ADJUSTMENT COSTS AND OPTIMAL TAXES IN A DYNAMIC MODEL OF POLLUTION

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1. Introduction

The effects of pollution upon the environment are causing increased concern in the developed countries. This concern centers on the policies to adopt to control the level of pollution. Some countries are also concerned in reducing the level of pollution. The developing and less developed countries can benefit from the experience of the developed countries in pollution control. The next century will probably experience stringent regulations on environmental pollution as the problem becomes enormous. Societies are becoming more and more aware of the environmental issues. This awareness will reflect itself in a stronger demand for strategic policies to deal with environmental issues.

Following Pigou, economists have regarded environmental pollution as an issue of externality. We point out that simple policies designed to deal with externalities may fail when used against the problem of environmental pollution which requires centralized control. Keller, Spence, and Zeckhauser (1971) suggests that optimal control of pollution may require restrictions on the consumption of certain type of products, limiting or completely abandoning the use of some inputs and productive processes, and possible restricting the growth of certain industries. The control of pollution may even require limiting the population growth. These intertemporal aspects of pollution are best addressed within the framework of optimal control theory. Therefore, we put the pollution problem into the framework of optimal control theory and analyze several issues addressed in this paper within this framework. Similar approaches can be found in Keller, Spence, and Zeckhauser (1971), Froster (1975), Gruver (1976), and Lee (1977).

It is observed that the standards set for controlling pollution are changing over time. For instance, the emission standards set for air pollution have changed significantly. Many countries have also changing water quality standards. Why it is optimal to set changing standards over time needs an economic explanation. Harford (1975) offers three explanations. First, the amount of wastes being created and diffused had an upward trend. This may be due to insufficient preventive or treatment standards. Therefore, governments faced with this upward trend set stringent affluent standards. Second, there have been costless improvements in the waste treatment and prevention technology. This second reason may lead to both tighter ambient and emission standards. Third, the rate of change of standards affects the level of costs. This means that to achieve a certain target in two years costs more than achieving it in three years. This phenomenon is called adjustment costs in the literature. Economists have become aware that firms may face adjustment costs when changing their level of capital stock. Adjustment costs can partly be explained by increases in the prices of capital and skilled labor due to rapidly increasing demand. In this study, we point out to another aspect of pollution prevention. In order to achieve socially optimal levels of pollution, economists propose marginal damage taxes. The pollution taxation is usually assumed to be costless. However, collection costs and costs of determining marginal damages make the pollution taxation costly. We argue that these costs will depend on the rate of change of the pollution as well as the level of pollution stock. Because, as the level of pollution increases new parties will be affected, this will require new marginal damage calculations. This explains the observed facts that preventive pollution measures usually come after pollution reaches a certain level. It is optimal to delay pollution taxes because of adjustment costs associated with taxation.

Pollution type market failures are usually analyzed in a static environment. Marginal damages obtained from static models form the basis of the Pigouvian corrective taxation. However, dynamic aspects of pollution have significant economic consequences. The optimal time path of pollution can only be derived from a dynamic model. In this study, we derive the time path of optimal pollution tax using a dynamic model. The Pigouvian pollution tax as a method of correcting market failure is usually assumed to be costless. We show that adjustment costs will affect the optimal path of the Pigouvian pollution tax. We further show that the corrective tax rate can be used to set a standard even though the optimal tax rate and the optimal pollution level are unknown to the policymaker.

This paper is organized as follows. Section 2 sets up the basic dynamic model of the economy. The adjustment cost function is also introduced in this section. In section 3, we derive the Pareto optimal allocations. The time path of optimal pollution tax is also derived in this section. Section 4 presents the steady state analysis of the model. In Section 5, we introduce an alternative formulation and present some numerical examples that support the predictions of the model. The Pigouvian pollution tax is discussed in Section 6. Section 7 concludes the paper. Mathematical derivations are given in the Appendix.

2. The Model

There is more than one model that can be used to analyze the economic impacts of pollution. Pollution has major effects on consumption, production, or both. In our model, society derives utility from consumption and production process generates externality type pollution. Although we are analyzing a pollution type externality, the model can be generalized to other kinds of externalities. The interdependence of consumption and pollution through production makes them joint products. This also allows us to measure pollution in the same units in which consumption is measured. In order to focus on the basic problem, we assume away the problems of population growth, technical change, and capital accumulation. These problems can easily be incorporated into the model.

