ЕДИНСТВЕННОСТЬ РЕШЕНИЙ ЛИНЕЙНЫХ ИНТЕГРАЛЬНЫХ УРАВНЕНИЙ ВОЛЬТЕРА ПЕРВОГО РОДА С ТРЕМЯ НЕЗАВИСИМЫМИ ПЕРЕМЕННЫМИ

Проф.др.Авыт АСАНОВ

Кыргызско-Турецкий Университет «Манас», Инженерный Факультет

Ж.А.ЗУЛПУКАРОВ

Ошский Технический Университет

Рассмотрим уравнение:

$$\int_0^t K_1(t,x,y,s)u(s,x,y)ds+\int_0^x K_2(t,x,y,z)u(t,z,y)dz+$$

$$+\int_0^y K_3(t,x,y,w)u(t,x,w)dw+$$

$$+\int_0^t\int_0^x K_4(t,x,y,s,z)u(s,z,y)dzds+\int_0^t\int_0^y K_5(t,x,y,s,w)u(s,x,w)dwds+$$

$$+\int_0^x\int_0^y K_6(t,x,y,z,w)u(t,z,w)dwdz+$$

$$+\int_0^t\int_0^x\int_0^y C(t,x,y,s,z,w)u(s,z,w)dwdzds=f(t,x,y), \tag{1}$$

где $(t,x,y)\in G=\{(t,x,y):\ 0\le t\le T,\ \ 0\le x\le l,\ \ 0\le y\le m\}.$

$K_1(t,x,y,s),\ K_2(t,x,y,z),\ K_3(t,x,y,w),\ K_4(t,x,y,s,z),\ K_5(t,x,y,s,w),$

$K_6(t,x,y,z,w),\ C(t,x,y,s,z,w)$ и $f(t,x,y)$ -известные функции,

$u(t,x,y)$ - неизвестная функция.

Интегральные уравнения Вольтера I рода с одной или двумя переменными широко исследованы в работах многих авторов [1-4]. Однако, при решении прикладных задач в пространстве, часто сталкиваемся с решениями с тремя переменными. В данной работе рассматривается вопрос о единственности решения в функциональном пространстве *L₂(G)*.

Пусть *u(t,x,y)* – решение уравнения (1) и $u(t,x,y)\in L_2(G)$, тогда уравнение (1) умножив на *u(t,x.y)* и интегрируя по области $G_{txy}=\{(s,z,w):\ 0\le s\le t,\ 0\le z\le x,\ 0\le w\le y\}$ получим:

$$\int_0^t\int_0^x\int_0^y\int_0^s K_1(s,z,w,\tau)u(\tau,z,w)u(s,z,w)d\tau dwdzds+$$

$$+\int_0^t\int_0^x\int_0^y\int_0^s K_2(s,z,w,\xi)u(s,\xi,w)u(s,z,w)\times$$

$$\times d\xi dwdzds+\int_0^t\int_0^x\int_0^y\int_0^s\int_0^w K_3(s,z,w,\eta)u(s,z,\eta)u(s,z,w)d\eta dwdzds+$$

$$+\int_0^t\int_0^x\int_0^y\int_0^s\int_0^z K_4(s,z,w,\tau,\xi)\times$$

$$u(\tau,\xi,w)u(s,z,w)d\xi d\tau dwdzds+$$

$$+\int_0^t\int_0^x\int_0^y\int_0^s\int_0^w K_5(s,z,w,\tau,\eta)u(\tau,z,\eta)u(s,z,w)d\eta d\tau dwdzds+$$

$$+\int_0^t\int_0^x\int_0^y\int_0^z\int_0^w K_6(s,z,w,\ ,\ )u(s,\ ,\ )u(s,z,w)d\ d\ dwdzds+$$

$$+\int_0^t\int_0^x\int_0^y\int_0^s\int_0^z\int_0^w C(s,z,w,\ ,\ ,\ )\times$$

$$\times u(\tau,\xi,\eta)u(s,z,w)d\eta d\xi d\tau dwdzds=\int_0^t\int_0^x\int_0^y f(s,z,w)u(s,z,w)dwdzds \quad (2)$$

Предположим выполнения следующих условий:

a) Функции $K_1(t,x,y,s)$, $K'_{1s}(t,x,y,s)$ -неотрицательны, а функции $K'_{1t}(t,x,y,s)$, $K''_{1ts}(t,x,y,s)$ -неположительны и непрерывны в области

$$G_1=\{(t,x,y,s):\ 0\le s\le t\le T,\ \ 0\le x\le l,\ 0\le y\le m\},$$

Функции $K_2(t,x,y,z)$, $K'_{1z}(t,x,y,z)$ -неотрицательны, а функции $K'_{2x}(t,x,y,z)$, $K''_{2xz}(t,x,y,z)$ -неположительны и непрерывны в области

$$G_2=\{(t,x,y,z):\ 0\le t\le T,\ \ 0\le z\le x\le l,\ 0\le y\le m\},$$

Функции $K_3(t,x,y,w)$, $K'_{3w}(t,x,y,w)$ -неотрицательны, а функции $K'_{3y}(t,x,y,w)$ и $K''_{3yw}(t,x,y,w)$ - неположительны и непрерывны в области

$$G_3=\{(t,x,y,w):\ 0\le t\le T,\ \ 0\le x\le l,\ 0\le w\le y\le m\},$$

Функции $K_4(t,x,y,s,z)$ и $K^{IV}_{4txsz}(t,x,y,s,z)$ непрерывны в области

$$G_4=\{(t,x,y,s,z):\ 0\le s\le t\le T,\ \ 0\le z\le x\le l,\ 0\le y\le m\},$$

Функции $K_5(t,x,y,s,w)$ и $K^{IV}_{5tysw}(t,x,y,s,w)$ непрерывны в области

$$G_5=\{(t,x,y,s,w):\ 0\le s\le t\le T,\ \ 0\le x\le l,\ 0\le w\le y\le m\},$$

Функции $K_6(t,x,y,z,w)$ и $K^{IV}_{6xyzw}(t,x,y,z,w)$ непрерывны в области

$$G_6=\{(t,x,y,z,w):\ 0\le t\le T,\ \ 0\le z\le x\le l,\ 0\le w\le y\le m\},$$

Функции $C_7(t,x,y,s,z,w)$ и $C^{VI}_{txyszw}(t,x,y,s,z,w)$ непрерывны в области

$$G_7=\{(t,x,y,s,z,w):\ 0\le s\le t\le T,\ \ 0\le z\le x\le l,\ 0\le w\le y\le m\}.$$

б)

$$\left|a_{i,11}\right|\ge 0,\ \begin{vmatrix}a_{i,11}&a_{i,12}\\a_{i,12}&a_{i,22}\end{vmatrix}\ge 0,\ \begin{vmatrix}a_{i,11}&a_{i,12}&a_{i,13}\\a_{i,12}&a_{i,22}&a_{i,23}\\a_{i,13}&a_{i,23}&a_{i,33}\end{vmatrix}\ge 0,\quad \mathrm{i}=1,2,...,12$$

