ON PERIODICAL SOLUTIONS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS OF THIRD ORDER

Prof. Dr. Murzabek İMANALİEV

Academician of NAS KR, Institute of Mathematics, National Academy of Sciences of Kyrgyz Republic

Prof. Dr. Taalaibek İMANALİEV

Kyrgyz State National University Chair of Applied Mathematics and İnformation Sciences

The review of the modern situation in development of periodical solutions of equations of Korteweg-De Vries type could be found in [1, pp. 112-186]. In this article the problem of existence of periodical solutions of regularized equations of KDF type is considered. Also, in this work the problem of existence and uniqueness of continuous and bounded solutions of initial problem of the same type equations is considered.

The present article searches the existence and uniqueness of periodical with period T by x solution of nonlinear partial differential equation of 3rd order as

$$\begin{aligned}&(\alpha^2+1)u_t(t,x)+\alpha(u_{xx}(t,x)+2\alpha u_x(t,x)+(\alpha^2+1)u(t,x))+2\alpha u_{xt}(t,x)\\&+u(t,x)(u_x(t,x)+\alpha u(t,x))+u_{xxt}(t,x)=f(t,x,u(t,x)),\end{aligned}$$

(1)

where $f(t, x, u)$ is known periodical by t with period T, and periodical with period 2π by x function, α -is some positive constant.

Periodical with period *t* and by *x* solution of nonlinear partial differential equation (1) is defined as

$$u(t,x)=\int_{t}^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-v)}\sin(x-s)Q(v,s)dsdv}{e^{\alpha T}-1},$$

(2)

where $Q(t,x)$ - is a new unknown periodical with period *T* by *t*, and periodical with period 2π by *x* function.

We have

$$u(t+T,x+2\pi)=\int_{t+T}^{t+2T}\int_{-\infty}^{x+2\pi}\frac{e^{-\alpha(x+2\pi-s+t+T-v)}\sin(x+2\pi-s)Q(v,s)dsdv}{e^{\alpha T}-1}=$$

$$=\int_{t}^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-\gamma+t-\rho)}\sin(x-\gamma)Q(\rho+T,\gamma+2\pi)d\gamma d\rho}{e^{\alpha T}-1}=u(t,x).$$

To determine the unknown function $Q(t,x)=Q(t+T,x+2\pi)$ (2) is substituted into (1). For this purpose differentiating (2) by *t*,

$$u_t(t,x)=\int_{-\infty}^{x}\frac{e^{-\alpha(x-s)}\sin(x-s)[e^{-\alpha(t-t-T)}-1]Q(t,s)ds}{e^{\alpha T}-1}-\alpha\int_{t}^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-v)}\sin(x-s)Q(v,s)dsdv}{e^{\alpha T}-1}=$$

$$=\int_{-\infty}^{x}e^{-\alpha(x-s)}\sin(x-s)Q(t,s)ds-\alpha u(t,x)$$

is received.

Therefore

$$u_t+\alpha u(t,x)=\int_{-\infty}^{x}e^{-\alpha(x-s)}\sin(x-s)Q(t,s)ds$$

(3)

is obtained.

Further, differentiating (3) by *x*,

$$u_{tx}+\alpha u_x+\alpha u_t+\alpha^2 u=\int_{-\infty}^{x} e^{-\alpha(x-s)}\cos(x-s)Q(t,s)ds$$

(4)

is received.

Differentiating (2) by *x*,

$$u_x+\alpha u=\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}\cos(x-s)Q(\nu,s)dsd\nu}{e^{\alpha T}-1}$$

(5)

is obtained.

Here differentiating once more (5),

$$u_{xx}+\alpha u_x(t,x)=\int_t^{t+T}\frac{e^{-\alpha(t-\nu)}Q(\nu,x)d\nu}{e^{\alpha T}-1}-\alpha(u_x+\alpha u)-u$$

(6)

is obtained.

Therefore, from (6) follows the inequity

$$u_{xx}+2\alpha u_x(t,x)+(\alpha^2+1)u=\int_t^{t+T}\frac{e^{-\alpha(t-\nu)}Q(\nu,x)d\nu}{e^{\alpha T}-1}.$$

(7)

Differentiating (7) by *t*,

$$u_{xxt}+2\alpha u_{xt}+(\alpha^2+1)u_t=Q(t,x)-\alpha[u_{xx}+2\alpha u_x+(\alpha^2+1)u]$$

(8)

is obtained.

Hence, the inequity

$$u_{xxt}+(\alpha^2+1)u_t+2\alpha u_{xt}+\alpha[u_{xx}+2\alpha u_x+(\alpha^2+1)u]=Q(t,x)$$

(9)

follows up.

Multiplying (5) by u

$$u(u_x+\alpha u)=\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}\sin(x-s)Q(\nu,s)dsd\nu}{e^{\alpha T}-1}\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}\cos(x-s)Q(\nu,s)dsd\nu}{e^{\alpha T}-1}$$

(10)

is obtained.

Adding the right and left parts of (9) and (10),

$$(\alpha^2+1)u_t+2\alpha u_{xt}+\alpha[u_{xx}+2\alpha u_x+(\alpha^2+1)u]+u_{xxt}=f(t,x,u)=$$
$$=Q(t,x)+\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}\sin(x-s)Q(\nu,s)dsd\nu}{e^{\alpha T}-1}\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}\cos(x-s)Q(\nu,s)dsd\nu}{e^{\alpha T}-1}$$

(11)

is obtained.

From (11) to define the unknown function $Q(t,x)$, the following nonlinear integral equation of

$$Q(t,x)=f(t,x,\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}\sin(x-s)Q(\nu,s)dsd\nu}{e^{\alpha T}-1})-$$
$$-\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}\sin(x-s)Q(\nu,s)dsd\nu}{e^{\alpha T}-1}\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}\cos(x-s)Q(\nu,s)dsd\nu}{e^{\alpha T}-1}$$

(12)

type is obtained.

Thus, the problem of existence of periodical with period T by argument t and periodical with period 2π by x solution of nonlinear differential equation (1) lead to the problem of existence and uniqueness of the solution of periodical with period T by argument t and periodical with period T_1 by x of nonlinear integral equation (12).

