

COMPARISON OF ATA METHOD AND CROSTON BASED METHODS ON FORECASTING OF INTERMITTENT DEMAND

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Abstract

Intermittent demand forecasting is crucial for firms and commercial activities. Recently, many researchers have focused on forecasting methods for intermittent demand and proposed various forecasting techniques. The most prominent methods among these proposed techniques are the Croston method, which is based on exponential smoothing, and its two popular variations: SBA (Syntetos-Boylan Approximation), SBJ (Shale-Boylan-Johnston Approximation). Croston method is widely used in forecasting of intermittent demand and inventory (stock) control. Since these demands usually include zero values, using the ground breaking method developed by Croston in this data becomes inevitable. Nevertheless, there are some shortcomings to this method such as producing biased forecasts and for this reason its variations have been proposed. ATA method is a recently developed forecasting method which is an alternative to exponential smoothing. In this paper we propose a modification of ATA method that can be used for forecasting of intermittent demand. We will compare the results of the proposed approach to those of Croston and other forecasting methods used for intermittent demand forecasting.

Keywords: Exponential smoothing, Demand forecasting, Time series, Croston method, Intermittent demand, Inventory control

KESİKLİ TALEPLERİN TAHMİNLENMESİNDE ATA METOT VE CROSTON TEMELLİ METOTLARIN KARŞILAŞTIRILMASI

Özet

Kesikli talep tahmini, şirketler ve ticari faaliyetler için çok önemlidir. Son zamanlarda, birçok araştırmacı kesikli talep için tahmin yöntemlerine odaklandı ve çeşitli tahmin teknikleri önerdiler. Bu önerilen teknikler arasında öne çıkan yöntemler, üssel düzleştirmeye dayanan Croston yöntemi ve bu yöntemin iki türevidir: SBA (Syntetos-Boylan Yaklaşımı) ve SBJ (Shale-Boylan-Johnston Yaklaşımı) metotlarıdır. Croston yöntemi, kesikli talep ve envanter (stok) kontrolünün tahmininde yaygın olarak kullanılmaktadır. Bu talepler genellikle sıfır değerini içerdiğinden, bu verilerde Croston tarafından geliştirilen öne çıkan metodun kullanılması kaçınılmaz hale gelir. Bununla birlikte; bu yöntemin, yanlış tahminler üretmek gibi bazı eksiklikleri vardır ve bu sebeple türevleri önerilmiştir. ATA metot, üssel düzleştirmeye alternatif olarak yeni geliştirilen bir tahmin metodudur. Bu çalışmada, kesikli talebin tahmin edilmesi için bir ATA yönteminin bir modifikasyonunu öneriyoruz. Önerilen yaklaşımın sonuçlarını, Croston ve kesikli talep tahmini için kullanılan diğer tahmin yöntemleriyle karşılaştıracaktır.

Anahtar Kelimeler: Croston metot, Envanter kontrolü, Kesikli talep, Talep tahminlemesi, Üssel düzleştirme, Zaman serisi

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1. Introduction

Intermittent demand that shows no demand any interval seems randomly. It is often called as sporadic demand. On intermittent demand data sets, demand which is observed during specific periods is zero interspersed by periods with regular or irregular non-zero demand [1]. These data set constitute of time series data most of which refer to non-negative values and the

others are zero [2]. These demands are referred often "lumpy" or "erratic" because there are great variabilities between the nonzero values [3]. This type of demand occurs in various realms. [3] described intermittent demand in various field, especially heavy machinery and aviation, maritime, electronics, etc.; while [4] studied intermittent demands in the automotive spare parts [5]. Similarly, [6] characterized a significance of intermittent

demands in the field of aviation, just as [7] mentioned that spare parts often exhibited intermittent demand and were in particular prevailing in the aerospace, automotive, military and information technology sectors [5].