неотрицательны в своих областях определения, где

$$a_{1,11}=(x-z)(y-w)K_1(s,z,w,0),\qquad a_{1,12}=-(t-s)(x-z)(y-w)K_4(s,z,w,0,0),$$

$$a_{1,22}=(t-s)(y-w)K_2(s,z,w,0),\qquad a_{1,13}=-(t-s)(x-z)(y-w)K_5(s,z,w,0,0),$$

$$a_{1,33}=(t-s)(x-z)K_3(s,z,w,0),\qquad a_{1,23}=-(t-s)(x-z)(y-w)K_6(s,z,w,0,0),$$

$$a_{2,11}=-\frac{(t-s)(x-z)(y-w)}{zw}K'_{1s}(s,z,w,0),$$

$$a_{2,12}=\frac{(t-s)(x-z)(y-w)}{w}K'_{4\xi}(s,z,w,0,\xi),$$

$$a_{2,22}=\frac{(t-s)(y-w)}{zw}K'_{2\xi}(s,z,w,\xi),$$

$$a_{2,13}=-\frac{(t-s)(x-z)(y-w)}{z}K'_{5\eta}(s,z,w,0,\eta),$$

$$a_{2,33} = \frac{(t-s)(x-z)}{zw} K'_{3\eta}(s,z,w,\eta),$$

$$a_{2,23} = -(t-s)(x-z)(y-w) K''_{6\xi\eta}(s,z,w,\xi,\eta),$$

$$a_{3,11} = \frac{(x-z)(y-w)}{zw} K'_{1\tau}(s,z,w,\tau),$$

$$a_{3,12} = -\frac{(t-s)(x-z)(y-w)}{w} K'_{4\tau}(s,z,w,\tau,0),$$

$$a_{3,22} = -\frac{(t-s)(x-z)(y-w)}{sw} K'_{2z}(s,z,w,0),$$

$$a_{3,13} = -(t-s)(x-z)(y-w) K''_{5\eta\tau}(s,z,w,\tau,\eta),$$

$$a_{3,33} = -\frac{(t-s)(x-z)(y-w)}{s} K''_{3\eta w}(s,z,w,\eta),$$

$$a_{3,23} = -\frac{(t-s)(x-z)(y-w)}{s} K'_{6\eta}(s,z,w,0,\eta),$$

$$a_{4,11} = -\frac{(t-s)(x-z)(y-w)}{z} K''_{1\tau s}(s,z,w,\tau),$$

$$a_{4,12} = -(t-s)(x-z)(y-w) K''_{4\xi\tau}(s,z,w,\tau,\xi),$$

$$a_{4,33} = -\frac{(t-s)(x-z)(y-w)}{sz} K'_{3w}(s,z,w,0),$$

$$a_{4,13} = -\frac{(t-s)(x-z)(y-w)}{z} K'_{5\tau}(s,z,w,\tau,0),$$

$$a_{4,22} = -\frac{(t-s)(x-z)(y-w)}{s} K''_{2\xi z}(s,z,w,\xi),$$

$$a_{4,23} = -\frac{(t-s)(x-z)(y-w)}{s} K'_{6\xi}(s,z,w,\xi,0),$$

$$a_{5,11} = (y-w)\left[K_4(s,z,w,0,0) - (t-s)K'_{4s}(s,z,w,0,0)\right],$$

$$a_{5,22} = (x-z)\left[K_5(s,z,w,0,0) - (y-w)K'_{5w}(s,z,w,0,0)\right],$$

$$a_{5,33} = (t-s)\left[K_6(s,z,w,0,0) - (x-z)K'_{6z}(s,z,w,0,0)\right],$$

$$a_{5,12} = -(t-s)(x-z)(y-w)\left[C(s,z,w,0,0,0) - C'_s(s,z,w,0,0,0)\right],$$

$$a_{5,13} = -(t-s)(x-z)(y-w)\left[C(s,z,w,0,0,0) - C'_w(s,z,w,0,0,0)\right],$$

$$a_{5,23} = -(t-s)(x-z)(y-w)\left[C(s,z,w,0,0,0) - C'_z(s,z,w,0,0,0)\right],$$

$$a_{6,11} = \frac{(z-x)(y-w)}{w}\left[(t-s)K''_{4zs}(s,z,w,0,0) - K'_{4z}(s,z,w,0,0)\right],$$

$$a_{6,22} = \frac{(t-s)(x-z)}{w}\left[(y-w)K'''_{5\eta ws}(s,z,w,0,\eta) - K''_{5\eta s}(s,z,w,0,\eta)\right],$$

$$a_{6,33} = (t-s)(y-w)\left[(x-z)K'''_{6\eta wz}(s,z,w,0,\eta) - K''_{6\eta w}(s,z,w,0,\eta)\right],$$

$$a_{6,12} = (t-s)(x-z)(y-w)\left[C'_\eta(s,z,w,0,0,\eta) - C''_{\eta s}(s,z,w,0,0,\eta)\right],$$

$$a_{6,13} = (t-s)(x-z)(y-w)\left[C_{\eta}(s,z,w,0,0,\eta) - C_{\eta w}(s,z,w,0,0,\eta)\right],$$

$$a_{6,23} = -(t-s)(x-z)(y-w)\left[C'_{\eta}(s,z,w,0,0,\eta) - C''_{\eta z}(s,z,w,0,0,\eta)\right],$$

$$a_{7,11} = \frac{y-w}{s}\left[K'_{4\tau}(s,z,w,\tau,0) - s(t-s)K''_{4\tau s}(s,z,w,\tau,0)\right],$$

$$a_{7,22} = (t-s)(x-z)\left[(y-w)K'''_{5\tau ws}(s,z,w,\tau,0) - K''_{5\tau s}(s,z,w,\tau,0)\right],$$

$$a_{7,33} = \frac{(t-s)(y-w)}{s}\left[(x-z)K''_{6wz}(s,z,w,0,0) - K'_{6w}(s,z,w,0,0)\right],$$

$$a_{7,12} = -(t-s)(x-z)(y-w)\left[C'_{\tau}(s,z,w,\tau,0,0) - C''_{\tau s}(s,z,w,\tau,0,0)\right],$$

$$a_{7,13} = -(t-s)(x-z)(y-w)\left[C'_{\tau}(s,z,w,\tau,0,0) - C''_{\tau w}(s,z,w,\tau,0,0)\right],$$

$$a_{7,23} = -(t-s)(x-z)(y-w)\left[C'_{\tau}(s,z,w,\tau,0,0) - C''_{\tau z}(s,z,w,\tau,0,0)\right],$$

$$a_{8,11} = \frac{y-w}{z}\left[K'_{4\xi}(s,z,w,0,\xi) - z(x-z)K''_{\xi z}(s,z,w,0,\xi)\right],$$

$$a_{8,22} = \frac{(t-s)(x-z)}{z}\left[(y-w)K''_{5ws}(s,z,w,0,0) - K'_{5s}(s,z,w,0,0)\right],$$

$$a_{8,33} = \frac{(t-s)(y-w)}{z}\left[(x-z)K'''_{6\xi zw}(s,z,w,\xi,0) - K''_{6\xi w}(s,z,w,\xi,0)\right],$$

$$a_{8,12} = -(t-s)(x-z)(y-w)\left[C'_{\xi}(s,z,w,0,\xi,0) - C''_{\xi s}(s,z,w,0,\xi,0)\right],$$

$$a_{8,13} = -(t-s)(x-z)(y-w)\left[C'_{\xi}(s,z,w,0,\xi,0) - C''_{\xi w}(s,z,w,0,\xi,0)\right],$$

$$a_{8,14} = -(t-s)(x-z)(y-w)\left[C'_{\xi}(s,z,w,0,\xi,0) - C''_{\xi z}(s,z,w,0,\xi,0)\right],$$

$$a_{9,11} = \frac{(t-s)(y-w)}{zw}\left[z(x-z)K_{4\xi zs}(s,z,w,0,\xi) - K_{4\xi s}(s,z,w,0,\xi)\right],$$

