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THE WAY-BELOW SOFT SET RELATION

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Abstract

Order is used frequently in mathematical objects and related fields. We use order in the soft set that is introduced by Molodtsov [1]. Babitha and Sunil [2] gave the definition of soft set relation, then Babitha and Sunil [3] defined partially ordered soft set. We introduced directed soft sets in [4]. Moreover, definition of soft lattice, complete soft lattice were given in [5]. We made a presentation, whose abstract was published only in the Abstract Book, in ICRAPAM 2014 [6] about the way-below soft set relation. In this paper, this study is extended and the proofs of all results of our presentation are given. Moreover, continuous soft lattices, L-soft domain are defined, and some important results are obtained and proved in this study.

Keywords: Soft set relation, Directed soft set, Soft domain, Way-below soft set relation

WAY-BELOW ESNEK KÜME BAĞINTISI

Özet

Sıralama, matematiksel objeler ve ilgili alanlarda sıklıkla kullanılmaktadır. Molodtsov [1] tarafında ortaya çıkartılan esnek kümelerde sıralama kullanıyoruz. İlk olarak Babitha and Sunil [2] esnek küme bağıntısı tanımını verdikten sonra Babitha and Sunil [3] kısmi sıralı esnek kümeyi tanımlamışlardır. "New structures on partially ordered soft sets and soft Scott topology" [4] isimli çalışmamızda, yönlendirilmiş esnek kümeleri tanıttık. Bundan başka, "Some new results on orderings on soft sets" [5] isimli çalışmamızda esnek latis, tam esnek latis tanımları verildi. Way-Below Esnek Küme bağıntısı hakkında ICRAPAM 2014'te Konferans Özet Kitabında sadece özetinin yayınlandığı bir sunum yaptık [6]. Bu makalede, bu çalışma genişletildi ve sunumumuzdaki tüm sonuçların ispatları verildi. Ayrıca, sürekli esnek latis, L-esnek domain tanımlandı ve bazı önemli sonuçlar elde edilerek ispatlandı.

Anahtar Kelimeler: Esnek küme bağıntısı, Yönlendirilmiş esnek küme, Esnek domain, Way-below esnek küme bağıntısı

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1. Introduction

In the social world, usually exact solution cannot be determined for the problems, so the social world problems have to be solved approximately. D. Molodtsov [1] introduced a new theory that overcomes uncertainties in the social world problem by approximating initial universe in 1999. Maji et. al. [7] defined many operators on soft sets and established some results on them. Çağman and Enginoğlu [8] introduced a matrix representation of soft sets and by using matrix representations they made soft sets have a rich computer application potential. In recent years, the properties and applications on the soft set theory [9,10,11,12,13,14,15,16,17,18] and the fuzzy soft set theory have been studied increasingly [19,20,21,22,23,24].

Order is used frequently in mathematical objects and related fields. We use order in the soft set that is introduced by Molodtsov [1]. Firstly, Babitha and Sunil [2] gave definition of soft set relation, then Babitha and Sunil [3] introduced partially ordered soft set. Ibrahim et.al. [25] gave the matrix representation of soft set relations and they studied soft set relations by using this matrix representations. Soft lattice was introduced by Li [26] by taking initial universe a complete lattice. Jun, et.al. [13] defined soft ordered semigroups and idealistic soft ordered semigroups on semigroups, but Ali [27] gave some results on soft ordered semi-groups, examined some results of soft ordered semigroups which are mentioned in [13], strengthened these results and gave the definition of soft filter on ordered semigroups. Yang and Guo [28] introduced some further definitions about soft set relations such as kernels and closures of soft sets. Zhang and Gao [29] obtained lattice and topological structures of soft sets over 2^U. Moreover Karaaslan, et. al. [30] defined soft lattices by using two binary operations on subsets of the all soft set over U and Jobish, et. al. [31] defined some basic operations in soft lattice. Park, et. al. [32] studied equivalence soft relation and they proved that "poset(ESSR(<F,A>), ⊆) of the equivalence soft set relations on given soft set <F,A> is a complete lattice with the least element and the greatest element". Furthermore Tanay and Yaylali introduced directed soft sets in [4] and gave the definition of soft lattice, complete soft lattice [5]. Sayed [33] established some characterization theorems for continuity for partially ordered soft sets. We refer to [34,35] for the fundamental concepts of the set theory used in this paper.