Further, let's state that the condition (A) is fulfilled if in area $R_1 = \{0 \le t \le T, -\infty < x, u < +\infty\}$ function $f(t,x,u)$ is continuous and periodic with period *T* by argument *t*, and this function is periodic with period 2π by argument *x*

$$\|f(t,x,u)\| \le M = const, \quad \|f\| = \max_{0\le t\le T} \sup_{-\infty<x<+\infty} |f(t,x,u)|$$

function $f(t,x,u)$ in area $R_1 = \{0 \le t \le T, -\infty < x, u < +\infty\}$ satisfies to Lipschitz's condition by argument *u*

$$\|f(t,x,u_2) - f(t,x,u_1)\| \le N\|u_2 - u_1\|, \quad 0 < N = const,$$

$$\frac{N\alpha^2 + 4M}{\alpha^4} \le \frac{1}{2}$$

To prove the existence of periodical with period *T* by *t* and periodical with period 2π by argument *x* of solution the principle of compressive mapping is implemented. For this purpose the right part of (12) is considered as some operator $H[Q]$ acting on function $Q(t,x)$.

We have

$$\|H[Q]\| \le \left\| f\left(t,x,\int_t^{t+T}\int_{-\infty}^{x} \frac{e^{-\alpha(x-s+t-v)}\sin(x-s)}{e^{\alpha T}-1} Q(v,s)\,ds\,dv\right) - f(t,x,0)\right\| + \|f(t,x,0)\| +$$

$$+\int_t^{t+T}\int_{-\infty}^{x} \frac{e^{-\alpha(x-s+t-v)}|\sin(x-s)|}{e^{\alpha T}-1}\|Q(v,s)\|\,ds\,dv \int_t^{t+T}\int_{-\infty}^{x} \frac{e^{-\alpha(x-s+t-v)}|\cos(x-s)|}{e^{\alpha T}-1}\|Q(v,s)\|\,ds\,dv \le$$

$$\le M + N\int_t^{t+T}\int_{-\infty}^{x} \frac{e^{-\alpha(x-s+t-v)}|\sin(x-s)|}{e^{\alpha T}-1}\|Q(v,s)\|\,ds\,dv +$$

$$+\int_t^{t+T}\int_{-\infty}^{x} \frac{e^{-\alpha(x-s+t-v)}\,ds\,dv}{e^{\alpha T}-1}\int_t^{t+T}\int_{-\infty}^{x} \frac{e^{-\alpha(x-s+t-v)}\,ds\,dv}{e^{\alpha T}-1} R^2 \le$$

$$\le M + \frac{NR}{\alpha^2} + \frac{R^2}{\alpha^4} = M + \frac{(N\alpha^2 + R)R}{\alpha^4} \le M + \frac{1}{2}R = R.$$

Therefore, let $R = 2M$.

Thus, we have shown; that the operator $H[Q]$ maps the point of sphere $\|Q(t,x)\| \le R$ the space of nonlinear periodical functions into point of the same sphere.

Let's show that operator $H[Q]$ maps the point of sphere $\|Q(t,x)\| \le R = 2M$ compressively.

$$\|Q(t,x)-G(t,x)\| = \|H[Q]-H[G]\| \le \| f\Big(t,x,\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}\sin(x-s)Q\,ds\,d\nu}{e^{\alpha T}-1}\Big)-$$

$$-f\Big(t,x,\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}\sin(x-s)G(\nu,s)\,ds\,d\nu}{e^{\alpha T}-1}\Big)\| + \int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}|\sin(x-s)|}{e^{\alpha T}-1}\|Q-G\|\,ds\,d\nu\times$$

$$\times\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}|\cos(x-s)|}{e^{\alpha T}-1}\|Q(\nu,s)\|\,ds\,d\nu+\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}|\sin(x-s)|}{e^{\alpha T}-1}\|G(\nu,s)\|\,ds\,d\nu\times$$

$$\times\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}|\cos(x-s)|}{e^{\alpha T}-1}\|Q(\nu,s)-G(\nu,s)\|\,ds\,d\nu \le N\int_t^{t+T}\int_{-\infty}^{x}\frac{e^{-\alpha(x-s+t-\nu)}}{e^{\alpha T}-1}\,ds\,d\nu\,\|Q-G\|+$$

$$+\frac{R}{\alpha^4}\|Q-G\|+\frac{R}{\alpha^4}\|Q-G\| \le \Big(\frac{N}{\alpha^2}+\frac{2R}{\alpha^4}\Big)\|Q-G\| \le \frac{N\alpha^2+4M}{\alpha^4}\|Q-G\| \le \frac{1}{2}\|Q-G\|.$$

Therefore, the operator $H[Q]$ maps the points of sphere $\|Q(t,x)\| \le R = 2M$ into the same sphere compressively. That is why on the basis of compressed mappings the equation (12) has the unique solution. Let's show that the function $Q(t,x)$ is periodical function

$$Q(t+T, x+2\pi) = Q(t,x).$$

We have

$$Q(t+T,x+2\pi)\equiv f\Big(t+T,x+2\pi,\int\limits_{t+T}^{t+2T}\int\limits_{-\infty}^{x+2\pi}\frac{e^{-\alpha(x+2\pi+t+T-\nu-s)}\sin(x+2\pi-s)Q(\nu,s)dsd\nu}{e^{\alpha T}-1}\Big)-$$

$$-\int\limits_{t+T}^{t+2T}\int\limits_{-\infty}^{x+2\pi}\frac{e^{-\alpha(x+2\pi-s+t+T-\nu)}\sin(x+2\pi-s)Q(\nu,s)dsd\nu}{e^{\alpha T}-1}\int\limits_{t+T}^{t+2T}\int\limits_{-\infty}^{x+2\pi}\frac{e^{-\alpha(x+2\pi-s+t+T-\nu)}\cos(x+2\pi-s)Q(\nu,s)dsd\nu}{e^{\alpha T}-1}\equiv$$

$$\equiv f\Big(t,x,\int\limits_{t}^{t+T}\int\limits_{-\infty}^{x}\frac{e^{-\alpha(x+2\pi+t+T-T-\rho-\gamma-2\pi)}\sin(t-s)Q(\rho+T,\gamma+2\pi)}{e^{\alpha T}-1}d\gamma d\rho\Big)-$$

$$-\int\limits_{t}^{t+T}\int\limits_{-\infty}^{x}\frac{e^{-\alpha(x-\gamma+t-\rho)}\sin(t-\rho)Q(\rho+T,\gamma+2\pi)d\gamma d\rho}{e^{\alpha T}-1}\int\limits_{t}^{t+T}\int\limits_{-\infty}^{x}\frac{e^{-\alpha(x-\gamma+t-\rho)}\cos(t-\gamma)Q(\rho+T,\gamma+2\pi)d\rho d\gamma}{e^{\alpha T}-1}=$$

$$=f\Big(t,x,\int\limits_{t}^{t+T}\int\limits_{-\infty}^{x}\frac{e^{-\alpha(x-\gamma+t-\rho)}\sin(t-s)Q(\rho,\gamma)}{e^{\alpha T}-1}d\gamma d\rho\Big)-\int\limits_{t}^{t+T}\int\limits_{-\infty}^{x}\frac{e^{-\alpha(x-\gamma+t-\rho)}\sin(t-\rho)Q(\rho,\gamma)d\gamma d\rho}{e^{\alpha T}-1}\times$$

$$\times\int\limits_{t}^{t+T}\int\limits_{-\infty}^{x}\frac{e^{-\alpha(x-\gamma+t-\rho)}\cos(t-\gamma)Q(\rho,\gamma)d\rho d\gamma}{e^{\alpha T}-1}\equiv Q(t,x).$$