$$a_{9,22} = \frac{(x-z)(y-w)}{z}\left[(t-s)K'''_{5\eta ws}(s,z,w,0,\eta) - K''_{5\eta w}(s,z,w,0,\eta)\right],$$

$$a_{9,33} = \frac{t-s}{zw}\left[K''_{6\xi\eta}(s,z,w,\xi,\eta) - z(x-z)K'''_{6\xi\eta z}(s,z,w,\xi,\eta)\right],$$

$$a_{9,12} = -(t-s)(x-z)(y-w)\left[C''_{\xi\eta}(s,z,w,0,\xi,\eta) - C'''_{\xi\eta s}(s,z,w,0,\xi,\eta)\right],$$

$$a_{9,13} = -(t-s)(x-z)(y-w)\left[C''_{\xi\eta}(s,z,w,0,\xi,\eta) - C'''_{\xi\eta w}(s,z,w,0,\xi,\eta)\right],$$

$$a_{9,23} = -(t-s)(x-z)(y-w)\left[C''_{\xi\eta}(s,z,w,0,\xi,\eta) - C'''_{\xi\eta z}(s,z,w,0,\xi,\eta)\right],$$

$$a_{10,11} = \frac{y-w}{sw}\left[K''_{4\xi\tau}(s,z,w,\tau,\xi) - s(t-s)K'''_{\tau\xi s}(s,z,w,\tau,\xi)\right],$$

$$a_{10,22} = \frac{x-z}{sz}\left[K'_{5\tau}(s,z,w,\tau,0) - (y-w)K''_{5\tau w}(s,z,w,\tau,0)\right],$$

$$a_{10,33} = \frac{t-s}{sz}\left[K'_{6\xi}(s,z,w,\xi,) - z(x-z)K''_{\xi z}(s,z,w,\xi,0)\right],$$

$$a_{10,12} = -(t-s)(x-z)(y-w)\left[C''_{\tau\xi}(s,z,w,\tau,\xi,0) - C'''_{\tau\xi s}(s,z,w,\tau,\xi,0)\right],$$

$$a_{10,13} = -(t-s)(x-z)(y-w)\left[C''_{\tau\xi}(s,z,w,\tau,\xi,0) - C'''_{\tau\xi w}(s,z,w,\tau,\xi,0)\right],$$

$$a_{10,23} = -(t-s)(x-z)(y-w)\left[C''_{\tau\xi}(s,z,w,\tau,\xi,0) - C'''_{\tau\xi z}(s,z,w,\tau,\xi,0)\right],$$

$$a_{11,11} = \frac{(x-z)(y-w)}{sw}\left[s(t-s)K'''_{4\tau zs}(s,z,w,\tau,0) - K''_{4\tau s}(s,z,w,\tau,0)\right],$$

$$a_{11,22} = \frac{x-z}{sw}\left[w(y-w)K'''_{5\eta\tau w}(s,z,w,\tau,\eta) - K''_{5\eta\tau}(s,z,w,\tau,\eta)\right],$$

$$a_{11,33} = \frac{t-s}{sw}\left[K'''_{6\eta w}(s,z,w,0,\eta) - (x-z)K''_{6\eta wz}(s,z,w,0,\eta)\right],$$

$$a_{11,12} = -(t-s)(x-z)(y-w)\left[C''_{\tau\eta}(s,z,w,\tau,0,\eta) - C'''_{\tau\eta s}(s,z,w,\tau,0,\eta)\right],$$

$$a_{11,13} = -(t-s)(x-z)(y-w)\left[C''_{\tau\eta}(s,z,w,\tau,0,\eta) - C'''_{\tau\eta w}(s,z,w,\tau,0,\eta)\right],$$

$$a_{11,23} = -(t-s)(x-z)(y-w)\left[C''_{\tau\eta}(s,z,w,\tau,0,\eta) - C'''_{\tau\eta z}(s,z,w,\tau,0,\eta)\right],$$

$$a_{12,11} = \frac{(x-z)(y-w)}{sw}\left[s(t-s)K^{IV}_{4\tau\xi zs}(s,z,w,\tau,\xi) - K'''_{4\xi\tau z}(s,z,w,\tau,\xi)\right],$$

$$a_{12,22}=\frac{(t-s)(x-z)}{zw}\left[w(y-w)K^{IV}_{5\tau\eta ws}(s,z,w,\tau,\eta)-K'''_{5\tau\eta s}(s,z,w,\tau,\eta)\right],$$

$$a_{12,33}=\frac{(t-s)(y-w)}{sz}\left[z(x-z)K^{IV}_{6\xi\eta wz}(s,z,w,\xi,\eta)-K'''_{6\xi\eta w}(s,z,w,\xi,\eta)\right]$$

,

$$a_{12,12}=-(t-s)(x-z)(y-w)\left[C'''_{\tau\xi\eta}(s,z,w,\tau,\xi,\eta)-C^{IV}_{\tau\xi\eta s}(s,z,w,\tau,\xi,\eta)\right],$$
$$a_{12,13}=-(t-s)(x-z)(y-w)\left[C'''_{\tau\xi\eta}(s,z,w,\tau,\xi,\eta)-C^{IV}_{\tau\xi\eta w}(s,z,w,\tau,\xi,\eta)\right],$$
$$a_{12,23}=-(t-s)(x-z)(y-w)\left[C'''_{\tau\xi\eta}(s,z,w,\tau,\xi,\eta)-C^{IV}_{\tau\xi\eta z}(s,z,w,\tau,\xi,\eta)\right],$$

в) Для любых $(t,x,y,s,z,w)\in G_7$ справедлива

$$C'_{\tau}(s,z,w,\tau,0,0)-(t-s)C''_{\tau s}(s,z,w,\tau,0,0)-$$

$$-(x-z)C''_{\tau z}(s,z,w,\tau,0,0)-(y-w)\times$$

$$\times C''_{\tau w}(s,z,w,\tau,0,0)+(t-s)(x-z)C'''_{\tau sz}(s,z,w,\tau,0,0)+$$

$$+(t-s)(y-w)C'''_{\tau sw}(s,z,w,\tau,0,0)+$$

$$+(x-z)(y-w)C'''_{\tau zw}(s,z,w,\tau,0,0)-$$

$$-(t-s)(x-z)(y-w)C^{IV}_{\tau szw}(s,z,w,\tau,0,0)\ge 0$$

$$C'_{\xi}(s,z,w,0,\xi,0)-(t-s)C''_{\xi s}(s,z,w,0,\xi,0)-$$

$$-(x-z)C''_{\xi z}(s,z,w,0,\xi,0)-(y-w)\times$$

$$\times C''_{\xi w}(s,z,w,0,\xi,0)+(t-s)(x-z)C'''_{\xi sz}(s,z,w,0,\xi,0)+$$

$$+(t-s)(y-w)C'''_{\xi sw}(s,z,w,0,\xi,0)+$$

$$+(x-z)(y-w)C'''_{\xi zw}(s,z,w,0,\xi,0)-$$

$$-(t-s)(x-z)(y-w)C^{IV}_{\xi szw}(s,z,w,0,\xi,0)\geq 0,$$

$$C'_{\eta}(s,z,w,0,0,\eta)-(t-s)C''_{\eta s}(s,z,w,0,0,\eta)-$$

$$-(x-z)C''_{\eta z}(s,z,w,0,0,\eta)-(y-w)\times$$

$$\times C''_{\eta w}(s,z,w,0,0,\eta)+(t-s)(x-z)C'''_{\eta sz}(s,z,w,0,0,\eta)+$$