Intermittent demand accompanies with important problems in the manufacturing and supply realm when forecasting and stock controls are handled. It is not only the variability of the demand sizes, but also the volatility of the demand pattern that make this demand so hard to forecast [1], [8]. The firms commonly confront a problem which has the proportionally high percentage of items which is called erratic or lumpy demand at the process of obtaining accurate forecasts. Hence, in order to promote inventory holding and replenishment decisions and overcome these shortcomings, accurate demand forecasts are crucial [9].

1.1. Literature Review

There are several approaches that deal with intermittent demand data one of them being traditional exponential smoothing. Conventional forecasting methods are frequently based on some assumptions that are charged of being unsuitable for items with an erratic demand pattern. The first systematic approach to cope with intermittent demand data was presented by Croston [10] and later it was improved by Rao [11]. [10] draw an attention the deficiencies of exponentially weighted moving averages to approach the underlying demand pattern. For these items, many of the observed demand during many periods would be zero [12]. Croston [10] proposed the decomposition of these data into two separate series i.e. corresponding to the nonzero demand sizes (Z) and the inter-demand intervals (\hat{r}) [2], [13].

Because intermittent time series have variability in demand size in addition to demand interval, there is special challenge to cope with these items. So, in this area (forecasting of intermittent demand) interested a substantial amount of academic researches [1], [3], [4], [9] etc. Many of these works on intermittent demand forecasting are established on Croston's innovative article. The studies on intermittent demand assume that there are independence between successive demand sizes and successive demand intervals, since most of the forecasting and inventory control theory, in particular Croston method, is established on the assumption that successive demand values are independent [1], [9]. Croston's method was alleged to be unbiased but despite its theoretical superiority modest benefits were registered in the literature [14]. Particularly, [8] proved that the Croston's method is positively biased (i.e. over-forecasting mean demand) and then [15] proposed a Modified Croston's method by using earlier work by [16]. The aforementioned method was only a modification of the original Croston's method and was proven by [4] to be more biased than the original

estimator. Right after, they proposed an "approximately" unbiased estimator: SBA (Syntetos-Boylan Approximation, [4]). Later, [17] revealed the expected bias in Croston's method and proposed an "exact" correction factor, SBJ (Shale-Boylan-Johnston Approximation). The only difference between these two approximations is the correction factor. The SBA procedure was further evaluated by [18] and it was found to perform better to previous approaches. Moreover, according to these studies, Croston's original method presents smaller bias if hardly ever demands are zero, whereas SBA modification has a smaller bias if many demands are zero [19]. Furthermore, the modification considerably enhances the forecasting performance of the original Croston method, [20]. However, it is a special case of it and some (negative) bias remains. In addition; it is later proven that the Syntetos - Boylan Approximation (SBA) may be more biased than the original Croston method [18], [21], [22]). Hence, [21] proposed a novel method which is the TSB (Teunter - Syntetos - Babai) method. This was not originally proposed for intermittent demand, but does update the forecast after every period, so it can be thought of as the naïve method that uses the last observation of demand as the forecast for future periods. Nevertheless, [21] suggested that their new method can be used as an alternative to Croston and SBA methods, but only presented results from simulated data [19].

To sum up, although there are several modifications of the original Croston method, all of them are built on similar concepts: either separating the non-zero demands from the intervals or modelling the probability to have a non-zero demand [13]. Thus, in this paper, we will compare the forecasting performances of the Croston method, which is an essential and the first systematic method for intermittent demand data forecasting, the SBA and SBJ methods that are widely used, and the proposed modification of the ATA method for the M4-competition inventory data set.

2. Intermittent Demand

Nowadays, many stock control systems may involve thousands of observations, many of which show very few demand. In addition, we can say that these items may be requested rarely. When events corresponding to positive demands (not negative) occur solely sporadically, we refer to demand as intermittent. [23] defined intermittent demand as "*infrequent in the sense that the average time between consecutive transactions is considerably larger than the unit time period, the latter being the interval of forecast updating*" (p. 127). Intermittent demand intends to be divided into two categories, namely erratic and lumpy. Both patterns are characterized by infrequent demand transactions. While a lumpy demand pattern is distinguished by low demand sizes, an erratic demand pattern is determined

by variable demand sizes [24]. Besides, lumpy demand is always sporadic while the opposite is not necessarily true.