THEOREM 1. Let the condition (A) is fulfilled, then the nonlinear differential equation in partial derivatives (1) has unique continuous solution *u (t, x),* which is periodical with period *T* by argument *t* and period 2π by argument *x*.

Let's consider now the system of nonlinear integro-differential equations of

$$u'(x)=\int\limits_{-\infty}^{+\infty}K(x-s)f(s,u(s))ds$$

(13)

type, where *u (x)* - *n*-dimensional vector, *f (x, u (x))* - *n*- dimensional vector function, *K (x-s)* - $n\times n$ –matrix function. To make further researches easy it is assumed that

$$K(x-s)=e^{-\frac{(x-s)^2}{2\alpha}}(x-s),$$

where α -some positive constant, and $2\alpha>1$.

It is assumed here, that *f (x, u (x))* - is periodical function by argument *x* with period *T*, and this function is continuous in $R_1 = (-\infty < x, u_1, \ldots, u_n < +\infty)$ area. Periodical with period *T* solution of the system (13) we will search as

$$u(x) = C + \int_{-\infty}^{+\infty} e^{-(x-s)^2\alpha} z(s)ds - \int_{-\infty}^{\infty} e^{-\alpha s^2} z(s)ds$$

(14)

where *z (x)* - is a new periodical function with period *T* to be determined,

C - is *n*-dimensioned arbitrary constant vector.

From (14) follows that under *x=0*

u (0)=C

Substituting (14) into (13),

$$u'(x) = -2\alpha\int_{-\infty}^{+\infty} e^{-\alpha(x-s)^2}(x-s)z(s)ds = \int_{-\infty}^{+\infty} e^{-\frac{(x-s)^2}{2\alpha}}(x-s)f(s, C + \int_{-\infty}^{+\infty} e^{-\alpha(s-\gamma)^2} z(\gamma)d\gamma -$$
$$- \int_{-\infty}^{+\infty} e^{-\alpha\gamma^2} z(\gamma)d\gamma)ds$$

(15)

is obtained.

The equation (15) is written as

$$\int_{-\infty}^{+\infty} e^{-\alpha(x-s)^2}(x-s)z(s)ds = -\frac{1}{2\alpha}\int_{-\infty}^{+\infty} e^{-\frac{(x-s)^2}{2\alpha}}(x-s)f(s, C + \int_{-\infty}^{+\infty} e^{-\alpha(s-\gamma)^2} z(\gamma)d\gamma - \int_{-\infty}^{+\infty} e^{-\alpha\gamma^2} z(\gamma)d\gamma)ds$$

(16)

Thus, researching the existence of all continuous periodical with period *T* solutions of system (13) is reduced to the existence of periodical with period *T* solutions of system of nonlinear integral equations of 1st order (16).

Implementing the transformation of Fourier to the both parts of (16)

$$\int_{-\infty}^{+\infty} z(\gamma)\int_{-\infty}^{+\infty} e^{-\alpha(s-\gamma)^2+iws}(s-\gamma)dsd\gamma = -\frac{1}{2\alpha}\int_{-\infty}^{+\infty} f(\gamma, C+\int_{-\infty}^{+\infty} e^{-\alpha(\gamma-\rho)^2} zd\rho - \int_{-\infty}^{+\infty} e^{-\alpha\rho^2} z(\rho)d\rho)\times$$

$$\times\int_{-\infty}^{+\infty} e^{iws-\frac{(s-\gamma)^2}{2\alpha}}(s-\gamma)dsd\gamma$$

is obtained.

From this substituting $s-\gamma=\sigma$,

$$\int_{-\infty}^{+\infty} e^{iw\gamma} z(\gamma)d\gamma\int_{-\infty}^{+\infty} e^{iw\sigma-\alpha\sigma^2}\sigma d\sigma = -\frac{1}{2\alpha}\int_{-\infty}^{+\infty} e^{iw\gamma} f(\gamma, C+\int_{-\infty}^{+\infty} e^{-\alpha(\gamma-\rho)^2} z(\rho)d\rho - \int_{-\infty}^{+\infty} e^{-\alpha\rho^2} z(\rho)d\rho)\times$$

$$\times\int_{-\infty}^{+\infty} e^{iw\sigma-\frac{\sigma^2}{2\alpha}}\sigma d\sigma d\gamma$$

(17)

is obtained.

As

$$\int_{-\infty}^{+\infty} e^{iw\sigma-\alpha\sigma^2}\sigma d\sigma = \frac{iw}{2\alpha}\frac{\sqrt{\pi}}{\sqrt{\alpha}} e^{-\frac{w^2}{4\alpha}}; \qquad \int_{-\infty}^{+\infty} e^{iw\sigma-\frac{\sigma^2}{2\alpha}}\sigma d\sigma = iw\alpha e^{-\frac{\alpha w^2}{2}}\sqrt{2\pi\alpha},$$

(18)

Then

$$\frac{iwe^{-\frac{w^2}{4\alpha}}\sqrt{\pi}}{2\alpha\sqrt{\alpha}} z(w) = -\frac{iw}{2\alpha} e^{-\frac{\alpha w^2}{2}}\sqrt{2\pi\alpha}\int_{-\infty}^{+\infty} e^{iw\gamma} f(\gamma, C+\int_{-\infty}^{+\infty}(e^{-\alpha(\gamma-\rho)^2} - e^{-\alpha\rho^2})z(\rho)d\rho)d\gamma$$

(19)

follows from (17) and (18).