$$+(t-s)(y-w)C'''_{\eta sw}(s,z,w,0,0,\eta)+$$

$$+(x-z)(y-w)C'''_{\eta zw}(s,z,w,0,0,\eta)-$$

$$-(t-s)(x-z)(y-w)C^{IV}_{\eta szw}(s,z,w,0,0,\eta)\geq 0$$

$$C''_{\tau\xi}(s,z,w,\tau,\xi,0)-(t-s)C'''_{\tau\xi s}(s,z,w,\tau,\xi,0)-$$

$$-(x-z)C'''_{\tau\xi z}(s,z,w,\tau,\xi,0)-(y-w)\times$$

$$\times C'''_{\tau\xi w}(s,z,w,\tau,\xi,0)+(t-s)(x-z)C^{IV}_{\tau\xi sz}(s,z,w,\tau,\xi,0)+$$

$$+(t-s)(y-w)C^{IV}_{\tau\xi sw}(s,z,w,\tau,\xi,0)+$$

$$+(x-z)(y-w)C^{IV}_{\tau\xi zw}(s,z,w,\tau,\xi,0)-$$

$$-(t-s)(x-z)(y-w)C^{V}_{\tau\xi szw}(s,z,w,\tau,\xi,0)\geq 0$$

$$C''_{\tau\eta}(s,z,w,\tau,0,\eta)-(t-s)C'''_{\tau\eta s}(s,z,w,\tau,0,\eta)-(x-z)C'''_{\tau\eta z}(s,z,w,\tau,0,\eta)-(y-w)\times$$

$$\times C'''_{\tau\eta w}(s,z,w,\tau,0,\eta)+(t-s)(x-z)C^{IV}_{\tau\eta sz}(s,z,w,\tau,0,\eta)+$$

$$+(t-s)(y-w)C^{IV}_{\tau\eta sw}(s,z,w,\tau,0,\eta)+$$

$$+(x-z)(y-w)C^{IV}_{\tau\eta zw}(s,z,w,\tau,0,\eta)-(t-s)(x-z)(y-w)C^{V}_{\tau\eta szw}(s,z,w,\tau,0,\eta)\ge 0$$

$$C''_{\xi\eta}(s,z,w,0,\xi,\eta)-(t-s)C'''_{\xi\eta s}(s,z,w,0,\xi,\eta)-(x-z)C'''_{\xi\eta z}(s,z,w,0,\xi,\eta)-(y-w)\times$$

$$\times C'''_{\xi\eta w}(s,z,w,0,\xi,\eta)+(t-s)(x-z)C^{IV}_{\xi\eta sz}(s,z,w,0,\xi,\eta)+$$

$$+(t-s)(y-w)C^{IV}_{\xi\eta sw}(s,z,w,0,\xi,\eta)+$$

$$+(x-z)(y-w)C^{IV}_{\xi\eta zw}(s,z,w,0,\xi,\eta)-(t-s)(x-z)(y-w)C^{V}_{\xi\eta szw}(s,z,w,0,\xi,\eta)\ge 0$$

$$C'''_{\tau\xi\eta}(s,z,w,\tau,\xi,\eta)-(t-s)C^{IV}_{\tau\xi\eta s}(s,z,w,\tau,\xi,\eta)-$$

$$-(x-z)C^{IV}_{\tau\xi zs}(s,z,w,\tau,\xi,\eta)-(y-w)\times$$

$$\times C^{IV}_{\tau\xi\eta w}(s,z,w,\tau,\xi,\eta)+(t-s)(x-z)C^{V}_{\tau\xi\eta sz}(s,z,w,\tau,\xi,\eta)+$$

$$+(t-s)(y-w)C^{V}_{\tau\xi\eta sw}(s,z,w,\tau,\xi,\eta)+$$

$$+(x-z)(y-w)C^{V}_{\tau\xi\eta zw}(s,z,w,\tau,\xi,\eta)-(t-s)(x-z)(y-w)C^{VI}_{\tau\xi\eta szw}(s,z,w,\tau,\xi,\eta)\ge 0.$$

г) Для любых $(t,x,y,s,z,w)\in G_7$ справедливо

$$C(s,z,w,0,0,0)-(t-s)C'_s(s,z,w,0,0,0)-(x-z)C'_z(s,z,w,0,0,0)-(y-w)\times$$

$$\times C'_w(s,z,w,0,0,0)+(t-s)(x-z)C''_{sz}(s,z,w,0,0,0)+(t-s)(y-w)C''_{sw}(s,z,w,0,0,0)+$$

$$+(x-z)(y-w)C''_{zw}(s,z,w,0,0,0)-(t-s)(x-z)(y-w)C'''_{szw}(s,z,w,0,0,0)\ge K>0$$

где K - известные числа.

Интегрируя уравнение (2) по области G_{txy} еще раз и используя формулы Дирихле получим:

$$\frac{1}{2}\int_0^t\int_0^x\int_0^y\{(x-z)(y-w)K_1(s,z,w,0)(\int_0^s u(v_1,z,w)dv_1)^2+(t-s)(y-w)K_2(s,z,w,0)\times$$

$$\times(\int_0^{\cdot}(\cdot,\cdot_2,w)dv_2)^2+(t-s)(x-z)K_3(s,z,w,0)(\int_0^{\cdot}u(s,z,v_3)dv_3)^2-2(t-s)\times$$

$$\times(x-z)(y-w)K_4(s,z,w,0,0)(\int_0^z u(s,v_2,w)dv_2)(\int_0^s u(v_1,z,w)dv_1)-2(t-s)\times$$

$$\times(x-z)(y-w)K_5(s,z,w,0,0)(\int_0^w u(s,z,v_3)dv_3(\int_0^s u(v_1,z,w)dv_1)-$$

$$-2(t-s)(x-z)(y-w)\times$$

$$\times(y-w)K_6(s,z,w,0,0)(\int_0^w u(s,z,v_3)dv_3)(\int_0^z u(s,v_2,w)dv_2)\}dwdzds+$$

$$+\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^z\int_0^w\frac{t-s}{zw}\{-(x-z)(y-w)K_{1s}'(s,z,w,0)(\int_0^s u(v_1,z,w)dv_1)^2+(y-w)\times$$

$$\times K_{2\xi}'(s,z,w,\xi)(\int_0^z u(s,v_2,w)dv_2)^2+(x-z)K_{3\eta}'(s,z,w,\eta)\times$$

$$(\int_\eta^w u(s,z,v_3)dv_3)^2-2z(x-z)\times$$

$$\times(y-w)K_{4\xi}'(s,z,w,0,\xi)(\int_0^s u(v_1,z,w)dv_1)(\int_\xi^z u(s,v_2,w)dv_2)-2w(x-z)(y-w)\times$$

$$\times K_{5\eta}'(s,z,w,0,\eta)(\int_0^s u(v_1,z,w)dv_1)(\int_\eta^w u(s,z,v_3)dv_3)-2zw(x-z)(y-w)\times$$

$$\times K_{6\xi\eta}''(s,z,w,\xi,\eta)(\int_\xi^z u(s,v_2,w)dv_2)(\int_\eta^w u(s,z,v_3)dv_3)\}d\eta d\xi dwdzds+$$

$$+\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^s\int_0^w\frac{(x-z)(y-w)}{sw}\{K_{1\tau}'(s,z,w,\tau)(\int_\tau^s u(v_1,z,w)dv_1)^2-(t-s)K_{2z}'(s,z,w,0)\times$$