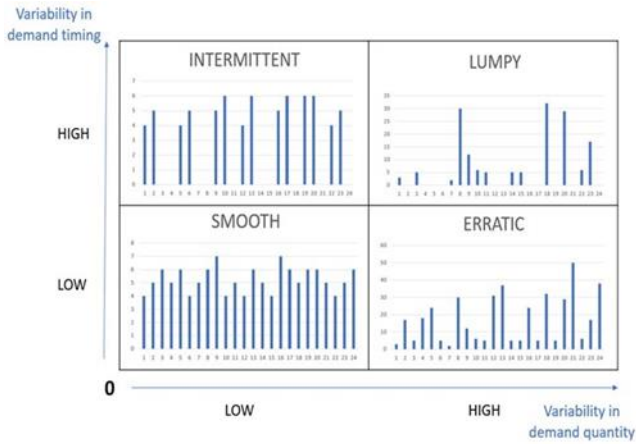


Figure 1. Graphics of demand patterns

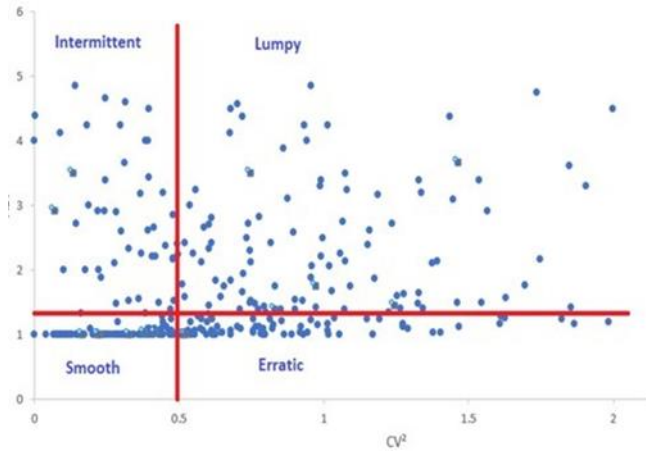


Figure 2. Scatter plot of demand patterns

[25] defined lumpy demand items as “items whose demand frequency is less than 4 times a year (p. 379)”. So, since it also is a type of demand, it can be represented in demand pattern classification tables or it can be thought of as similar to intermittent demand. These demand types are pictured in Figure 1. Besides the accuracy of a forecast depends on the properties of the demand history. In order to determine the characteristics of a demand history, there are two coefficients being used and given above Table 1:

- The first coefficient is the Average Demand Interval (**ADI**), it calculates the average interval between two demands and measures the regularity of a demand in time.
- The second coefficient is the square of the Coefficient of Variation (**CV²**), it measures the variation in the amount of demand.

Table 1. Coefficients of demands

	ADI	CV ²
Smooth Demand	<1.32	<0.49
Intermittent Demand	≥1.32	<0.49
Erratic Demand	>1.32	≥0.49
Lumpy Demand	≥1.32	≥0.49

Intermittent demand items, also known as volatile, variable, sporadic or unpredictable demand, have many zero values interspersed with random spikes of demand that are often many times larger than the average. These items occur if there are no demand in any interval. Often in these situations, when small demand occurs sometimes highly variable in size [26].

3. Methods

3.1. Croston Method

[10] proposed a new method that establishes demand estimates considering both demand size and the interval between demand incidences, then later this method was verified by Rao [11]. Croston method is a classical method that specifically deals with intermittent demand, which is based on simple exponential smoothing. The main difference that Croston introduced the forecast that is updated only when there is a demand and not. When the forecast time interval has a similar manner to ordinary exponential smoothing. Croston’s method not just focuses on the size of the order, but his model also consider the time between consecutive orders. This made the model suitable for forecasting items that have intermittent demand patterns [15]. It was advocated by Croston [10] and then later as approved by Rao [11] that exponential smoothing, one of the major technique of the forecasting methods, is not appropriate when dealing with intermittent demands.