Therefore

$$z(w) = -e^{-\frac{\alpha w^2}{2}+\frac{w^2}{4\alpha}}\alpha\sqrt{2}\int_{-\infty}^{+\infty} e^{iw\gamma} f(\gamma, C+\int_{-\infty}^{+\infty}(e^{-\alpha(\gamma-\rho)^2} - e^{-\alpha\rho^2})z(\rho)d\rho)d\gamma.$$

Consequently, passing from mapping to original

$$z(x)=\frac{\alpha\sqrt{2}}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}e^{-\frac{w^2(2\alpha^2-1)}{4\alpha}-iw(x-\gamma)}f(\gamma,C+\int_{-\infty}^{+\infty}(e^{-\alpha(\gamma-\rho)^2}-e^{-\alpha\rho^2})z(\rho)d\rho)d\gamma dw$$

(20)

is obtained.

Thus, we have shown that the researches on periodic with period *T* of all solutions of system of nonlinear integro-differential equations (13) is reduced to researching of existence of all periodical with period *T* systems of nonlinear integral equations (20).

Let's state that the condition (A) is fulfilled, if in area $R_1=(-\infty<x,u_1,...,u_n<+\infty)$ vector function $f(x,u_1,...,u_n)\equiv f(x,u)$ is continuous and periodic with period *T* by *x* and bounded, and $\|f(x,u)\|\le Me^{-\alpha_1 x^2}$, where α_1 and M are some positive constants; beside this, function satisfies to Lipschitz 's condition by vector argument *u*

$$\|f(x,u_2)-f(x,u_1)\|\le Ne^{-\alpha_1 x^2}\|u_2-u_1\|,$$

where *N* – is some positive constant, and

$$\frac{4\alpha\sqrt{\pi}N}{\sqrt{2\alpha(2\alpha-1)}}\le\frac{1}{2},\qquad 2\alpha-1>0.$$

Let's consider the right part of (20) as operator $H[z]$ acting on vector function *z (x).*

We have

$$\|z(x)\|=\|H[z]\|\le\frac{\alpha}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left|e^{-iw(x-s)}\right|e^{\frac{w^2(2\alpha^2-1)}{4\alpha}}\left\|f(s,C+\int_{-\infty}^{+\infty}(e^{-\alpha(s-\gamma)^2}-e^{-\alpha\gamma^2})z(\gamma)d\gamma)\right\|dsdw\le$$

$$\le\frac{\alpha}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}e^{\frac{w^2(2\alpha^2-1)}{4\alpha}}Me^{\alpha_1 s^2}dsdw=\frac{\alpha M}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{\frac{w^2(2\alpha^2-1)}{4\alpha}}dw\int_{-\infty}^{+\infty}e^{-\alpha_1 s^2}ds=\alpha\sqrt{\frac{2\alpha}{\alpha_1(2\alpha^2-1)}}M=R$$

Let's show now that the operator $H[z]$ is the compressing operator. Indeed,

$$\|z(x)-g(x)\| = \|H[z]-H[g]\| \le \frac{\alpha}{\sqrt{2\pi}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left|e^{iw(x-s)}\right| e^{-\frac{w^2(2\alpha-1)}{4\alpha}} \| f(s, C+\int_{-\infty}^{+\infty}(e^{-\alpha(s-\gamma)^2}-e^{-\alpha\gamma^2})z(\gamma)d\gamma)-$$

$$-f(s, C+\int_{-\infty}^{+\infty}(e^{-\alpha(s-\gamma)^2}-e^{-\alpha\gamma^2})g(\gamma)d\gamma) \| \, ds dw \le$$

$$\le \frac{\alpha}{\sqrt{2\pi}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{w^2(2\alpha-1)}{4\alpha}} N e^{-\alpha_1 s^2} [\int_{-\infty}^{+\infty}(e^{-\alpha(s-\gamma)^2}-e^{-\alpha\gamma^2})ds \|z(\gamma)-g(\gamma)\| d\gamma dw \le$$

$$\le \frac{\alpha N}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{w^2(2\alpha-1)}{4\alpha}} dw \int_{-\infty}^{+\infty} e^{-\alpha_1 s^2} ds 2\sqrt{\frac{\pi}{\alpha}} \|z-g\| \le \frac{2\alpha\sqrt{\pi}N\sqrt{\pi}}{\sqrt{2\pi}\sqrt{\alpha}\,\alpha_1} \frac{2\alpha}{\sqrt{2\alpha-1}} \|z-g\| =$$

$$= \frac{4\alpha^{3/2}\sqrt{\pi}N}{\sqrt{2\alpha_1(2\alpha-1)}} \|z-g\| \le \frac{1}{2}\|z-g\|.$$

Thus, based on the principles of compressed mappings the system of nonlinear integral equations (2) has the unique solution, which could be determined by successive approximation method.

Let's show now the periodical with period T solution of the system (20). Let vector - function $z(x)$ transfer the system (20) into identity. Then,

$$z(x+T) = \frac{\alpha\sqrt{2}}{2\pi} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{w^2(2\alpha-1)}{4\alpha} - iw(x+T-s)} f(s, C+\int_{-\infty}^{+\infty}(e^{-\alpha(s-\gamma)^2} z(\gamma)d\gamma - \int_{-\infty}^{+\infty} e^{-\alpha\rho^2} z(\rho)d\rho) ds dw$$

Substituting $s = T + \beta$, then,

$$z(x+T) = \frac{\alpha\sqrt{2}}{2\pi} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{w^2(2\alpha-1)}{4\alpha} - iw(x-\beta)} f(\beta+T, C+\int_{-\infty}^{+\infty}(e^{-\alpha(\beta+T-\gamma)^2} z(\gamma)d\gamma - \int_{-\infty}^{+\infty} e^{-\alpha\rho^2} z(\rho)d\rho) d\beta dw$$

In this identity let's substitute $\gamma = T + \sigma$, then from the last identity follows

$$z(x+T) = \frac{\alpha\sqrt{2}}{2\pi} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-\frac{w^2(2\alpha-1)}{4\alpha} - iw(x-\beta)} f(\beta+T, C+\int_{-\infty}^{+\infty}(e^{-\alpha(\beta-\sigma)^2} z(\sigma)d\sigma - \int_{-\infty}^{+\infty} e^{-\alpha\rho^2} z(\rho)d\rho) d\beta dw \equiv z(x)$$

Thus the following theorem is proved.