In the International Conferences on Recent Advances in Pure and Applied Mathematics, Antalya, Turkey (2014) we introduced the definitions of "way-below soft set relation, soft continuous directed complete soft sets, soft domains" and gave some results without any proofs in our oral presentation whose abstract was published only in the Abstract Book of ICRAPAM 2014. One of these new soft set definitions, way-below soft set relation, was expressed simultaneously by Sayed [33] as "F(x) is approximate to F(y)". In this paper we extend the abstract and give the proofs of all results of this presentation. Moreover, continuous soft lattices, L-soft domain are defined and some important results are obtained and proved in this study. Some definitions and results in [6] which are presented in ICRAPAM 2014, were used in [36] to obtain some results about the relation between "way-below soft set relation" and "Soft Scott Topology".

2. Material and Method

Molodtsov [1] defined soft set as "a pair (F, A), by taking U as an initial universe, E as the set of parameters where $F: A \rightarrow P(U)$ is a set-valued function". Maji et. al. (2003) called "a soft set (F,A) is a null soft set if for every ε∈A, F(ε)=∅ and it is denoted by Φ". In this study Maji et. al. [7] called "(F, A) is a soft subset of (G, B) over a common universe U , if

i. $A \subseteq B$
ii. $\forall a \in A, F(a)$ and $G(a)$ are identical approximations.

And it is denoted by $(F, A) \widetilde{\subseteq} (G, B)$."

After these studies, Babitha and Sunil [2] introduced the cartesian product of two soft sets and soft set relation. "Let (F,A) and (G,B) be two soft sets over U. Then the cartesian product of (F,A) and (G,B) is defined as a soft set (H,A×B)=(F,A)×(G,B) where H:A×B→P(U×U) and H(a,b)=F(a)×G(b), for (a,b)∈A×B e.i. H(a,b)={(h_i,h_j)|h_i∈F(a), h_j∈G(b)}". "A soft set relation R from (F,A) to (G,B) is a soft subset of (F,A)×(G,B)". Also Babitha and Sunil [2] were called a soft set relation "R is reflexive if $H_1(a, a) \in R$", "R is symmetric if $H_1(a, b) \in R \Rightarrow H_1(b, a) \in R$" and "R is transitive if $H_1(a, b) \in R$, $H_1(b, c) \in R \Rightarrow H_1(a, c) \in R$" for every a,b,c ∈ A.

Babitha and Sunil (2011), in their latter study, called "a soft set relation R is an antisymmetric on a soft set (F,A) if $H_1(a, b) \in R$, $H_1(b, a) \in R \Rightarrow F(a) = F(b)$". Beside, "with a reflexive, antisymmetric and transitive soft set relation R, (F,A,R) is called a partially ordered soft set relation" which is defined by Babitha and Sunil [3]. Moreover Babitha and Sunil [3] gave the least and greatest element definitions for a partially ordered soft set (F,A, ≤) as follows:,

"For a∈A, F(a) is the least element of (F,A) in the ordering '≤' if F(a)≤F(x) for all x∈A. For a∈A, F(a) is the greatest element of (F,A) in the ordering '≤' if for every x∈A, F(x)≤F(a)."

According to the previous studies, "preordered soft set which is a soft set (F, A) equipped with reflexive, transitive soft set relation ≤", is given by Tanay and Yaylalı [4]. Also in a preordered soft set (F,A), "let (G, B) be a soft subset of (F,A). For a ∈ A, F(a) is called a lower bound of (G,B) if F(a)≤G(x) for all x∈B and F(a) is called infimum of (G,B) if it is the greatest element of the set of all lower bounds of (G,B) in (F,A,≤)". Similarly, "for a∈A, F(a) is called an upper bound of (G,B) if G(x)≤F(a) for all x∈B," and "F(a) is called supremum of (G,B) in (F,A,≤) if it is the least element of the set of all upper bounds of (G,B) in (F,A,≤)". Beside these, Tanay and Yaylalı [4] defined "finite soft set which is a soft set with a finite parameter set". And Tanay and Yaylalı [4] use this definition to introduce a directed soft set (G,B) in a preordered soft set (F,A), such that "$(G, B) \neq \Phi$ and every finite soft subset of (G,B) has an upperbound in (G,B)". Furthermore Tanay and Yaylalı [4,5] gave some basic definitions such as "lower soft set, soft ideal, soft lattice, directed complete partially ordered soft set" in orderings on the soft sets as follows. Let (G,B) is a soft subset of a preordered soft set (F,A).