The Croston’s method comprises of two main parts which decompose the original intermittent data into two different series. The first one contains all non-zero demand sizes, while the other involves the respective intervals between two successive non-zero demands. The Croston method uses exponential smoothing method by differentiating between these demand size and the demand interval that are separately updated after each period with a positive demand. Then, the ratio of the demand size over the estimated interval provides a forecast for the future demand per period [21]. When demand occurs every period namely time

periods contain no zero demand, Croston's method is the same as the conventional exponential smoothing method. Most of the forecasting and inventory control theory like Croston's method assume that consecutive demand values are independent [9].

Let \hat{Z}_t be the estimated demand at time t , \hat{n}_t be the estimate of time interval between nonzero demands, \hat{X}_t be the estimate of the mean size of nonzero demands and n_t be the time interval since last transaction and also X_t be the observed value of original demand.

If $X_t=0$;

$$\hat{Z}_t = \hat{Z}_{t-1} \quad (1)$$

$$\hat{n}_t = \hat{n}_{t-1} \quad (2)$$

Else, ($X_t \neq 0$);

$$\hat{Z}_t = \alpha X_t + (1 - \alpha) \hat{Z}_{t-1} \quad (3)$$

$$\hat{n}_t = \beta n_t + (1 - \beta) \hat{n}_{t-1} \quad (4)$$

Forecast equation is:

$$\hat{X}_t = \frac{\hat{Z}_t}{\hat{n}_t} \quad (5)$$

An initial value problem of this method is unimportant. Because contrary to exponential smoothing method, choosing an initial value is rather easy. Croston method chooses a first non-zero demand from an intermittent demand as an initial value of demand. Similarly, an initial value of a time interval is set equal to the time until the first non-zero demand.

3.2. ATA Method

The development of accurate, fast, robust and simple forecasting methods for time series is very important especially if there are large numbers of time series in the modelling and forecasting process. When the existing methods for forecasting time series are considered, there are still major drawbacks that interfere with accurate forecast. On the other hand; forecast competitions have played a critical role in both showcasing the effects of these drawbacks on accuracy of forecasts and in helping develop new forecasting techniques that could perform well while forecasting large numbers of time series. Recently, ATA method has been proposed which is a new forecasting method that is simple, fast and accurate [27]–[29]. ATA method is very similar to exponential smoothing in mathematical form but its weighting scheme is quite different. In exponential smoothing the most recent observation receives the same weight no matter where in time the

prediction is being made but with ATA the weights assigned to observations are always dynamic depending on where in time a prediction is needed. This is accomplished by making the weights dependent on both the smoothing parameter and time t .

A similar approach can be used when modelling intermittent demand data using ATA as below:

If $X_t=0$;

$$\hat{Z}_t = \hat{Z}_{t-1} \quad (6)$$

$$\hat{n}_t = \hat{n}_{t-1} \quad (7)$$

Else, ($X_t \neq 0$);

$$\hat{Z}_t = \left(\frac{p}{t}\right) X_t + \left(\frac{t-p}{t}\right) \hat{Z}_{t-1} \quad (8)$$

$$\hat{n}_t = \left(\frac{q}{t}\right) n_t + \left(\frac{t-q}{t}\right) \hat{n}_{t-1} \quad (9)$$

Again, forecast equation is below:

$$\hat{X}_t = \frac{\hat{Z}_t}{\hat{n}_t} \quad (10)$$

For $p \in (1, \dots, n)$, $q \in (0, \dots, p)$ and $t > p \geq q$ and $h = 1, 2, \dots$. For $t \leq p$, let $Z_t = X_t$ and for $t \leq q$ let $n_t = X_t - X_{t-1}$. Here X_t is the original demand of the series, \hat{Z}_t is the smoothed value of demand, \hat{n}_t is the estimate of time interval between nonzero demands, n_t is the time interval since last transaction and finally \hat{X}_t is the estimate of the average size of nonzero demands.