Our assumption about the nature of pollution implies that pollution enters directly into the social utility function. Pollution may have adverse effect on production as well. We assume that social welfare of society at time t is a function of consumption in time t, $C(t)$, and pollution in time t, $P(t)$.The time invariant social utility function and its properties are as follows:

(1) $U = U\big(C(t), P(t)\big)$,

(2) $U_C > 0, \quad U_{CC} < 0, \quad C(t) > 0,$

(3) $P_C < 0, \quad P_{CC} < 0, \quad P(t) \geq 0,$

(4) $U_{CC}U_{PP} - U_{CP} \geq 0, \quad U_{CP} < 0,$

(5) $\lim_{C \to 0} U_C\big(C(t), P(t)\big) = 0,$ for all $P(t) > 0,$

(6) $\lim_{P \to 0} U_P\big(C(t), P(t)\big) = 0,$ for all $C(t) > 0.$

Assumptions stated in (2)-(5) guarantee a well behaving utility function. Although we assumed

$U_{CP} < 0$, this assumption is not essential for the analysis. One can certainly assume a separable utility function for simplicity and we will do so when assumption of a separable utility function is not damaging for the analysis. A separable utility function can be specified by assuming $U_{CP} = 0$.

In static models of pollution, the distinction between a stock and flow pollution is not important and usually ignored. In a dynamic model, it is possible to make a distinction between a stock and flow variable. A pollutant may have both stock and flow impacts. In some cases, such as river pollution, stock concept with a decay rate is appropriate. In other cases, flow concept better describes the nature of pollution. Noise pollution is such a case. Noise pollution has a decay rate of one. A decay rate equal to one makes pollution a perfect flow variable. We believe that pollution control is usually a stock control problem. However, many pollutants have a nonzero decay rate. Recognizing this fact we assume that pollution evolves according to the following differential equation,

(7) $\dot{P} = F\big(C(t)\big) - dP(t),$

(8) $0 \leq d \leq 1, \quad F(0) = 0, \quad C(t) > 0,$

(9) $F' > 0, \quad F'' > 0, \quad \lim_{C \to 0} F'\big(C(t)\big) = 0,$

where $F\big(C(t)\big)$ is the flow of pollution at any point in time. $F\big(C(t)\big)$ can also bethought as the pollution control function. The particular specification given in (7)-(9) is fairly standard in the literature. Societies usually have no direct control over the quantity of pollution. This implies that pollution should be treated as a state variable, not a choice. There is no choice in regard to pollution. By selecting the level of $C(t)$, society uniquely determines the net pollution level it desires. d in expression (7) is the constant decay rate. The assumption of constant decay rate is fairly common in the literature and we offer no justification for it. If $0 \leq d < 1$, but not large enough to reduce pollution level to the desired level in desired time, present consumption must be foregone in order to have less pollution in the future. There is no need to forego present consumption in order to reduce future pollution level when d=1. If d=0, foregoing present consumption is the only way of reducing pollution.

In order to get around the problems of capital accumulation, population growth, etc., we assume that a fixed amount of net of replacement investment output Y^0 is produced in each period. We further assume that markets are perfectly competitive and there is no Coasian solution to pollution. We do not think the Coasian solution will be accessible even for the solution of externality type pollution. One reason for this may be high transaction costs, among others.

When there is no government intervention society would choose $C(t)=Y^0$ in each time period. This is the competitive solution. However, this solution is not Pareto optimal in the sense that $C(t)<Y^0$ is a Pareto improvement over $C(t)=Y^0$. In order to get Pareto optimal allocations government may impose a Pigouvian tax on either *P* or *C*. It is not unrealistic to assume that some resources should be allocated to antipollution activities. The literature on environmental pollution mostly concentrated on the damage and abatement costs. We agree on the importance of these costs. However, we believe that the societies should concentrate on preventive measures rather than abatement. When abatement and prevention are equally costly, we think that prevention should be preferred. Preventive measures traditionally include installment of filters, abandoning a certain type of production process, adaptation a of a new technology, etc. We also include taxation among the preventive measures. As it is true for the other type of preventive measures, society should forgo some resources for the collection and imposition of taxes. These foregone resources can be interpreted as the cost of taxation. Total taxation costs may include collection costs, costs incurred in determining marginal damages, etc. As pointed out before, these costs depend on both the level and rate of change of pollution stock. The dependence on the rate of change is known as adjustment costs.

