k^*)^2\right)\right\}}. \tag{15}$$

Step 4: The ranking order of all alternatives and the best one are given corresponding to the values of the weighted correlation coefficient.

Step 5: End.

5. Decision making example with LNN information

This section presents a decision making example regarding the MCGDM problem to illustrate the applicability of the proposed MCGDM method in LNN setting.

A hospital requires the human resources department to recruit a nurse. When the five candidates (the five alternatives) S_1, S_2, S_3, S_4, and S_5 are selected preliminarily from all applicants by the human resources department, a group of three experts/decision makers $D = \{D_1, D_2, D_3\}$ is invited to assess the five candidates corresponding to the three requirements (criteria): (a) E_1 is nursing skill; (b) E_2 is past nursing experience; (c) E_3 is self-confidence. The weight vector of the three criteria is provided by ρ = (0.4, 0.3, 0.3) and the weight vector of the three experts is given by λ = (0.35, 0.35, 0.3).

Then, the three experts are requested to suitably evaluate the five candidates from the predefined linguistic term set V = {v_0 = extremely poor, v_1 = very poor, v_2 = poor, v_3 = slightly poor, v_4 = fair, v_5 = slightly good, v_6 = good, v_7 = very good, v_8 = extremely good} for q = 8 in LNN setting, and then they give the following three LNN matrices:

$$R^1 = \begin{bmatrix} <v_5, v_1, v_2> & <v_6, v_2, v_2> & <v_6, v_2, v_3> \\ <v_7, v_2, v_2> & <v_7, v_2, v_2> & <v_7, v_3, v_2> \\ <v_5, v_1, v_2> & <v_6, v_2, v_4> & <v_7, v_1, v_3> \\ <v_6, v_2, v_3> & <v_6, v_2, v_4> & <v_6, v_2, v_3> \\ <v_4, v_3, v_4> & <v_6, v_3, v_4> & <v_6, v_4, v_4> \end{bmatrix},$$

$$R^2 = \begin{bmatrix} <v_6, v_2, v_3> & <v_5, v_3, v_4> & <v_6, v_2, v_3> \\ <v_6, v_1, v_3> & <v_7, v_2, v_3> & <v_7, v_2, v_1> \\ <v_6, v_2, v_1> & <v_6, v_2, v_3> & <v_7, v_1, v_3> \\ <v_5, v_1, v_2> & <v_6, v_2, v_1> & <v_6, v_2, v_2> \\ <v_5, v_3, v_4> & <v_6, v_3, v_3> & <v_6, v_3, v_3> \end{bmatrix},$$

$$R^3 = \begin{bmatrix} <v_6, v_2, v_3> & <v_5, v_4, v_3> & <v_6, v_1, v_3> \\ <v_7, v_2, v_2> & <v_6, v_3, v_2> & <v_7, v_2, v_1> \\ <v_6, v_1, v_1> & <v_5, v_1, v_4> & <v_7, v_1, v_2> \\ <v_7, v_2, v_3> & <v_6, v_2, v_2> & <v_6, v_2, v_3> \\ <v_5, v_3, v_4> & <v_6, v_3, v_2> & <v_6, v_3, v_3> \end{bmatrix}.$$

Thus, the proposed MCGDM approach can be applied to the decision making example, which is depicted by the following steps:

Step 1: By using Eq. (13), the aggregated matrix of LNNs is yielded as follows:

$$R = \begin{bmatrix} <v_{5.6478}, v_{1.5157}, v_{2.5508}> & <v_{5.4492}, v_{2.7808}, v_{2.7808}> & <v_{6.0000}, v_{1.6245}, v_{3.000}> \\ <v_{6.7689}, v_{1.6245}, v_{2.2587}> & <v_{6.7689}, v_{2.2587}, v_{2.2587}> & <v_{7.0000}, v_{2.3522}, v_{1.3195}> \\ <v_{5.6478}, v_{1.2311}, v_{1.3195}> & <v_{5.7413}, v_{1.6245}, v_{3.6693}> & <v_{7.0000}, v_{1.0000}, v_{2.6564}> \\ <v_{6.1654}, v_{1.6245}, v_{2.6564}> & <v_{6.0000}, v_{2.0000}, v_{2.1435}> & <v_{6.0000}, v_{2.0000}, v_{2.6564}> \\ <v_{4.6341}, v_{3.0000}, v_{4.0000}> & <v_{6.0000}, v_{3.0000}, v_{2.9804}> & <v_{6.0000}, v_{3.3659}, v_{3.3659}> \end{bmatrix}.$$

Step 2: Corresponding to the ideal LNN $s_k^* = <v_{a_k^*}, v_{b_k^*}, v_{c_k^*}> = <\max_i(v_{a_{ik}}), \min_i(v_{b_{ik}}), \min_i(v_{c_{ik}})>$ (k = 1, 2, 3; i = 1, 2, 3, 4, 5), the ideal solution is yielded from the aggregated matrix R as follows:

$$S^* = \{s_1^*, s_2^*, s_3^*\} = \{<v_{6.7689}, v_{1.2311}, v_{1.3195}>, <v_{6.7689}, v_{1.6245}, v_{2.1435}>, <v_{7.0000}, v_{1.0000}, v_{1.3195}>\}.$$

Step 3: By using Eq. (14) or Eq. (15), we can obtain the following weighted correlation coefficient values:

$W_1(S_1, S^*) = 0.9661$, $W_1(S_2, S^*) = 0.9908$, $W_1(S_3, S^*) = 0.9790$, $W_1(S_4, S^*) = 0.9787$, and $W_1(S_5, S^*) = 0.9082$; or $W_2(S_1, S^*) = 0.8985$, $W_2(S_2, S^*) = 0.9545$, $W_2(S_3, S^*) = 0.9334$, $W_2(S_4, S^*) = 0.9313$, and $W_2(S_5, S^*) = 0.8956$.