It is worth pointing out that ATA method does not require initial values for level and trend patterns. Since it is very similar in form the exponential smoothing, ATA can be adapted to handle different forms of trend and seasonality like its exponential smoothing counterparts [30]. It can be easily seen that the smoothed value at time t is a weighted average of all past observations and the initial value. From the equation (6), it is clearly evident that the initial value relies on the smoothing parameter of p . Thus, the smoothing parameter and initial value are optimized at the same time [27], [28]. ATA method uses the p^{th} and q^{th} values as the initial value of demand and inter-demand intervals respectively where p and q are the optimum smoothing parameters.

4. Application

In this paper, we use the 1,500 monthly inventory data sets from the M4-competition. These data sets from the M4-competition are used (<https://www.m4.unic.ac.cy/the-dataset/>). Each data set consists of 78 in-sample and 6 out-sample observations. 926 of these data have intermittent demand where the remaining and 574 can be categorized as lumpy demand. Only 176 of these data sets have non-zero initial values where the remaining data sets all start at zero. The analysis of these data sets has been carried out using R-Studio (for ATA method the "ATAforecasting" package is used and for Croston method and its derivation SBA and SBJ the "tsintermittent" package is used). The performances of these methods are compared based on average of various error metrics i.e. MAE, MSE, sMAPE etc in in-sample optimization. Thus, in in-sample optimization MSE is selected both ATA and Croston methods, because when MSE is applied in in-sample optimization, then out-sample error is comparatively smaller than the others. Also, for an initial value of Croston method (both demand and inter-demand interval), the best results are obtained by mean technique among two initial value techniques: naïve and mean. On the other hand, since there is not any information about to scale of data, then we use symmetric mean absolute percentage error (sMAPE) for comparing forecast accuracy.

Table 2. Original demand sizes for I0001 data from left to right sequentially

0	0	15	0	50	0	0
0	0	0	0	0	0	130
20	0	0	17	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	150	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	10	0	0
0	0	0	0	0	0	35
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

In Table 2, original demand sizes from M4-competition inventory data set's I0001 data are given row by row

and from left to right sequentially (first data row among the data set).

In Table 3, Croston's decomposition of I0001 series are given. Besides its periods, non-zero demand sizes, the interval between two consecutive demands and also cumulative non-zero demand sizes are given.

Table 3. Croston's decomposition of I0001 series from inventory data set

Period	Non-zero demands	Inter-demand intervals	Cumulative Non-zero demands
3	15	3	15
5	50	2	65
14	130	9	195
15	20	1	215
18	17	3	232
34	150	16	382
54	10	20	392
63	35	9	427

In Table 4, average in-sample sMAPE for the I0001 data for all mentioned methods are given. Besides, one step ahead forecast value, initials and weights for all methods examined in this paper are given for the I0001 data.

Table 4. In-sample results for I0001 data using MSE metrics for optimization

	Croston	SBA	SBJ	ATA
Forecasts	2.6766	2.6766	2.6766	9.6916
Weight Z	0.0463	0.0463	0.0463	1
Weight n	0.8925	0.8925	0.8925	1
Initial Z	15	15	15	25.4816
Initial n	7.875	7.875	7.875	2.6292
sMAPE	1.9537	1.9537	1.9537	1.9371

Table 5. Out-sample sMAPE values for M4-competition inventory data set

	1	2	3	4	5	6	1-2	1-4	1-6
Croston	1.9583	1.9574	1.9509	1.9442	1.9469	1.9646	1.9578	1.9527	1.9537
SBA	1.9583	1.9574	1.9509	1.9442	1.9469	1.9646	1.9578	1.9527	1.9537
SBJ	1.9583	1.9574	1.9509	1.9442	1.9469	1.9646	1.9578	1.9527	1.9537
ATA	1.9419	1.9423	1.9302	1.9260	1.9286	1.9535	1.9420	1.9351	1.9371