THEOREM 2. Let the known periodical with period T by x vector-function $f(x,u)$ satisfies the condition (A_1). Then, the system of nonlinear integro-differential equation (20) has the unique periodical with period T solution as:

$$u(x)=C+\int_{-\infty}^{+\infty}e^{-\alpha(x-s)^2}z(s)ds-\int_{-\infty}^{+\infty}e^{-\alpha\rho^2}z(\rho)d\rho\equiv C+\int_{-\infty}^{+\infty}e^{-\alpha(x+T-s)^2}z(s)ds-\int_{-\infty}^{+\infty}e^{-\alpha\rho^2}z(\rho)d\rho\equiv$$

$$\equiv C+\int_{-\infty}^{+\infty}e^{-\alpha(x-\gamma)^2}z(T+\gamma)d\gamma-\int_{-\infty}^{+\infty}e^{-\alpha\rho^2}z(\rho)d\rho\equiv C+\int_{-\infty}^{+\infty}e^{-\alpha(x-\gamma)^2}z(\gamma)d\gamma-\int_{-\infty}^{+\infty}e^{-\alpha\rho^2}z(\rho)d\rho\equiv u(x+T)$$

Thus, $u(x)\equiv u(x+T)$.

Let's consider now the differential equation of

$$u_{xxx}+u_{txx}(t,x)+(\alpha^2+1+u(t,x))(u_x(t,x)+u_t(t,x)+2\alpha(u_{tx}(t,x)+u_{xx}(t,x))=f(t,x,u)$$

(21)

type, with the initial condition of

$$u(0,x)=\varphi(x)$$

(22)

Here $\varphi(x)$ and $f(t,x,u)$ are known functions, α - is some positive constant. Let's show that the problem (21) and (22) has near soliton solution.

The near soliton solution of the problem (21)-(22) is searched as

$$u(t,x)=c(t,x)+\int_0^t\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}\sin(x-t+\nu-s)Q(\nu,s)dsd\nu\ ,$$

(23)

where $Q(t,x)$ is a new unknown function to be determined, and $C(t,x)$ is soliton solution of known nonlinear differential equation of Korteweg-De Vries type

$$(\alpha^2+1)c_t+(\alpha^2+1+c(t,x)c_x(t,x)+c_{xxx}(t,x)=0$$

(24)

with the initial condition

$c(0,x)=\varphi(x)$.

The main problem is to determine the unknown function $Q(t,x)$. For this purpose (23) is substituted into equation (21). We have

$$u_t(t,x)=c_t(t,x)+\int_{-\infty}^{x} e^{-\alpha(x-s)}\sin(x-s)Q(t,s)ds+\alpha\int_0^t\int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}\sin(x-t+\nu-s)Q(\nu,s)dsd\nu-$$
$$-\int_0^t\int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}\cos(x-t+\nu-s)Q(\nu,s)dsd\nu$$

(25)

And derivative of (23) by x has the following type:

$$u_x(t,x)=c_x(t,x)-\alpha\int_0^t\int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}\sin(x-t+\nu-s)Q(\nu,s)dsd\nu+$$
$$+\int_0^t\int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}\cos(x-t+\nu-s)Q(\nu,s)dsd\nu$$

(26)

From (25) and (26) the equity follows:

$$u_t(t,x)+u_x(t,x)=c_t(t,x)+c_x(t,x)+\int_{-\infty}^{x-t+\nu} e^{-\alpha(x-s)}\sin(x-s)Q(t,s)ds.$$

(27)

Differentiating the both parts of (27) by x,

$$u_{tx}(t,x)+u_{xx}(t,x)=c_{tx}(t,x)+c_{xx}(t,x)-\alpha(u_t+u_x-c_t-c_x)+\int_{-\infty}^{x-t+\nu} e^{-\alpha(x-s)}\cos(x-s)Q(t,s)ds$$ is received.

Therefore it follows that

$$u_{tx}(t,x)+u_{xx}(t,x)+\alpha(u_t+u_x)=c_{tx}(t,x)+c_{xx}(t,x)+\alpha(c_t+c_x)+\int_{-\infty}^{x-t+\nu} e^{-\alpha(x-s)}\cos(x-s)Q(t,s)ds \quad (28)$$

Differentiating ones more time by x both parts of (28), taking into account (27) and (28),

$$u_{txx}(t,x)+u_{xxx}(t,x)+\alpha(u_{tx}+u_{xx})=c_{txx}(t,x)+c_{xxx}(t,x)+\alpha(c_{tx}+c_{xx})+Q(t,x)-$$
$$-\alpha[u_{tx}+u_{xx}+\alpha(u_t+u_x)-c_{tx}-c_{xx}-\alpha(c_t+c_x)]-[u_t+u_x-c_t-c_x]$$

(29)

is obtained.

Therefore, from (27)

$$u_{txx}(t,x)+u_{xxx}(t,x)+2\alpha(u_{tx}+u_{xx})+(\alpha^2+1)(u_t+u_x)=Q(t,x)+c_{txx}(t,x)+c_{xxx}(t,x)+$$
$$+2\alpha(c_{tx}+c_{xx})+\alpha^2+1)(c_t+c_x)$$

(30)

is obtained.

We have

$$u(u_t+u_x)=[c(t,x)+\int_0^t\int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}\sin(x-t+\nu-s)Q(\nu,s)dsd\nu][c_t+c_x+$$
$$+\int_{-\infty}^{x} e^{-\alpha(x-s)}\sin(x-s)Q(t,s)ds]=c(t,x)(c_t+c_x)+$$
$$+(c_t+c_x)\int_0^t\int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}\sin(x-t+\nu-s)Q(\nu,s)dsd\nu+$$
$$+c\int_{-\infty}^{x} e^{-\alpha(x-s)}\sin(x-s)Q(t,s)ds+$$
$$+\int_{-\infty}^{x} e^{-\alpha(x-s)}\sin(x-s)Q(t,s)ds\int_0^t\int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}\sin(x-t+\nu-s)Q(\nu,s)dsd\nu.$$

(31)

Further, the left and right parts of the equality (30), (31), and (24) considering (21),

$$u_{txx}(t,x)+u_{xxx}(t,x)+2\ (u_{tx}+u_{xx})+(\ ^{2}+1+u)(u_t+u_x)=f(t,x,u)=Q(t,x)+c_{txx}(t,x)+$$

$$+2\ (c_{tx}+c_{xx}(t,x))+c(t,x)c_t(t,x)+c\int_{-\infty}^{x}e^{-\alpha(x-s)}\sin(x-s)Q(t,s)ds+$$

$$+(c_t+c_x)\int_0^t\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}\sin(x-t+\ -s)Q(\ ,s)dsd\ +\int_{-\infty}^{x}e^{-\alpha(x-s)}\sin(x-s)Q(t,s)ds\times$$

$$\times\int_0^t\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}\sin(x-t+\ -s)Q(\ ,s)dsd$$

(32)

is obtained.