Step 4: Based on the above values, all the alternatives are ranked as $S_2 > S_3 > S_4 > S_1 > S_5$, and then the best candidate with the biggest value is S_2.

Clearly, the ranking orders of the candidates/alternatives and the best one corresponding to the proposed two correlation coefficients of LNNs are the same in this MCGDM example.

6. Comparison with MCGDM methods based on existing correlation coefficients of LNNs

To demonstrate the effectiveness of the proposed method in LNN setting, this section indicates the comparison of the proposed approach with the ones based on existing correlation coefficients of LNNs [32] by the above MCGDM example.

Thus, the correlation coefficient values between S_i and S^* are obtained by applying Eqs. (2)-(4), and then all the decision results based on various correlation coefficients of LNNs are tabulated in Table 1.

Table 1. Decision results based on various correlation coefficients of LNNs

Correlation coefficient	Correlation coefficient value	Ranking order	The best one
$M_1(S_i, S^*)$ [32]	0.8651, 0.9517, 0.9239, 0.8998, 0.8033	$S_2 > S_3 > S_4 > S_1 > S_5$	S_2
$M_2(S_i, S^*)$ [32]	0.2466, 0.2967, 0.2938, 0.2499, 0.2180	$S_2 > S_3 > S_4 > S_1 > S_5$	S_2
$M_3(S_i, S^*)$ [32]	0.7412, 0.8950, 0.8462, 0.8035, 0.6326	$S_2 > S_3 > S_4 > S_1 > S_5$	S_2
$W_1(S_i, S^*)$	0.9661, 0.9908, 0.9790, 0.9787, 0.9082	$S_2 > S_3 > S_4 > S_1 > S_5$	S_2
$W_2(S_i, S^*)$	0.8985, 0.9545, 0.9334, 0.9313, 0.8956	$S_2 > S_3 > S_4 > S_1 > S_5$	S_2

From Table 1, we can see that all the ranking orders and the best one are identical regarding the decision results based on various correlation coefficients of LNNs. Obviously, the proposed approach indicates its effectiveness. Thus, the proposed MCGDM approach provides another new effective way for the linguistic neutrosophic decision making problems in LNN setting.

7. Conclusion

As the complement/extension of our previous work [32], this study first presented two correlation coefficients of LNNs based on the correlation and information energy of LNNs. Then we presented a MCGDM approach using the weighted correlation coefficients in LNN setting. A decision making example regarding the MCGDM problem was presented to demonstrate the applicability of the proposed MCGDM approach in LNN setting. By comparison with the MCGDM approaches based on the existing correlation coefficients of LNNs, the decision results demonstrated the developed new approach is effective. Hence, the proposed MCGDM approach provides another new effective way for linguistic neutrosophic decision making problems. In the next work, we shall extend the proposed correlation coefficients to develop the refined linguistic neutrosophic correlation coefficients based on the refined neutrosophic concept [34] and to use them for decision making, pattern recognition, and medical diagnosis problems in refined linguistic neutrosophic setting.

Acknowledgement

This work was supported by the National Natural Science Foundation of China, Grant number: 61703280.

References

[1] F. Smarandache, Neutrosophy: Neutrosophic probability, set, and logic. American Research Press, Rehoboth, USA, 1998.

[2] Y. H. Guo, C. Zhou, H. P. Chan, A. Chughtai, J. Wei, L. M. Hadjiiski, E. A. Kazerooni, Automated iterative neutrosophic lung segmentation for image analysis in thoracic computed tomography. Medical Physics 40 (2013) 081912.

[3] Y. H. Guo, A. Sengur, J. W. Tian, A novel breast ultrasound image segmentation algorithm based on