"↓(G,B)=(H,C) where C ={a∈A:F(a)≤G(b) for some b∈B} and H=F|C ."

"(G,B) is a called lower soft set iff (G,B)=↓(G,B)". "(G,B) is called an ideal iff it is a directed lower soft set". And "an ideal is called principal iff it has a maximum element".

"A partially ordered soft set (F,A,≤) is called soft inf-semilattice (soft sup-semilattice) if F(a), F(b) have infimum (supremum) for any two elements a, b ∈ A". "If a partially ordered soft set is both soft inf-semilattice and soft sup-semilattice, it is called a soft lattice."

"If every directed soft subset of a partially ordered soft set has a supremum, it is called directed complete soft sets". "A partially ordered soft set which is an soft inf-semilattice and directed complete is called a directed complete soft inf-semilattice". And "if every soft subset of a soft lattice has a supremum and infimum, it is called complete soft lattice".

3. Results

We will use the notation F(a)F(b) for inf{F(a),F(b)}.

Definition 1. Let (F,A) be an inf soft set semilattice. If (F,A) is directed complete soft set and if it satisfies

$$F(x)\sup(G,B)=\sup(F(x),(G,B)) \qquad (1)$$

for all x∈A and all directed soft subsets (G,B) of (F,A), it is called soft meet continuous.

We will say that a sup soft set semilattice (F,A) is soft join continuous iff (F,A,$\leq^{op}$) is soft meet continuous. A soft lattice (F,A)is soft meet continuous if it is complete soft lattice satisfying (1).

Example 1. Consider the soft set (F,A) over the initial universe U, where U = $\{h_1, h_2, h_3, h_4, h_5, h_6\}$, $A = \{a_1, a_2, a_3\}$ and F(a_1) = $\{h_2, h_5\}$, F(a_2) = $\{h_1\}$, F(a3) = $\{h_1, h_4, h_5\}$ and soft set relation ≤= { F(a_1)×F(a_2), F(a_2)×F(a_3), F(a_1)× F(a_3), F(a_1)×F(a_1), F(a_2)×F(a_2), F(a_3)×F(a_3)} be given.

One can easily show that (F,A) is an inf-semi soft lattice and it is directed complete soft set. Now let us show that (F,A) is meet continuous soft set.

For all $x \in A$ and for all directed soft subset (G,B) of (F,A), in this example all soft subsets of (F,A) are directed, $F(x)sup(G,B) = sup(F(x)(G,B))$ need to be demonstrated.

For a directed soft subset {F(a_2)} and F(a_1), $F(a_1)sup\{F(a_2)\}=inf(F(a_1),F(a_2))=F(a_1)=sup\{inf(F(a_1),\{F(a_2)\})\}$.

For a directed soft subset $\{F(a_2),F(a_3)\}$ and $F(a_1)$,
$F(a_1)sup\{F(a_2),F(a_3)\} = inf\{F(a_1),F(a_3)\} = F(a_1)$ and
$sup(F(a_1)\{F(a_2),F(a_3)\}) =$
$sup\{inf\{F(a_1),F(a_2)\},inf\{F(a_1),F(a_3)\}\} =$
$sup\{F(a_1),F(a_1)\}$=F(a_1). Thus
$F(a_1)sup\{F(a_2),F(a_3)\}=sup(F(a_1)\{F(a_2),F(a_3)\})$ is obtained.

Other situations are obtained in a similar way.

Since $F(x)sup(G,B) = sup(F(x)(G,B))$, for all $x \in A$ and for all directed soft subset (G,B) of (F,A), (F,A) is meet continuous soft set.

Following theorems were presented in ICRAPAM 2014 without any proof. In this study proofs are given.