To determine the unknown function $Q(t,x)$ the nonlinear integral equation of

$$Q(t,x)=f(t,x,c(t,x)+\int_0^t\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}\sin(x-t+\nu-s)Q(\nu,s)dsd\nu)-$$

$$-c(t,x)\int_{-\infty}^{x}e^{-\alpha(x-s)}\sin(x-s)Q(t,s)ds-(c_t(t,x)+$$

$$+c_x(t,x)\int_0^t\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}\sin(x-t+\nu-s)Q(\nu,s)dsd\nu-$$

$$-\int_{-\infty}^{x}e^{-\alpha(x-s)}\sin(x-s)Q(t,s)ds*$$

$$\int_0^t\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}\sin(x-t+\nu-s)Q(\nu,s)dsd\nu-c_{xxt}-2\alpha(c_{tx}+c_{xx})-cc_t(t,x)$$

(33)

type is obtained

Further, it is assumed, that the condition (A_2) is fulfilled, if:

1) In area $R_2=\{0\le t\le T,-\infty<x,u<+\infty\}$ function $f(t,x,u)$ is continuous and bounded, $\|f(t,x,u)\|\le M_0=const$

and satisfies to the Lipschitz condition by argument u

$$\left\| f(t,x,u_2) - f(t,x,u_1) \right\| \le N \left\| u_2 - u_1 \right\|,$$

where N – is some positive constant,

in area $R_2 = \{0 \le t \le T, -\infty < x < +\infty\}$ soliton function $c(t,x)$ is continuous and bounded with derivatives

$c_x(t,x)$, $c_t(t,x)$, $c_{xx}(t,x)$, $c_{tx}(t,x)$, $c_{xxx}(t,x)$, $c_{txx}(t,x)$ and

$$\sup_{\substack{0\le t\le T\\ -\infty<x<+\infty}}\{|c_t(t,x)|+|c_x(t,x)|, |c(t,x)|, |c_{xxx}(t,x)|, (|c_{tx}(t,x)|+|c_{xx}(t,x)|)2\alpha, |c(t,x)|, |c_t(t,x)|\} \le c_0 = cons$$

2) $$\frac{NT\alpha + c_0(T+1)\alpha + 4(M_0 + c_0)T}{\alpha^2} \le \frac{1}{2}.$$

Let's prove the existence of unique solution of nonlinear integral equation (33). For this purpose, as it was done above, the right part of (33) is considered as operator $H[Q]$ acting on function $Q(t,x)$. We have

$$\begin{aligned}
&\|Q(t,x)\| = \|H[Q]\| \le \| f(t,x,c(t,x)+\int_0^t\int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)}\sin(x-t+v-s)Q(v,s)dsdv) - f(t,x,c(t,x))\| + \\
&+\| f(t,x,c(t,x))\| + (\|c_t\| + \|c_x\|)\int_0^t\int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)}|\sin(x-t+v-s)|\|Q\|dsdv + \\
&+\|c(t,x)\|\int_{-\infty}^{x} e^{-\alpha(x-s)}|\sin(x-s)|\|Q(t,s)\|ds + \int_{-\infty}^{x} e^{-\alpha(x-s)}|\sin(x-s)|\|Q(t,s)\|ds \times \\
&\times\int_0^t\int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)}|\sin(x-t+v-s)|\|Q(v,s)\|dsdv + \|c_{txx}(t,x)\| + 2\alpha(\|c_{tx}(t,x)\| + \|c_{xx}(t,x)\|) + \\
&+\|c(t,x)\|\|c_t(t,x)\| \le M + N\int_{-\infty}^{x} e^{-\alpha(x-s)}|\sin(x-s)|\|Q(t,s)\|ds + \\
&+c_0[\int_0^t\int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)}\|Q\|ds + \int_{-\infty}^{x} e^{-\alpha(x-s)}\|Q(t,s)\|ds] + \int_{-\infty}^{x} e^{-\alpha(x-s)}\|Q(t,s)\|ds \times \\
&\times\int_0^t\int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)}\|Q(v,s)\|ds + c_0 \le M + c_0 + \frac{NTP}{\alpha} + \frac{c_0(T+1)}{\alpha}P + \frac{P^2T}{\alpha^2} \le M + c_0 + \\
&+\frac{[NT\alpha + c_0\alpha(T+1) + 2TP]}{\alpha^2}P \le M_0 + c_0 + \frac{1}{2}P = P, \quad P = 2(M_0 + c_0)
\end{aligned}$$

Let's show that $H[Q]$ is compressive operator.

$$\|Q(t,x)-G(t,x)\| = \|H[Q]-H[G]\| \le \| f(t,x,c+\int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} \sin(x-t+v-s)Q(v,s)\,ds\,dv)-$$

$$-f(t,x,c+\int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} \sin(x-t+v-s)G(v,s)\,ds\,dv)\|+\|c(t,x)\| \int_{-\infty}^{x} e^{-\alpha(x-s)} |\sin(x-s)|\,\|Q-G\|\,ds+$$

$$+(\|c_t(t,x)\|+\|c_x(t,x)\|)\int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} |\sin(x-t+v-s)|\,\|Q(v,s)-G(v,s)\|\,ds\,dv+$$

$$+\int_{-\infty}^{x} e^{-\alpha(x-s)} |\sin(x-s)|\,\|Q(t,s)-G(t,s)\|\,ds \int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} |\sin(x-t+v-s)|\,\|Q(v,s)\|\,ds\,dv+$$

$$+\int_{-\infty}^{x} e^{-\alpha(x-s)} |\sin(x-s)|\,\|G(t,s)\|\,ds \int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} |\sin(x-t+v-s)|\,\|Q(v,s)-G(v,s)\|\,ds\,dv \le$$

$$\le N\int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} |\sin(x-t+v-s)|\,\|Q(v,s)-G(v,s)\|\,ds\,dv+[c_0\int_{-\infty}^{x} e^{-\alpha(x-s)}\,ds+$$

$$+c_0\int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)}\,ds\,dv]\,\|Q-G\|+\frac{2P}{\alpha}\int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)}\,ds\,dv\,\|Q-G\| \le [\frac{NT}{\alpha}+\frac{c_0(T+1)}{\alpha}+$$

$$+\frac{2PT}{\alpha^2}]\,\|Q-G\| = \frac{[NT\alpha+c_0\alpha(T+1)+4T(M_0+c_0)]}{\alpha^2} \le \frac{1}{2}\|Q-G\|$$

Therefore, by the principle of compressed mappings of nonlinear integral equation (33) have the unique continuous and bounded solution $Q(t,x)$ under $0 \le t \le T, \quad -\infty < x < +\infty$, and the evaluation

$\| Q(t,x) \| \le R = 2(M_0 + c_0)$ takes place.