There is more than one way to explain adjustment costs in the literature. Following Treadway (1969) we can treat adjustment costs as a loss in output arising from the new capital investment for reducing pollution. This is an intra-firm cost arising from the difficulty of undertaking greater and greater levels of investment in a certain time interval. A second way of explaining adjustment costs is due to Eisner and Strotz (1963). They treat adjustment costs as arising from a rising supply curve of capital goods as the demand for capital goods increases. Because of an upward sloping supply curve, the price of new capital goods will increase as all firms demand larger amounts. This type of adjustment costs are called extra-firm, because it is caused by the behavior of all firms together. As firms are required to invest in pollution reduction, they will face these types of adjustment costs. Investment in waste treatment plants will of course be subject to these types of adjustment costs. We further argue that taxation when used as a preventive measure against pollution will also be subject to adjustment costs. This means that cost of taxation will depend on both the level and rate of change of pollution stock. It depends on the level of pollution since collection costs depend on the level of total taxes collected. Dependence on the rate of change of pollution can be explained as follows: As pollution grows faster, new parties will be affected. This will require new marginal damage calculations and extending the coverage of pollution tax. Effect of this on the cost of taxation will be the same as the effect of increased investment on costs. Adjustment costs may also arise from the entrance of new firms, which brings extra costs to taxation. Based on these reasons, we may write pollution cost function as

(10) $$G = G\left(P(t), \dot{P}(t)\right),$$

(11) $$G_P > 0, \quad G_{PP} > 0, \quad G_{\dot{P}} > 0, \quad G_{\dot{P}\dot{P}} > 0, \quad G_{P\dot{P}} = 0,$$

(12) $$\lim_{\substack{P\to 0 \\ \dot{P}\to 0}} G_P\left(P(t), \dot{P}(t)\right) = \lim_{P\to 0} G_P\left(P(t), \dot{P}(t)\right) = \lim_{\dot{P}\to 0} G_{\dot{P}}\left(P(t), \dot{P}(t)\right) = 0,$$
$$\lim_{P\to 0} G_{PP}\left(P(t), \dot{P}(t)\right) = \lim_{\dot{P}\to 0} G_{\dot{P}\dot{P}}\left(P(t), \dot{P}(t)\right) = 0,$$

(13) $$\lim_{\dot{P}\to 0} G\left(P(t), \dot{P}(t)\right) = \min\left\{G\left(P(t), \dot{P}(t)\right)\right\}.$$

Conditions in (11) yield a separable and nondecreasing cost function in its arguments. The evidence suggests that cost function is an increasing function both in the level and rate of change of pollution. However, very little is known about the curvature of such a cost function. The conditions given in (11)-(13) except the separability assumption are minimum requirements for a well behaving cost function that includes adjustment costs. The condition in (13) states that costs are at minimum when $\dot{P}=0$. Therefore, the level of costs given in (13) can be interpreted as the long-run cost of maintaining the pollution stock at the socially optimal level. After introducing $G\big(P(t),\dot{P}(t)\big)$, society should allocate output over $C(t)$ and $G(t)$, namely,

(14) $Y^0 = C(t) + G\big(P(t),\dot{P}(t)\big).$

In writing (14), we assume that tax revenues are rebated to consumers in a nondistortionary way. Equivalently, we can assume that tax revenues are spent on imports by the government and imported goods are distributed to consumers. However, this last assumption requires that the foreign country from which the goods are imported does not create any externality for the importing country. We further assume that government budget is balanced.

3. The Pareto Optimal Solution

Now we seek a Pareto optimal solution that maximizes social welfare subject to the constraints given in (7) and (14). Another question we are trying to answer is what should be the per unit Pigouvian tax that yields a Pareto optimal allocation. In order to determine the Pareto optimal solution we assume that a social planner maximizes the discounted flow of utility over an infinite horizon. Assume that future utility streams are discounted at a constant exponential rate s>0. Then, the social planner needs to find solution of the following problem:

$$\max_{C(t)} \int_0^\infty U\big(C(t),P(t)\big)e^{-st}\,dt,$$

(15) $$\text{s.t. } \dot{P}(t) = F\big(C(t)\big) - dP(t), P(0)=P_0, P(\infty) \text{ free},$$

$$Y^0(t) = C(t) + G\big(P(t),\dot{P}(t)\big), P(t)\geq 0, C(t)>0.$$

This is a fairly standard optimal control problem, which can be solved using the Pontryagin's maximum principle. The solution to this problem will give the socially optimal time paths of $P(t)$ and $C(t)$. Then, government's objective is to choose these same time paths of $P(t)$ and $C(t)$ which could be chosen by a social planner. In practice, the government needs to impose a Pigouvian tax on pollution in order to obtain the Pareto optimal time paths of $P(t)$ and $C(t)$. The maximization problem in (15) is an optimal control problem in one state variable, $P(t)$, and one control variable, $C(t)$. First order conditions from the Pontryagin's maximum principle, which are derived in the Appendix, yield