$$(\int_0^z u(s,v_2,w)dv_2)^2 - w(t-s)K''_{3\eta}(s,z,w,\eta)(\int_\eta^w u(s,z,v_3)dv_3)^2 - 2s(t-s)K'_{4\tau}(s,z,w,\tau,0)$$

$$\times(\int_\tau^s u(v_1,z,w)dv_1)(\int_w^z u(s,v_2,w) - 2sw(t-s)K''_{s\eta\tau}(s,z,w,\tau,\eta)(\int_\tau^s u(v_1,z,w)dv_1)\times$$

$$\times(\int_\eta^w u(s,z,v_3)dv_3) - 2w(t-s)K'_{6\eta}(s,z,w,0,\eta)\times$$

$$(\int_0^z u(s,v_2,w)dv_2)(\int_\eta^w u(s,z,v_3)dv_3)\}d\eta d\tau dwdzds +$$

$$\int_0^t\int_0^x\int_0^y\int_0^s\int_0^z \frac{(t-s)(x-z)(y-w)}{sz}\{-sK''_{1\tau s}(s,z,w,\tau)(\int_\tau^s u(v_1,z,w)dv_1)^2 -$$

$$zK''_{2\xi z}(s,z,w,\xi)$$

$$\times(\int_\xi^z u(s,v_2,w)dv_2)^2 - K'_{3w}(s,z,w,0)\times$$

$$(\int_0^w u(s,z,v_3)dv_3)^2 - 2szK''_{4\tau\xi}(s,z,w,\tau,\xi)(\int_\tau^s u(v_1,z,w)dv_1)$$

$$\times(\int_\xi^z u(s,z,v_3)dv_3) - 2sK'_{5\tau}(s,z,w,\tau,0)(\int_\tau^s u(v_1,z,w)dv_1)(\int_0^w u(s,z,v_3)dv_3) -$$

$$- 2zK'_{6\xi}(s,z,w,\xi,0)(\int_\xi^z u(s,v_2,w)dv_2)(\int_0^w u(s,z,v_3)dv_3)\}d\xi d\tau dwdzds +$$

$$\frac{1}{2}\int_0^t\int_0^x\int_0^y\{(y-w)[K_4(s,z,w,0,0) - (t-s)K'_{4s}(s,z,w,0,0)](\int_0^s\int_0^z u(v_1,v_2,w)dv_2dv_1)^2$$

$$+(x-z)[K_5(s,z,w,0,0) - (y-w)K'_{5w}(s,z,w,0,0)](\int_0^s\int_0^w u(v_1,z,v_3)dv_3dv_1)^2 +$$

$$+(t-s)[K_6(s,z,w,0,0) - (x-z)K'_{6z}(s,z,w,0,0)](\int_0^z\int_0^w u(s,v_2,v_3)dv_3dv_2)^2 -$$

$$-(t-s)(x-z)(y-w)[C(s,z,w,0,0,0) - C'_s(s,z,w,0,0,0)](\int_0^s\int_0^z u(v_1,v_2,w)dv_2dv_1)\times$$

$$\times(\int_0^{\cdot}\int_0^{\cdot}(\cdot_1,z,v_3)dv_3dv_1)-(t-s)(x-z)(y-w)\left[C(s,z,w,0,0,0)-C'_{\cdot}(s,z,w,0,0,0)\right]\times$$

$$\times(\int_0^s\int_0^w u(v_1,z,v_3)dv_3dv_1)(\int_0^z\int_0^w u(s,v_2,v_3)dv_3dv_2)-(t-s)(x-z)(y-w)\times$$

$$\left[C(s,z,w,0,0,0)-C'_z(s,z,w,0,0,0)\right](\int_0^s\int_0^z u(v_1,v_2,w)dv_2dv_1)(\int_0^s\int_0^w u(s,v_2,v_3)dv_3dv_2)\}$$

$$\times dwdzds+\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^w\left\{\frac{(x-z)(y-w)}{w}\left[(t-s)K''_{4zs}(s,z,w,0,0)-K'_{4z}(s,z,w,0,0)\right]\times\right.$$

$$(\int_0^s\int_0^z u(v_1,v_2,w)dv_2dv_1)^2+\frac{(t-s)(x-z)}{w}\left[(y-w)K'''_{5\eta ws}(s,z,w,0,\eta)-K''_{5\eta s}(s,z,w,0,\eta)\right]$$

$$\times(\int_0^s\int_\eta^w u(v_1,z,v_3)dv_3dv_1)^2+(t-s)(y-w)\left[(x-z)K'''_{6\eta wz}(s,z,w,0,\eta)-K''_{6\eta w}(s,z,w,0,\eta)\right]$$

$$\times(\int_0^z\int_\eta^w u(s,v_2,v_3)dv_3dv_2)^2-(t-s)(x-z)(y-w)\left[C'_\eta(s,z,w,0,0,\eta)-C''_{\eta s}(s,z,w,0,0,\eta)\right]\times$$

$$\times(\int_0^s\int_0^z u(v_1,v_2,w)dv_2dv_1)(\int_0^s\int_\eta^w u(v_1,z,v_3)dv_3dv_1)-$$

$$-(t-s)(x-z)(y-w)\left[C'_\eta(s,z,w,0,0,\eta)-\right.$$

$$\left.-C''_{\eta w}(s,z,w,0,0,\eta)\right](\int_0^s\int_\eta^w u(v_1,z,v_3)dv_3dv_1)\times$$

$$(\int_0^z\int_\eta^w u(s,v_2,v_3)dv_3dv_2)-(t-s)(x-z)(y-w)\times$$

$$\left[C'_\eta(s,z,w,0,0,\eta)-C''_{\eta z}(s,z,w,0,0,\eta)\right](\int_0^s\int_0^z u(v_1,v_2,w)dv_2dv_1)(\int_0^z\int_\eta^w u(s,v_2,v_3)dv_3dv_2)\}$$

$$\times d\eta dwdzds+\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^s\left\{\frac{y-w}{s}\left[K'_{4\tau}(s,z,w,\tau,0)-s(t-s)K''_{4\tau s}(s,z,w,\tau,0)\right]\times\right.$$

$$(\int_\tau^s\int_0^z u(v_1,v_2,w)dv_2dv_1)^2+(t-s)(x-z)\left[(y-w)K'''_{5\tau ws}(s,z,w,\tau,0)-K''_{5\tau s}(s,z,w,\tau,0)\right]$$

$$\times(\int_\tau^s\int_0^w u(v_1,z,v_3)dv_3dv_1)^2+\frac{(t-s)(y-w)}{s}\left[(x-z)K''_{6wz}(s,z,w,0,0)-K'_{6w}(s,z,w,0,0)\right]$$

$$\times(\int_0^z\int_0^w u(s,v_2,v_3)dv_3dv_2)^2-(t-s)(x-z)(y-w)\left[C'_\tau(s,z,w,\tau,0,0)-C''_{\tau s}(s,z,w,\tau,0,0)\right]$$

$$\times(\int_\tau^s\int_0^z u(v_1,v_2,w)dv_2dv_1)(\int_\tau^s\int_0^w u(v_1,z,v_3)dv_3dv_1)-(t-s)(x-z)(y-w)\left[C'_\tau(s,z,w,\tau 0,0)\right.$$

$$\left.-C''_{\tau w}(s,z,w,\tau 0,0)\right](\int_\tau^s\int_0^w u(v_1,z,v_3)dv_3dv_1)(\int_0^z\int_0^w u(s,v_2,v_3)dv_3dv_2)-(t-s)(x-z)(y-w)$$