Let's search the differential properties of solution $u(t,x)$ of the problem (21) and (22). Here, it is assumed that the condition (A_3) if in area $R_1 = \{0 \le t \le T, -\infty < x, u < +\infty\}$ of function $f(t,x,u)$ has continuous and bounded derivative of 1[st] order by arguments *x* and $u(t,x)$, i.e.

$$\|f_x(t,x,u)\| \le H_0 = const, \quad \|f_u(t,x,u)\| \le H_1 = const, \qquad H_0, H_1 > 0.$$

For this purpose the derivative of the 1[st] order by argument x of the solution of equation (33) is needed.

We have

$$Q_x(t,x) = f_x(t,x,c(t,x) + \int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} \sin(x-t+v-s) Q(v,s)\,ds\,dv) +$$

$$+ f_x(t,x,u) \int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} [-\alpha \sin(x-t+v-s) + \cos(x-t+v-s)] G(v,s)\,ds\,dv) \,\|-$$

$$-c_x(t,x) \int_{-\infty}^{x} e^{-\alpha(x-s)} \sin(x-s) Q(t,s)\,ds - c(t,x) \int_{-\infty}^{x} e^{-\alpha(x-s)} [-\alpha \sin(x-s) + \cos(x-s)] Q(t,s)\,ds +$$

$$+[c_{tx}(t,x) + c_{xx}(t,x)] \int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} \sin(x-t+v-s) Q(v,s)\,ds\,dv - [c_t(t,x) + c_x(t,x)] \times$$

$$\times \int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} [-\alpha \sin(x-t+v-s) + \cos(x-t+v-s)] Q(v,s)\,ds\,dv -$$

$$-\int_{-\infty}^{x} e^{-\alpha(x-s)} [-\alpha \sin(x-s) + \cos(x-s)] Q(t,s)\,ds \int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} \sin(x-t+v-s) Q(v,s)\,ds\,dv -$$

$$-\int_{-\infty}^{x} e^{-\alpha(x-s)} \sin(x-s) G(t,s)\,ds \int_0^t \int_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} [-\alpha \sin(x-t+v-s) + \cos(x-t+v-s)] Q(v,s)\,ds\,dv -$$

$$-c_{txxx}(t,x) - 2\alpha(c_{tx} + c_{xx}) - [c_{tx}(t,x)c(t,x) + c_t(t,x)c_x(t,x)].$$

From this identity under $\{0 \le t \le T, -\infty < x,u < +\infty\}$ the inequity follows up:

$$\|Q_x(t,x)\| \le \|f_x(t,x,u)\| + \|f_u(t,x,u)\| \int_0^t \int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}(\alpha+1)\,ds\,d\nu P +$$

$$+\|c_x(t,x)\| \int_{-\infty}^{x} e^{-\alpha(x-s)}ds P + \|c(t,x)\| \int_{-\infty}^{x} e^{-\alpha(x-s)}(\alpha+1)ds P + (\|c_{tx}\| + \|c_{xx}\|) \int_0^t \int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}ds\,d\nu P +$$

$$+(\|c_t\| + \|c_x\|) \int_0^t \int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}(\alpha+1)ds\,d\nu P + \int_{-\infty}^{x} e^{-\alpha(x-s)}(\alpha+1)ds \int_0^t \int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}ds\,d\nu P^2 +$$

$$+\int_{-\infty}^{x} e^{-\alpha(x-s)}ds \int_0^t \int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}(\alpha+1)ds\,d\nu P^2 + \|c_{txx}(t,x)\| + 2\alpha(\|c_{tx}\| + \|c_{xx}\|) + \|c_{tx}(t,x)\|\|c(t,x)\| +$$

$$+\|c_t\|\|c_x\| \le H_0 + H_1 \frac{TP(\alpha+1)}{\alpha} + c_0 \frac{1}{\alpha} P + \frac{c_0(\alpha+1)P}{\alpha} + \frac{2c_0(\alpha+1)T}{\alpha} P + \frac{2(\alpha+1)T^2}{\alpha^2} P^2 = M_{00} = const$$

Therefore, in area $R_1 = \{0 \le t \le T, -\infty < x < +\infty\}$ function $Q_x(t,x)$ is uniformly bounded $\|Q_x(t,x)\| \le M_{00} = const$

From identities (23) and (25) the inequities follows up:

$$\|u(t,x)\| \le \|c(t,x)\| + \int_0^t \int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)} |\sin(x-t+\nu-s)| \|Q(\nu,s)\| ds\,d\nu \le c_0 + \int_0^t \int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)} ds\,d\nu P \le$$

$$\le c_0 + \frac{1}{\alpha} P + \frac{PT(\alpha+1)}{\alpha} = M_{20} = const$$

We have

$$\|u_x(t,x)\| \le \|c_x(t,x)\| + \int_0^t \int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)} [\alpha \sin(x-t+\nu-s) + |\cos(x-t+\nu-s)|] \|Q\| ds\,d\nu \le$$

$$\le c_0 + \frac{(\alpha+1)T}{\alpha} P = M_{30} = const$$

From identity (25) the identity

$$u_{tx}(t,x)\equiv c_{tx}(t,x)+\int_{-\infty}^{x}e^{-\alpha(x-s)}[-\alpha\sin(x-s)+\cos(x-s)]Q(t,s)ds+$$

$$+\alpha\int_{0}^{t}\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}[-\alpha\sin(x-t+\nu-s)+\cos(x-t+\nu-s)]Q(\nu,s)dsd\nu+\int_{0}^{t}Q(\nu,x-t+\nu)d\nu-$$

$$-\int_{0}^{t}\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}[\alpha\cos(x-t+\nu-s)+\sin(x-t+\nu-s)]Q(\nu,s)dsd\nu\,,$$

$$u_{txx}(t,x)\equiv c_{txx}(t,x)+Q(t,x)+\int_{-\infty}^{x}e^{-\alpha(x-s)}[\alpha^{2}\sin(x-s)-2\alpha\cos(x-s)-\sin(x-s)]Q(t,s)ds+$$

$$+2\alpha\int_{0}^{t}Q(\nu,x-t+\nu)d\nu-\alpha\int_{0}^{t}\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}[\alpha^{2}\sin(x-t+\nu-s)-2\alpha\cos(x-t+\nu-s)-$$

$$-\sin(x-t+\nu-s)]Q(\nu,s)dsd\nu+\int_{0}^{t}Q_{x}(\nu,x-t+\nu)d\nu+\int_{0}^{t}\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}[(1-\alpha^{2})\cos(x-t+\nu-s)-$$

$$-2\alpha\sin(x-t+\nu-s)]Q(\nu,s)dsd\nu$$

follows up.