5. Conclusion

Intermittent demand data arise in many contexts and therefore forecasting methods have been developed in order to deal with this special type of time series data. The main difference that lead to the idea that special care should be taken when dealing with such data is that the data contains many zeros. The most famous methods that are used in forecasting intermittent demand data are the Croston method and its two variations: SBA and SBJ. These methods are based on the exponential smoothing which is one of the major forecasting techniques. Similarly; ATA method has similar form to exponential smoothing (ES) too and similar ideas can be applied to ATA so that it can be used for forecasting intermittent demand. Even though ATA and ES can be thought of as similar methods ATA method distinguishes itself from ES in many important aspects. ES suffers from initialization and optimization problems whereas ATA avoids these issues thanks to its different parameterization. For more details on the advantages of ATA's differences from ES see [28]. Since ATA performs better in terms of forecasting accuracy for regular time series forecasting, the variation proposed in this paper produces better forecasts for intermittent demand data when compared to Croston based models since these are all ES based.

In this paper; we proposed that a modification of ATA method could be used in a similar sense that exponential smoothing is adapted by Croston for this type of data. In order to show that this modification works we compared the out sample forecasting accuracy of this modification to the performance of the existing methods for the inventory data sets in the M4-competition. It is shown that the method proposed in this paper produced much more accurate forecasts for both short and long term forecasting horizons and therefore should be considered as a candidate model by the forecasting community whenever intermittent data needs to be forecasted.

6. References

- [1] A. A. Syntetos, "Forecasting of intermittent demand," *Brunel Univ.*, pp. 1-495, 2001.
- [2] B. Vasumathi and A. Saradha, "Forecasting Intermittent Demand for Spare Parts 1," *Int. J. Comput. Appl.*, vol. 75, no. 11, pp. 12-16, 2013.
- [3] T. R. Willemain, C. N. Smart, and H. F. Schwarz, "A new approach to forecasting intermittent demand for service parts inventories," *Int. J. Forecast.*, vol. 20, no. 3, pp. 375-387, 2004.
- [4] A. A. Syntetos and J. E. Boylan, "The accuracy of intermittent demand estimates," *Int. J. Forecast.*, vol. 21, no. 2, pp. 303-314, 2005.
- [5] N. Kourentzes, "Intermittent demand forecasts with neural networks," *Int. J. Prod. Econ.*, vol. 143, no. 1, pp. 198-206, 2013.
- [6] A. A. Ghobbar and C. H. Friend, "Evaluation of forecasting methods for intermittent parts demand in the field of aviation: A predictive model," *Comput. Oper. Res.*, vol. 30, no. 14, pp. 2097-2114, 2003.
- [7] F. R. Johnston, J. E. Boylan, and E. A. Shale, "An examination of the size of orders from customers, their characterisation and the implications for inventory control of slow moving items," *J. Oper. Res. Soc.*, vol. 54, no. 8, pp. 833-837, 2003.
- [8] J. E. Boylan and A. A. Syntetos, "On the bias of intermittent demand estimates," *Int. J. Prod. Econ.*, vol. 71, pp. 457-466, 2001.
- [9] N. Altay, L. A. Litteral, and F. Rudisill, "Effects of correlation on intermittent demand forecasting and stock control," *Int. J. Prod. Econ.*, vol. 135, no. 1, pp. 275-283, 2012.
- [10] J. D. Croston, "FORECASTING AND STOCK CONTROL FOR INTERMITTENT DEMANDS," *Oper. Res. Q.*, 1972.
- [11] A. V. Rao, "A Comment on: Forecasting and Stock Control for Intermittent Demands," *J. Oper. Res. Soc.*, vol. 24, no. 4, pp. 639-640, Dec. 1973.
- [12] F. R. Johnston and J. E. Boylan, "Forecasting for items with intermittent demand," *J. Oper. Res. Soc.*, vol. 47, no. 1, pp. 113-121, 1996.
- [13] F. Petropoulos, N. Kourentzes, and K. Nikolopoulos, "Another look at estimators for intermittent demand," *Int. J. Prod. Econ.*, vol. 181, pp. 154-161, 2016.
- [14] A. A. Syntetos, M. Z. Babai, Y. Dallery, and R. Teunter, "Periodic control of intermittent demand items: theory and empirical analysis," *J. Oper. Res. Soc.*, vol. 60, no. 5, pp. 611-618, May 2009.
- [15] E. Levén and A. Segerstedt, "Inventory control with a modified Croston procedure and Erlang distribution," *Int. J. Prod. Econ.*, vol. 90, no. 3, pp. 361-367, Aug. 2004.
- [16] A. Segerstedt, "Inventory control with variation in lead times, especially when demand is intermittent," *Int. J. Prod. Econ.*, vol. 35, no. 1-3, pp. 365-372, Jun. 1994.
- [17] E. A. Shale, J. E. Boylan, and F. R. Johnston, "Forecasting for intermittent demand: the estimation of an unbiased average," *J. Oper. Res. Soc.*, vol. 57, no. 5, pp. 588-592, May 2006.
- [18] R. Teunter and B. Sani, "On the bias of Croston's forecasting method," *Eur. J. Oper. Res.*, 2009.
- [19] F. Petropoulos, K. Nikolopoulos, G. P. Spithourakis, and V. Assimakopoulos, "Empirical heuristics for improving intermittent demand forecasting," *Ind. Manag. Data Syst.*, vol. 113, no. 5, pp. 683-696, 2013.
- [20] R. H. Teunter and L. Duncan, "Forecasting intermittent demand: a comparative study," *J. Oper. Res. Soc.*, vol. 60, no. 3, pp. 321-329, Mar. 2009.