$$\times\left[C'_{\tau}(s,z,w,\tau,0,0)-C''_{\tau z}(s,z,w,\tau,0,0)\right]\left(\int_{\tau}^{s}\int_{0}^{z}u(v_1,v_2,w)dv_2dv_1\right)\left(\int_{0}^{z}\int_{0}^{w}u(s,v_2,v_3)dv_3dv_2\right)\Big\}$$

$$\times d\tau dwdzds+\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^z\left\{\frac{y-w}{z}\left[K'_{4\xi}(s,z,w,0,\xi)-z(x-z)K''_{4\xi z}(s,z,w,0,\xi)\right]\times\right.$$

$$\times\left(\int_0^s\int_{\xi}^{z}u(v_1,v_2,w)dv_2dv_1\right)^2+\frac{(t-s)(x-z)}{z}\left[(y-w)K''_{5ws}(s,z,w,0,0)-K'_{5s}(s,z,w,0,0)\right]$$

$$\times\left(\int_0^s\int_0^w u(v_1,z,v_3)dv_3dv_1\right)^2+\frac{(t-s)(y-w)}{z}\left[(x-z)K'''_{6\xi w}(s,z,w,\xi,0)-K''_{6\xi w}(s,z,w,\xi,0)\right]$$

$$\times\left(\int_{\xi}^{z}\int_0^w u(s,v_2,v_3)dv_3dv_2\right)^2-(t-s)(x-z)(y-w)\left[C'_{\xi}(s,z,w,0,\xi,0)-C''_{\xi s}(s,z,w,0,\xi,0)\right]$$

$$\times\left(\int_0^s\int_{\xi}^{z}u(v_1,v_2,w)dv_2dv_1\right)\left(\int_0^s\int_0^w u(v_1,z,v_3)dv_3dv_1\right)-(t-s)(x-z)(y-w)\left[C'_{\xi}(s,z,w,0,\xi,0)\right.$$

$$\left.-C''_{\xi w}(s,z,w,0,\xi,0)\right]\left(\int_0^s\int_0^w u(v_1,z,v_3)dv_3dv_1\right)\times$$

$$\left(\int_{\xi}^{z}\int_0^w u(s,v_2,v_3)dv_3dv_2\right)-(t-s)(x-z)(y-w)\times$$

$$\left[C'_{\xi}(s,z,w,0,\xi,0)-C''_{\xi z}(s,z,w,0,\xi,0)\right]\left(\int_0^s\int_{\xi}^{z}u(v_1,v_2,w)dv_2dv_1\right)\left(\int_{\xi}^{z}\int_0^w u(s,v_2,v_3)dv_3dv_2\right)\Big\}$$

$$\times d\xi dwdzds+\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^z\int_0^w\left\{\frac{(t-s)(y-w)}{zw}\left[z(x-z)K'''_{4\xi zs}(s,z,w,0,\xi)-K''_{4\xi s}(s,z,w,0,\xi\right]\right.$$

$$\times\left(\int_0^s\int_{\xi}^{z}u(v_1,v_2,w)dv_2dv_1\right)^2+\frac{(x-z)(y-w)}{z}\left[(t-s)K'''_{5\eta ws}(s,z,w,0,\eta)-K''_{5\eta w}(s,z,w,0,\eta)\right]$$

$$\times\left(\int_0^s\int_{\eta}^{w}u(v_1,z,v_3)dv_3dv_1\right)^2+\frac{t-s}{zw}\left[K''_{6\xi\eta}(s,z,w,\xi,\eta)-z(x-z)K'''_{6\xi\eta z}(s,z,w,\xi,\eta)\right]\times$$

$$\left(\int_{\xi}^{z}\int_{\eta}^{w}u(s,v_2,v_3)dv_3dv_2\right)^2-(t-s)(x-z)(y-w)\left[C''_{\xi\eta}(s,z,w,0,\xi,\eta)-C'''_{\xi\eta s}(s,z,w,0,\xi,\eta)\right]$$

$$\times\left(\int_0^s\int_{\xi}^{z}u(v_1,v_2,w)dv_2dv_1\right)\left(\int_0^s\int_{\eta}^{w}u(v_1,z,v_3)dv_3dv_1\right)-(t-s)(x-z)(y-w)\left[C''_{\xi\eta}(s,z,w,0,\xi,\eta)\right.$$

$$\left.-C'''_{\xi\eta w}(s,z,w,0,\xi,\eta)\right]\left(\int_0^s\int_{\eta}^{w}u(v_1,z,v_3)dv_3dv_1\right)\times$$

$$\left(\int_{\xi}^{z}\int_{\eta}^{w}u(s,v_2,v_3)dv_3dv_2\right)-(t-s)(x-z)(y-w)\times$$

$$\left[C''_{\xi\eta}(s,z,w,0,\xi,\eta)-C'''_{\xi\eta\xi}(s,z,w,0,\xi,\eta)\right]\left(\int_0^s\int_\xi^z u(v_1,v_2,w)dv_2dv_1\right)\left(\int_\xi^z\int_\eta^w u(s,v_2,v_3)dv_3dv_2\right)\Big\}$$

$$\times d\eta d\xi dwdzds+\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^s\int_0^z\left\{\frac{y-w}{sz}\left[K''_{4\xi\tau}(s,z,w,\tau,\xi)-s(t-s)K'''_{\tau\xi s}(s,z,w,\tau,\xi\right]\times\right.$$

$$\times\left(\int_\tau^s\int_\xi^z u(v_1,v_2,w)dv_2dv_1\right)^2+\frac{x-z}{sz}\left[K'_{5\tau}(s,z,w,\tau,0)-(y-w)K''_{5\tau w}(s,z,w,\tau,0,)\right]\times$$

$$\times\left(\int_\tau^s\int_0^w u(v_1,z,v_3)dv_3dv_1\right)^2+\frac{t-s}{sz}\left[K'_{6\xi}(s,z,w,\xi,0)-z(x-z)K''_{6\xi z}(s,z,w,\xi,0)\right]\times$$

$$\left(\int_\xi^z\int_0^w u(s,v_2,v_3)dv_3dv_2\right)^2-(t-s)(x-z)(y-w)\left[C''_{\tau\xi}(s,z,w,\tau,\xi,0)-C''_{\tau\xi z}(s,z,w,\tau,\xi,0)\right]$$

$$\times\left(\int_\tau^s\int_\xi^z u(v_1,v_2,w)dv_2dv_1\right)\left(\int_\tau^s\int_0^w u(v_1,z,v_3)dv_3dv_1\right)-$$

$$-(t-s)(x-z)(y-w)\left[C''_{\tau\xi}(s,z,w,\tau,\xi,0)-\right.$$

$$\left.C''_{\tau\xi w}(s,z,w,\tau,\xi,0)\right]\left(\int_\tau^s\int_0^w u(v_1,z,v_3)dv_3dv_1\right)\left(\int_\xi^z\int_0^w u(s,v_2,v_3)dv_3dv_2\right)-(t-s)(x-z)(y-w)$$

$$\times\left[C''_{\tau\xi}(s,z,w,\tau,\xi,0)-C'''_{\tau\xi\xi}(s,z,w,\tau,\xi,0)\right]\left(\int_\tau^s\int_\xi^z u(v_1,v_2,w)dv_2dv_1\right)\left(\int_\xi^z\int_0^w u(s,v_2,v_3)dv_3dv_2\right)\Big\}$$