Theorem 1. "In a directed complete soft semilattice (F,A) the following conditions are equivalent:

1. (F,A) is soft meet continous;
2. For each directed soft set (D,C) and each F(a)≤sup(D,C) we have F(a)≤sup(F(a)(D,C)) (hence F(a)=sup(F(a)(D,C)))."

Proof.

1⇒2 Let (D,C) be a directed soft set and F(a)≤sup(D,C) then F(a)≤F(a)sup(D,C)=sup(F(a)(D,C)).

2⇒1 Let (D,C) be a directed soft set and F(a)≤sup(D,C) then F(a)=F(a)sup(D,C), by (2) F(a)=F(a) sup(D,C)=sup(F(a)(D,C)).

□

Definition 2. Let (F,A,≤) be a partially ordered soft set. F(a) is way-below F(b) if and only if all directed soft subsets $(D,C) \tilde{\subseteq} (F,A)$ for which sup(D,C) exists, F(b) ≤ sup(D,C) implies the existence of D(c) in (D,C) with F(a) ≤ D(c). It is denoted by F(a) ≪ F(b).

This definition was expressed concurrently by Sayed [30] as "Let (F,A) be a posset. For any two elements F(x),F(y) ∈ (F,A). F(x) is approximate to F(y), and write F(x) ≪ F(y), if for any directed soft subset (G,B) ⊂ (F,A) with ∨(G, B) existing and F (y) ≤ ∨(G, B), there exists G(z) ∈ (G, B) such that F(x)≤G(z). "

Example 2. Consider the soft set (F,A) given in the Example 1. Now we will show F(a_1) ≪ F(a3). By Definition 2, we need to show for all directed soft subsets (D,B) of (F,A) with F(a3)≤sup(D,B), there exists an element D(b) in (D,B) such that F(a_1)≤D(b).

- (D_1,B_1) ⊆̃ (F,A) such that B_1 = $\{a_1, a_3\}$.sup(D_1,B_1) = F(a_3) such that F(a_3) ≤ sup(D_1, B_1) and F (a_1) ≤ F (a_3).
- (D_2,B_2) ⊆̃ (F,A) such that B_2 = $\{a_2, a_3\}$. sup(D_2,B_2) = F(a_3) such that Fa_3≤sup(D_2,B_2) and F(a_1)≤F(a_2)
- (D_3,B_3) ⊆̃ (F,A) such that B_3 = $\{a_3\}$. sup(D_3, B_3) = F(a_3) such that F(a_3) ≤ sup(D_3,B_3) and F(a_1) ≤ F(a_3).
- (F,A) ⊆̃ F,A). sup(F,A)=F(a_3) such that F(a_3)≤sup(F,A) and F(a_1)≤F(a_2).

For this example the way below soft set relation is obtained as

≪={F(a_1)×F(a_1), F(a_2)×F(a_2), F(a_3)×F(a_3), F(a_1)×F(a_2), F(a_2)×F(a_3), F(a_1)× F(a_3)}.

Theorem 2. "Let (F,A) be a partially ordered soft set. For all F(x),F(y),F(z) in (F,A), the following statements hold.

1. F(x)≪F(y)⇒F(x)≤F(y);
2. F(u)≤F(x)≪F(y)≤F(v)⇒F(u)≪F(v);
3. F(x) ≪ F(z) and F(y) ≪ F(z) ⇒ sup{F(x), F(y)} ≪ F(z) whenever sup{F(x), F(y)} exists.
4. If (F,A) has a smallest element F(0); F(0)≪F(x)."

Proof.

1. Suppose F (a) ≪ F (b). Take directed soft subset (G, B) ⊆̃ (F, A) for which sup(G, B) exists and F (b) = sup(G, B). Then since F (a) ≪ F (b), there exists G(d) in (G,B) such that F(a) ≤ G(d) and also we know G(d) ≤ sup(G, B) hence we get F(a)≤F(b).
2. Since for all directed (G,B) which has sup(G,B), F(a) ≤ sup(G,B) then there exists c ∈ B such that F(a) ≤ G(c). Now take directed (H,C) which has a supremum such that F (v) ≤ sup(H, C). This implies F (b) ≤ sup(H, C), then there exists d ∈ C such that F (a) ≤ F (v) by definition.
3. Take a directed soft set (G, B) such that F (c)≤sup(G,B), then there exists G(d_1) and G(d_2) in (G,B) such that F(a)≤G(d_1) and F(b)≤G(d_2). Since sup{F(a), F(b)} ≤ sup{G(d_1), G(d_2)} and since (G, B) is a directed soft set sup{G(d_1), G(d_2)}∈(G, B). Thus sup{F(a), F(b)}≪F(c).
4. Take (G,B) directed soft subset of (F,A) such that F (a) ≤ sup(G,B). Since F(0) is the smallest element hence F(0)≤F(x) for all x∈B. Thus there exists b∈B such that F(0)≤G(b), then F(0)≪F(a). □