- [21] R. H. Teunter, A. A. Syntetos, and M. Z. Babai, "Intermittent demand: Linking forecasting to inventory obsolescence," *Eur. J. Oper. Res.*, vol. 214, no. 3, pp. 606–615, 2011.
- [22] P. Wallström and A. Segerstedt, "Evaluation of forecasting error measurements and techniques for intermittent demand," *Int. J. Prod. Econ.*, vol. 128, no. 2, pp. 625–636, 2010.
- [23] E. A. Silver, "Operations Research in Inventory Management: A Review and Critique Author (s): Edward A . Silver Published by: INFORMS Stable URL : <http://www.jstor.org/stable/170382> REFERENCES Linked references are available on JSTOR for this article : You may need," *INFORMS - Oper. Res.*, vol. 29, no. 4, pp. 628–645, 1981.
- [24] A. H. C. Eaves, "Forecasting for the ordering and stock-holding of consumable spare parts," no. July, 2002.
- [25] A. Vereecke and P. Verstraeten, "An inventory management model for an inventory consisting of lumpy items, slow movers and fast movers," *Int. J. Prod. Econ.*, vol. 35, no. 1, pp. 379–389, 1994.
- [26] J. K. Ord, R. D. Snyder, and A. Beaumont, "Forecasting the Intermittent Demand for Slow-Moving Items," *Res. Program. Forecast.*, p. 38, 2010.
- [27] G. Yapar, "Modified simple exponential smoothing," *Hacettepe J. Math. Stat.*, 2018.
- [28] G. Yapar, I. Yavuz, and H. T. Selamlar, "Why and How Does Exponential Smoothing Fail? An In Depth Comparison of ATA- Simple and Simple Exponential Smoothing .," vol. 01, no. 1, pp. 30–39, 2017.
- [29] G. Yapar, S. Capar, H. T. Selamlar, and I. Yavuz, "Modified holt's linear trend method," *Hacettepe J. Math. Stat.*, 2018.
- [30] R. Hyndman, A. Koehler, K. Ord, and R. Snyder, *Forecasting with Exponential Smoothing The state Space Approach*. 2008.