$$\times d\xi d\tau dwdzds+\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^s\int_0^w\left\{\frac{(x-z)(y-w)}{sw}\left[s(t-s)K'''_{4\tau zs}(s,z,w,\tau,0)-K''_{4\tau s}(s,z,w,\tau,0)\right]\right.$$

$$\times\left(\int_\tau^s\int_0^z u(v_1,v_2,w)dv_2dv_1\right)^2+\frac{x-z}{sw}\left[w(y-w)K'''_{5\eta\tau w}(s,z,w,\tau,\eta)-K''_{5\eta\tau}(s,z,w,\tau,\eta)\right]\times$$

$$\left(\int_\tau^s\int_\eta^w u(v_1,z,v_3)dv_3dv_1\right)^2+\frac{(t-s)(y-w)}{sw}\left[K''_{6\eta w}(s,z,w,0,\eta)-(x-z)K'''_{6\eta zw}(s,z,w,0,\eta)\right]$$

$$\times\left(\int_\xi^z\int_\eta^w u(s,v_2,v_3)dv_3dv_2\right)^2-(t-s)(x-z)(y-w)\left[C''_{\tau\eta}(s,z,w,\tau,0,\eta)-C'''_{\tau\eta\xi}(s,z,w,\tau,0,\eta)\right]$$

$$\times\left(\int_\tau^s\int_0^z u(v_1,v_2,w)dv_2dv_1\right)\left(\int_\tau^s\int_\eta^w u(v_1,z,v_3)dv_3dv_1\right)-(t-s)(x-z)(y-w)\left[C''_{\tau\eta}(s,z,w,\tau,0,\eta)\right.$$

$$\left.-C'''_{\tau\eta w}(s,z,w,\tau,0,\eta)\right]\left(\int_\tau^s\int_\eta^w u(v_1,z,v_3)dv_3dv_1\right)\left(\int_\xi^z\int_\eta^w u(s,v_2,v_3)dv_3dv_2\right)-(t-s)(x-z)(y-w)$$

$$\times\left[C''_{\tau\eta}(s,z,w,\tau,0,\eta)-C'''_{\tau\eta\xi}(s,z,w,\tau,0,\eta)\right]\left(\int_\tau^s\int_0^z u(v_1,v_2,w)dv_2dv_1\right)\left(\int_0^z\int_\eta^w u(s,v_2,v_3)dv_3dv_2\right)\Big\}$$

$$\times d\eta d\tau dw dz ds + \frac{1}{2}\times$$

$$\int_0^t\int_0^x\int_0^y\int_0^s\int_0^z\int_0^w\left\{\frac{(x-z)(y-w)}{sw}\left[s(t-s)K^{IV}_{4\tau\xi zs}(s,z,w,\tau,\xi)-K'''_{4\tau\xi z}(s,z,w,\tau,\xi)\right]\times\right.$$

$$\left(\int_\tau^s\int_\xi^z u(v_1,v_2,w)dv_2dv_1\right)^2+\frac{(t-s)(x-z)}{zw}\left[w(y-w)K^{IV}_{5\eta\tau ws}(s,z,w,\tau,\eta)-K'''_{5\tau\eta w}(s,z,w,\tau,\eta)\right]$$

$$\times\left(\int_\tau^s\int_\eta^w u(v_1,z,v_3)dv_3dv_1\right)^2+\frac{(t-s)(y-w)}{sz}\times$$

$$\left[z(x-z)K^{IV}_{6\xi\eta wz}(s,z,w,\xi,0)-K'''_{6\xi\eta w}(s,z,w,\xi,\eta)\right]\times$$

$$\left(\int_\xi^z\int_\eta^w u(s,v_2,v_3)dv_3dv_2\right)^2-(t-s)(x-z)(y-w)\left[C'''_{\tau\xi\eta}(s,z,w,\tau,\xi,\eta)-C^{IV}_{\tau\xi\eta z}(s,z,w,\tau,\xi,\eta)\right]$$

$$\times\left(\int_\tau^s\int_\xi^z u(v_1,v_2,w)dv_2dv_1\right)\left(\int_\tau^s\int_\eta^w u(v_1,z,v_3)dv_3dv_1\right)-$$

$$-(t-s)(x-z)(y-w)\left[C'''_{\tau\eta\xi}(s,z,w,\tau,\xi,\eta)\right.$$

$$\left.-C^{IV}_{\tau\xi\eta w}(s,z,w,\tau,\xi,\eta)\right]\left(\int_\tau^s\int_\eta^w u(v_1,z,v_3)dv_3dv_1\right)\times$$

$$\left(\int_\xi^z\int_\eta^w u(s,v_2,v_3)dv_3dv_2\right)-(t-s)(x-z)(y-w)\times$$

$$\left[C'''_{\tau\xi\eta}(s,z,w,\tau,\xi,\eta)-C^{IV}_{\tau\xi\eta z}(s,z,w,\tau,\xi,\eta)\right]\left(\int_\tau^s\int_\xi^z u(v_1,v_2,w)dv_2dv_1\right)\left(\int_\xi^z\int_\eta^w u(s,v_2,v_3)dv_3dv_2\right)\Big\}$$

$$\times d\eta d\xi d\tau dw dz ds+\frac{1}{2}\times$$

$$\int_0^t\int_0^x\int_0^y\left\{C(s,z,w,0,0,0)-(t-s)C'_s(s,z,w,0,0,0)-(x-z)C'_z(s,z,w,0,0,0)-\right.$$

$$-(y-w)C'_w(s,z,w,0,0,0)+(t-s)(x-z)C''_{zs}(s,z,w,0,0,0)+(t-s)\times$$

$$\times(y-w)C''_{sw}(s,z,w,0,0,0)+(x-z)(y-w)C''_{zw}(s,z,w,0,0,0)-(t-s)(x-z)(y-w)\times$$

$$\left.\times C'''_{szw}(s,z,w,0,0,0)\right\}\left(\int_0^s\int_0^z\int_0^w u(v_1,v_2,v_3)dv_3dv_2dv_1\right)^2dwdzds+$$

$$+\frac{1}{2}\int_0^t\int_0^x\int_0^y\left\{C'_\tau(s,z,w,\tau,0,0)-(t-s)C''_{\tau s}(s,z,w,\tau,0,0)-(x-z)C''_{\tau z}(s,z,w,\tau,0,0)-\right.$$

$$\left.-(y-w)C''_{\tau w}(s,z,w,\tau,0,0)+(t-s)(x-z)C'''_{\tau zs}(s,z,w,\tau,0,0)+(t-s)\times\right.$$

$$\times(y-w)C'''_{\tau sw}(s,z,w,\tau,0,0)+(x-z)(y-w)C'''_{\tau zw}(s,z,w,\tau,0,0)-(t-s)(x-z)(y-w)$$

$$\times C^{IV}_{\tau szw}(s,z,w,\tau,0,0)\}\{\int_\tau^s\int_0^z\int_0^w u(v_1,v_2,v_3)dv_3dv_2dv_1)^2\,dwdzds+$$

$$\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^z\{C'_\xi(s,z,w,0,\xi,0)-(t-s)C''_{\xi s}(s,z,w,0,\xi,0)-(x-z)C''_{\xi z}(s,z,w,0,\xi,0)$$

$$-(y-w)C''_{\xi w}(s,z,w,0,\xi,0)+(t-s)(x-z)C'''_{\xi sw}(s,z,w,0,\xi,0)+(t-s)\times$$

$$(y-w)C'''_{\xi sw}(s,z,w,0,\xi,0)+(x-z)(y-w)C'''_{\xi zw}(s,z,w,0,\xi,0)-(t-s)(x-z)(y-w)$$