neutrosophic similarity score and level set. Computer Methods and Programs in Biomedicine 123 (2016) 43–53.
[4] K. M. Amin, A. I. Shahin, Y. H. Guo, A novel breast tumor classification algorithm using neutrosophic score features. Measurement 81 (2016) 210–220.
[5] J. Ye, Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. Artificial Intelligence in Medicine 63(3) (2015) 171–179.
[6] J. Ye, J. Fu, Multi-period medical diagnosis method using a single valued neutrosophic similarity measure based on tangent function, Computer Methods and Programs in Biomedicine 123 (2016) 142-149.
[7] J. Fu, J. Ye, Simplified neutrosophic exponential similarity measures for the initial evaluation/diagnosis of benign prostatic hyperplasia symptom, Symmetry 9(8) (2017) 154.
[8] J. Ye, Fault diagnoses of steam turbine using the exponential similarity measure of neutrosophic numbers, Journal of Intelligent & Fuzzy Systems 30 (2016) 1927–1934.
[9] J. Ye, Single valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine, Soft Computing 21(3) (2017) 817-825.
[10] J. Ye, Fault diagnoses of hydraulic turbine using the dimension root similarity measure of single-valued neutrosophic sets. Intelligent Automation & Soft Computing 24(1) (2018) 1-8.
[11] J. Ye, Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. International Journal of General Systems 42 (2013) 386–394.
[12] J. Ye, Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making, Journal of Intelligent & Fuzzy Systems 16 (2014) 204–211.
[13] P. D. Liu, Y. M. Wang, Multiple attribute decision making method based on single-valued neutrosophic normalized weighted Bonferroni mean, Neural Computing & Applications 25 (2014) 2001–2010.
[14] P. D. Liu, Y. C. Chu, Y. W. Li, Y. B. Chen, Some generalized neutrosophic number Hamacher aggregation operators and their application to group decision making, Journal of Intelligent & Fuzzy Systems 16 (2014) 242–255.
[15] A. W. Zhao, J. G. Du, H. J. Guan, Interval valued neutrosophic sets and multi-attribute decision-making based on generalized weighted aggregation operator, Journal of Intelligent & Fuzzy Systems 29 (2015) 2697–2706.
[16] H. X. Sun, H. X. Yang, J. Z. Wu, O. Y. Yao, Interval neutrosophic numbers Choquet integral operator for multi-criteria decision making, Journal of Intelligent & Fuzzy Systems 28 (2015) 2443–2455.
[17] J. J. Peng, J. Q. Wang, J. Wang, H. Y. Zhang, X. H. Chen, Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. International Journal of Systems Science 47 (2016) 2342–2358.
[18] P. D. Liu, Y. M. Wang, Interval neutrosophic prioritized OWA operator and its application to multiple attribute decision making. Journal of Systems Science and Complexity 29 (2016) 681–697.
[19] P. Biswas, S. Pramanik, B. C. Giri, TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. Neural Computing & Applications 27 (2016) 727–737.
[20] J. Ye, Simplified neutrosophic harmonic averaging projection-based method for multiple attribute decision making problems. International Journal of Machine Learning and Cybernetics 8 (2017) 981–987.
[21] A. Tu, J. Ye, B. Wang, Symmetry measures of simplified neutrosophic sets for multiple attribute decision-making problems. Symmetry 10 (2018) 144.
[22] W. H. Cui, J. Ye, Improved symmetry measures of simplified neutrosophic sets and their decision-making method based on a sine entropy weight model. Symmetry 10(6) (2018) 225.
[23] Y. X. Ma, J. Q. Wang, J. Wang, X. H. Wu, An interval neutrosophic linguistic multi-criteria group decision-making method and its application in selecting medical treatment options. Neural Computing & Applications 28(9) (2017) 2745–2765.
[24] J. Ye, An extended TOPSIS method for multiple attribute group decision making based on single valued neutrosophic linguistic numbers. Journal of Intelligent & Fuzzy Systems 28(1) (2015) 247–255.
[25] J. Ye, Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making. Journal of Intelligent & Fuzzy Systems 27(5) (2014) 2231–2241.
[26] J. Ye, Hesitant interval neutrosophic linguistic set and its application in multiple attribute decision making. International Journal of Machine Learning and Cybernetics 10(4) (2017) 667-678.
[27] S. Broumi, F. Smarandache, Single valued neutrosophic trapezoid linguistic aggregation operators based multi-attribute decision making. Bull Pure Applied Sciences-Mathematics & Statistics 33(2) (2014) 135–155.
[28] S. Broumi, J. Ye, F. Smarandache, An extended TOPSIS method for multiple attribute decision making based on interval neutrosophic uncertain linguistic variables. Neutrosophic Sets and Systems 8 (2015)

23–32.
[29] Z. B. Fang, J. Ye, Multiple attribute group decision-making method based on linguistic neutrosophic numbers. Symmetry 9(7) (2017) 111.
[30] C. X. Fan, J. Ye, K. L. Hu, E. Fan, Bonferroni mean operators of linguistic neutrosophic numbers and their multiple attribute group decision-making methods. Information 8(3) (2017) 107.
[31] L. L. Shi, J. Ye, Cosine measures of linguistic neutrosophic numbers and their application in multiple attribute group decision-making. Information 8(4) (2017) 117.
[32] L. L. Shi, J. Ye, Multiple attribute group decision-making method using correlation coefficients between linguistic neutrosophic numbers, Journal of Intelligent & Fuzzy Systems 35(1) (2018) 917-925.
[33] P. D. Liu, X. L. You, Bidirectional projection measure of linguistic neutrosophic numbers and their application to multi-criteria group decision making, Computers & Industrial Engineering 128 (2019) 447-457.
[34] F. Smarandache, n-Valued refined neutrosophic logic and its applications in physics, Progress in Physics 4 (2013) 143-146.