Note that 1. We write

" ↡F(a)={F(x)|x∈A with F(x) ≪ F(a)}"

"↟F(a) = {F(x)|x∈A with F(a) ≪ F(x)}"

Note that 2. "For all F (x) in a complete soft semilattice (F, A), the set ↡ F (x) is an ideal contained in ↓ F (x). If F(x) ≤ F(y) then ↡ F(x) ⊆̃ ↡ F(y)."

Proof. First , show that ↡F(x) is an soft ideal (directed lower soft set). We know ↡F(x)={F(a)|a∈A with F(a)≪F(x)}. Take F(a) and F(b) in ↡ F(x). By the Theorem 2 sup{F(a), F(b)}≪F(x), then we get sup{F(a), F(b)}∈↡F(x). So ↡F(x) is a directed soft set.

↡ F(x) ⊆̃↓(↡F(x)). Now we need to show ↓(↡F(x)) ⊆̃↡F(x). Take F(u)∈↓(↡F(x)) then F(u)≪F(v) for some F(v)∈↡F(x) then F(v)≪F(x). Take a directed soft subset (G,D) of (F,A) such that F(x) ≤sup(G,D), then there exists F(d), d∈D with F(v)≤F(d) then F(u)≤F(v)≤F(d). This implies F(u)≪F(x) then F(u)∈↡F(x). So ↓(↡F(x)) ⊆̃ ↡F(x). Thus ↡ F(x) is a lower soft set. Let F(x)≤F(y). Take F(u)∈↡F(x), then F(u)≪F(x). Now take a directed soft subset (G,B) of (F,A) such that F(y)≤sup(G,D) then F(x)≤sup(G,D) then there exists d∈D such that F(u)≤F(d) then F(u)≪F(y). So F(u)∈↡F(y). ▯

Theorem 3. "Let (F,A) be partially ordered soft set.

i. The conditions (1) and (2) are equivalent;
 1. F(a)≪F(b),
 2. F(a) in (I,B) for every soft ideal (I,B) of (F,A) such that F(b)≤sup(I,B).
 Conditions (1) and (2) are equivalent to (3), if (F,A) is a meet continuous soft semilattice.
 3. F(a) in (I,B) for every soft ideal (I,B) of (F,A) such that F(b)=sup(I,B).

ii. Let there exists a directed soft subset (D,C) of ↡F(a) with sup(D,C)=F(a). Moreover, F(b)≪F(a) if F(b)≪F(a) in a partially ordered soft set ↓F(a) with the induced order."

Proof.

i. (1)⇒ (2): Let F(a)≪F(b), F(b)≤sup(I,B) for a soft ideal (I,B) of (F,A) then there exists I(b') in (I,B) such that F(a)≤I(b'), since (I,B) is a soft ideal then ↓I(b') ⊆̃(I,B) then F(a)∈↓I(b') ⊆̃(I,B).
 (2) ⇒ (1): Assume (2) and let (D,C) be a directed soft subset with F(b)≤sup(D,C). Then F(a) in (I,B) by (2), i.e., there is a D(c) in (D,C) such that F(a) ≤D(c). Hence F(a)≪F(b).
 For (3) we have only to remark that F(b)≤sup(I,B) is equivalent to F(b) = supF(b)(I,B) in a meet continuous lattice.

ii. Let F(b)≪F(a), F(c)≪F(a). Then F(b)≤D(b') and F(c)≤D(c') for some D(b'), D(c') in (D,C). Pick D(e) in (D,C) such that D(b')≤D(e) and D(c')≤D(e). Then F(b)≤D(e), F(c)≤D(e), and D(e)≪F(a). Thus, ↡F(a) is a directed soft set. If F(b)≪F(a) in ↓F(a), then F(b)≤D(e)≪F(a) for some D(e) in (D,C), so F(b)≪F(a) in (F,A). ▯

Definition 3.