$$\times C^{IV}_{\xi szw}(s,z,w,0,\xi,0)\}\{\int_0^s\int_0^z\int_0^w u(v_1,v_2,v_3)dv_3dv_2dv_1)^2\,d\xi dwdzds+$$

$$\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^w\{C'_\eta(s,z,w,0,0,\eta)-(t-s)C''_{\eta s}(s,z,w,0,0,\eta)-(x-z)C''_{\eta z}(s,z,w,0,0,\eta)$$

$$-(y-w)C''_{\eta w}(s,z,w,0,0,\eta)+(t-s)(x-z)C'''_{\eta sz}(s,z,w,0,0,\eta)+(t-s)\times$$

$$(y-w)C'''_{\eta sw}(s,z,w,0,0,\eta)+(x-z)(y-w)C'''_{\xi zw}(s,z,w,0,0,\eta)-(t-s)(x-z)(y-w)$$

$$\times C^{IV}_{\eta szw}(s,z,w,0,0,\eta)\}\{\int_0^s\int_0^z\int_\eta^w u\,(v_1,v_2,v_3)dv_3dv_2dv_1)^2d\eta d\xi dwdzds+\frac{1}{2}\times$$

$$\int_0^t\int_0^x\int_0^y\int_0^s\int_0^z\{C''_{\xi\eta}(s,z,w,\tau,\xi,0)-(t-s)\,C'''_{\tau\xi s}(s,z,w,\tau,\xi,0)-(x-z)\,C'''_{\tau\xi z}(s,z,w,\tau,\xi,0)$$

$$(y-w)C^{IV}_{\tau\xi sw}(s,z,w,\tau,\xi,0)+(x-z)(y-w)C^{IV}_{\tau\xi zw}(s,z,w,\tau,\xi,0)-(t-s)(x-z)(y-w)$$

$$\times C^{V}_{\tau\xi szw}(s,z,w,\tau,\xi,0)\}(\int_0^s\int_\xi^z\int_\eta^w u\,(v_1,v_2,v_3)dv_3dv_2dv_1)^2d\eta d\xi dwdzds=$$

$$\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^s\int_0^w\{C''_{\tau\eta}(s,z,w,\tau,0,\eta)-(t-s)C'''_{\tau\eta s}(s,z,w,\tau,0,\eta)-(x-z)C'''_{\tau\eta z}(s,z,w,\tau,0,\eta)$$

$$-(y-w)C'''_{\tau\eta w}(s,z,w,\tau,0,\eta)+(t-s)(x-z)C^{IV}_{\tau\eta sz}(s,z,w,\tau,0,\eta)+(t-s)\times$$

$$(y-w)C^{IV}_{\tau\eta sw}(s,z,w,\tau,0,\eta)+(x-z)(y-w)C^{IV}_{\tau\eta zw}(s,z,w,\tau,0,\eta)-(t-s)(x-z)(y-w)$$

$$\times C^{V}_{\tau\eta szw}(s,z,w,\tau,0,\eta)\}\{\int_\tau^s\int_0^z\int_\eta^w u(v_1,v_2,v_3)dv_3dv_2dv_1)^2\,d\eta d\tau dwdzds+$$

$$\frac{1}{2}\int_0^t\int_0^x\int_0^y\int_0^z\int_0^w\{C''_{\xi\eta}(s,z,w,0,\xi,\eta)-(t-s)C'''_{\xi\eta s}(s,z,w,0,\xi,\eta)-(x-z)C'''_{\xi\eta z}(s,z,w,0,\xi,\eta)$$

$$-(y-w)C'''_{\xi\eta w}(s,z,w,0,\xi,\eta)+(t-s)(x-z)C^{IV}_{\xi\eta sz}(s,z,w,0,\xi,\eta)+(t-s)\times$$

$$(y-w)C^{IV}_{sw}(s,z,w,0,\xi,\eta)+(x-z)(y-w)C^{IV}_{zw}(s,z,w,0,\xi,\eta)-(t-s)(x-z)(y-w)$$

$$\times C^{V}_{\xi\eta zw}(s,z,w,0,\xi,\eta)\}\Big(\int_0^s\int_\xi^z\int_\eta^w u(v_1,v_2,v_3)dv_3dv_2dv_1\Big)^2 d\eta d\xi dwdzds+\frac{1}{2}\times$$

$$\int_0^t\int_0^x\int_0^y\int_0^s\int_0^z\int_0^w\{C'''_{\tau\xi\eta}(s,z,w,\tau,\xi,\eta)-(t-s)C^{IV}_{\tau\xi\eta z}(s,z,w,\tau,\xi,\eta)-(x-z)C^{IV}_{\tau\xi\eta x}(s,z,w,\tau,\xi,\eta)$$

$$-(y-w)C^{IV}_{\tau\xi\eta w}(s,z,w,\tau,\xi,\eta)+(t-s)(x-z)C^{V}_{\tau\xi\eta sz}(s,z,w,\tau,\xi,\eta)+(t-s)\times$$

$$(y-w)C^{V}_{\tau\xi\eta sw}(s,z,w,\tau,\xi,\eta)+(x-z)(y-w)C^{V}_{\tau\xi\eta xw}(s,z,w,\tau,\xi,\eta)-(t-s)(x-z)(y-w)$$

$$\times C^{VI}_{\tau\xi\eta szw}(s,z,w,\tau,\xi,\eta)\}\Big(\int_\tau^s\int_\xi^z\int_\eta^w u(v_1,v_2,v_3)dv_3dv_2dv_1\Big)^2 d\eta d\xi d\tau dwdzds=$$

$$=\int_0^t\int_0^x\int_0^y\int_0^s\int_0^z\int_0^w f(\tau,\xi,\eta)u(\tau,\xi,\eta)d\eta d\xi d\tau dwdzds. \qquad (3)$$

При выполнении условий а) – г), квадратичные формы в фигурных скобках соотношения (3) неотрицательны. Поэтому из (3) получим:

$$\frac{K}{2}\Big(\int_0^t\int_0^x\int_0^y\Big)\Big(\int_0^s\int_\xi^z\int_\eta^w u\,(v_1,v_2,v_3)dv_3dv_2dv_1)^2dwdzds\le$$

$$\le\Big|\int_0^t\int_0^x\int_0^y\int_0^s\int_0^z\int_0^w f(\tau,\xi,\eta)u(\tau,\xi,\eta)d\eta d\xi d\tau dwdzds\Big|. \qquad (4)$$

Пусть $f(t,x,y)\equiv 0,\quad (t,x,y)\in G.$ Тогда из (4) следует, что

$\int_0^t\int_0^x\int_0^y u(v_1,v_2,v_3)dv_3dv_2dv_1\equiv 0,$ т.е $u(t,x,y)\equiv 0,\quad (t,x,y)\in G.$.

Таким образом, доказана следующая, теорема.

Теорема. Пусть выполняются условия а) –г). Тогда решение уравнения (1) в пространстве $L_2(G)$ единственно.

ЛИТЕРАТУРА.

1. А.Асанов Функц. анализ и его приложения ,17, № 4 (1983) 73-74.

2. М. И. Иманалиев, А. Асанов Исслед. по интегро-дифференц. уравнениям, 13 (1988) 3-38.

3. А.Асанов Исследов. по интегро-дифференц. уравнениям, 13 (1980) 207-215.

4. А.Л. Бухгейм Докл. АН.СССР, 242, №2 (1987) 272-275.