Therefore in area $R_2=\{0\le t\le T,-\infty<x<+\infty\}$ the inequities

$$\|u_{tx}(t,x)\|\le\|c_{tx}(t,x)\|+\int_{-\infty}^{x}e^{-\alpha(x-s)}(\alpha+1)\|Q(\nu,s)\|ds+\alpha\int_{0}^{t}\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}(\alpha+1)\|Q(\nu,s)\|dsd\nu+$$

$$+\int_{0}^{t}\|Q(\nu,x-t+\nu)\|d\nu+\int_{0}^{t}\int_{-\infty}^{x-t+\nu}e^{-\alpha(x-t+\nu-s)}(\alpha+1)\|Q(\nu,s)\|dsd\nu\le c_{0}+\frac{(\alpha+1)}{\alpha}P+\alpha(\alpha+1)TP+$$

$$+PT+\frac{(\alpha+1)}{\alpha}PT\le M_{40}=const$$

$$\|u_{txx}(t,x)\| \le \|c_{txx}(t,x)\| + \|Q(t,x)\| + \int_{-\infty}^{x} e^{-\alpha(x-s)}(\alpha+1)^2 \|Q(t,s)\| ds + 2\alpha\int_0^t \|Q(\nu,x-t+\nu)d\nu\| +$$

$$+\alpha\int_0^t \int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}(\alpha+1)^2 \|Q(\nu,s)\| ds d\nu + \int_0^t \|Q_x(\nu,x-t+\nu)\| d\nu +$$

$$+\int_0^t \int_{-\infty}^{x-t+\nu} e^{-\alpha(x-t+\nu-s)}(\alpha+1)^2 \|Q(\nu,s)\| ds d\nu \le c_0 + P + \frac{(\alpha+1)^2}{\alpha}P + 2\alpha PT + (\alpha+1)^2 TP +$$

$$+M_{00}T + \frac{(\alpha+1)^2}{\alpha}TP \le M_{50} = const$$

follows up.

From (28) the inequity

$$\|u_{xx}(t,x)\| \le \|u_{tx}(t,x)\| + \alpha(\|u_t(t,x)\| + \|u_x(t,x)\|) + \|c_{tx}(t,x)\| + \alpha(\|c_t(t,x)\| + \|c_x(t,x)\|) +$$

$$+\int_{-\infty}^{x} e^{-\alpha(x-s)} |\cos(x-s)| \|Q(t,s)\| ds \le M_{40} + \alpha(M_{20}+M_{30}) + c_0(1+\alpha) + \frac{1}{\alpha}P = M_{60} = cons$$

follows.

From (30) in area $R_2 = \{0 \le t \le T, -\infty < x < +\infty\}$ the inequities

$$\|u_{xxx}\| \le \|Q(t,x)\| + \|u_{txx}(t,x)\| + 2\alpha[\|u_{tx}(t,x)\| + \|u_{xx}(t,x)\|] + (\alpha^2+1)[\|u_t\| + \|u_x\|] +$$

$$+\|c_{txx}(t,x)\| + \|c_{xxx}(t,x)\| + 2\alpha[\|c_{tx}(t,x)\| + \|c_{xx}(t,x)\|] + (\alpha^2+1)[\|c_t\| + \|c_x\|] \le$$

$$\le P + M_{50} + 2\alpha(M_{40}+M_{60}) + (\alpha^2+1)(M_{20}+M_{30}) = M_{70} = const$$

follows up.

THEOREM 3. Let the known functions $f(t,x,u)$ and $c(t,x)$ satisfy the conditions (A_2) and (A_3). Then the Cauchy problem (22) for nonlinear differential equation (21) has the unique continuous solution, which could be introduced as (22), and this solution has continuous and bounded derivatives

$u_t(t,x)$, $u_x(t,x)$, $u_{tx}(t,x)$, $u_{xx}(t,x)$, $u_{txx}(t,x)$, $u_{xxx}(t,x)$, and

$$\| u(t,x) - c(t,x) \| \leq \int\limits_{0}^{t} \int\limits_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} \, | \sin(x-t+v-s) | \, \| Q(v,s) \| \, ds dv \leq$$

$$\leq \int\limits_{0}^{t} \int\limits_{-\infty}^{x-t+v} e^{-\alpha(x-t+v-s)} ds dv P = \frac{P}{\alpha} T,$$

where $c(t,x)$ is the soliton solution of Cauchy problem $c(0,x) = \varphi(x)$ of the equation (24).

This solution $u(t,x)$ is near soliton solution of Cauchy problem (22) for the equation (21).

REFERENCES

1. S. Novikov, Theory of Solitons: Method of Inverse Problem, Nauka, Moscow, 1980, 319 pp.
2. T. Imanaliev, On the Theory of Multyperiodical Solutions of Nonlinear Differential Equations in Partial Derivatives Korteweg-De Vries Type, Ilim, Bishkek, 1998, pp.26-30
3. T. Imanaliev, Periodical Solutions of Nonlinear Differential Equations of Korteweg-De Vries-Burgers Type, Ilim, Bishkek, 1998, pp. 31-37
4. T. Imanaliev, On the Theory of Periodical Solutions of Regularized Nonlinear Differential Equations Korteweg-De Vries Type, Vestnik KGNU, Serials of Natural Sciences, Issue 1, Part 1, Bishkek, 1999, pp.5-9
5. T. Imanaliev, On the Theory of Periodical Solutions of Regularized Nonlinear Differential Equations Korteweg-De Vries Type, Vestnik KGNU, Serials of Natural Sciences, Issue 1, Part 1, Bishkek, 1999, pp.285-289
6. M. Imanaliev, T. Imanaliev, Soliton Solutions of the Nonlinear Regularized Korteweg-De Vries Differential Equations, 1st Turkish World Mathematics Symposium, Elazig, Turkey, 1999, pp. 140-141.