1. [33] "If a partially ordered soft set (F,A,≤) satisfies the axiom of approximation:
 $\forall F(a)\ in\ (F,A), F(a) = \vee^{\uparrow}$↡ F(a)
 i.e. for all $a \in A$, the soft set ↡F(a)=(G,B) such that B={b∈A|F(b)≪F(a)}, G=F|B, and ↡F(a) is directed and F(a)=sup(G,B), then (F,A) is called soft continuous."
2. [33] "A continuous directed complete partially ordered soft set is called soft set domain."
3. A soft set domain which is complete soft set lattice is called continuous soft lattice.
4. A complete soft semilattice which is a soft set domain as a partially ordered soft set is called a complete continuous soft semilattice or alternatively bounded complete soft domain.
5. A soft domain in which every principal soft ideal ↡F(x) is complete soft lattice is called an L-soft domain.

Example 3. Consider the soft set (F,A) given in Example 1.

$\vee^{\uparrow}$↡ F(a_1) $= sup^{\uparrow}$\{F(a_1)\} $= F(a_1)$

$\vee^{\uparrow}$↡ F(a_2) $= sup^{\uparrow}$\{F(a_1), F(a_2)\} $= F(a_2)$

$\vee^{\uparrow}$↡ F(a_3) $= sup^{\uparrow}$\{F(a_1), F(a_2), F(a_3)\} $= F(a_3)$

So for all $a \in A, , F(a) = sup$ ↡ F(a) and ↡F(a) is directed. Therefore (F,A) is soft continuous. Further it is a soft set domain, since it is directed complete soft set.

Theorem 4. "Every continuous soft semilattice, hence every continuous soft lattice is meet continuous soft set."

Proof. Let (F, A) be a continuous soft semilattice. Assume that F(a)≤sup(D,C), where (D,C) is a directed soft set. We need to show ↡F(a) ⊆̃↓F(a)(D,C). If F(b)≪F(a), then F(b)≤F(a) but F(b)≤D(c) for some D(c) in (D,C). As F(b)≤F(a)D(c)∈F(a)(D,C), it follows that F(b)≤supF(a)(D,C). Then F(a)≤supF(a)(D,C). This implies by Theorem 3 (F,A) is a meet lattice.▯

The following theorem exhibits an important property of the way-below soft set relation on soft continuous domains, the soft interpolation property.

Theorem 5. "

1. For a directed soft set (D,C) in a continuous partially ordered soft set (F,A), if F(a)≪F(c) and F(c)≤sup(D,C), then F(a)≪D(b) for some D(b) in (D,C).
2. [30] Let (F,A) be a continuous partially ordered soft set. The way-below soft set relation satisfies the following soft interpolation property
 F(a)≪F(c) implies (∃F(b)) F(a)≪F(b)≪F(c) (SINT)"

Proof.

1. Let (D,C) be a directed soft set with F(c)≤sup(D,C), and let (I,B)=⋒̃{↡D(b)} in (D,C)}. By continuity, sup(I,B)=sup(D,C) and, being a soft union of directed soft family of soft ideals, (I,B) is a soft ideal. Hence, if F(a)≪F(c) then F(a) in (I,B), which means that F(a)≪D(b) for some D(b) in (D,C).
2. Follows from (1) by choosing (D,C) =↡F(c) and recalling that F(c)=sup↡F(c) by the continuity of (F,A). ▯

4. Conclusion

In this study, we introduced meet continuous soft set which was defined by the help of infimum and supremum. Also way below soft set relation, continuous soft lattices, L-soft domain and some examples were given. Furthermore we proved some properties about way-below soft set relation such as the link between partial order ≤ and way below soft set relation ≪ which is obtained from ≤. In addition to these comparison of continuous soft set and meet continuous soft set was reported. Following this study, one can study further properties of way below soft set relation and obtain new soft set relations. Beside these one can obtain a soft topology pertaining to the way below soft set relation.

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