(16) $U_C\big(C(t),P(t)\big) = -l(t)F'\big(C(t)\big) + m(t)\big[1 + G_{\dot{P}}\big(P(t),\dot{P}(t)\big)F'\big(C(t)\big)\big],$

(17) $\dot{l}(t) = (s+d)\,l(t) - U_P\big(C(t),P(t)\big) + m(t)\Big[G_P\big(P(t),\dot{P}(t)\big) - dG_{\dot{P}}\big(P(t),\dot{P}(t)\big)\Big].$

Equation (16) states that marginal utility of consumption should be equal to marginal cost. Costate variable $l(t)$ can be interpreted as the shadow price of pollution. This shadow price can be determined by solving the linear differential equation (17). This solution yields

$$l(t)=\int_t^{\infty}\left\{U_P\left(C(t),P(t)\right)-m(s)\left[G_P\left(P(s),\dot{P}(s)\right)-dG_{\dot{P}}\left(P(s),\dot{P}(s)\right)\right]\right\}e^{-(s+d)(s-t)}ds.$$

This expression states that $-l(t)$ is the discounted value as of time *t* of the later marginal disutility plus the marginal taxation cost of pollution. From this last expression the appropriate time path of damage tax imposed on pollution flow can be determined as $t(t)=-l(t)$. This pollution tax yields the Pareto optimal allocations. Alternatively a tax which is equal to $-l(t)F'(C(t))$ can be imposed on consumption. The per unit Pigouvian tax on pollution for each time period *t* can be determined from (17) as

(18) $$t(t)=\int_t^{\infty}\left\{m(s)\left[G_P\left(P(s),\dot{P}(s)\right)-dG_{\dot{P}}\left(P(s),\dot{P}(s)\right)\right]-U_P\left(C(s),P(s)\right)\right\}e^{-(s+d)(s-t)}ds.$$

Therefore, $t(t)$ is set equal to the discounted value as of time *t* of later marginal disutility plus the marginal taxation cost of the pollution.

One thing that needs an explanation in (18) is the negative effect of $G_{\dot{P}}$ on the per unit tax. The intuition behind this is simple. When there is an increase in current consumption away from the Pareto optimal level, $\dot{P}(t)$ will be positive. However, *d* percent of increased pollution will dissolve by itself. Therefore, the current pollution tax rate should be reduced by $dG_{\dot{P}}$. This is simply the decay effect and will disappear when *d*=0. This does not mean that the tax rate will be lower when consumption has increased. The overall tax rate will always be higher, because *P*(*t*) is now higher. Unfortunately, it is not optimal for the government to adjust the new tax rate immediately. During the period of positive $\dot{P}(t)$ firms and government incur extra costs at an increasing rate. Therefore, it is optimal for government to spread the tax increase over a longer period to save on costs. This is shown in Figure 1.

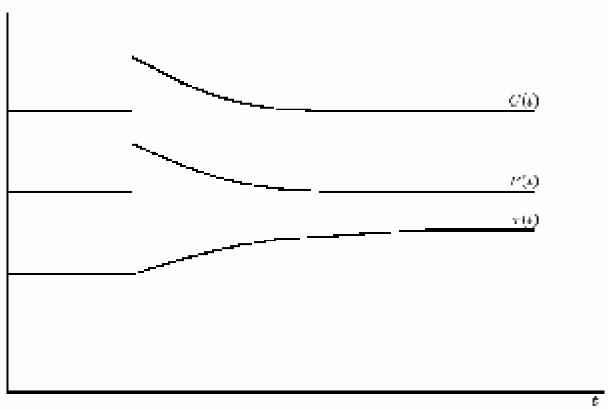

Figure 1: Effect of Adjustment Costs on the Time Path of the Pollution Tax

Assume that there is an unexpected increase in consumption at time t_0. For simplicity set *d*=1. While *P*(*t*) jumps simultaneously with consumption, $t(t)$ needs to be adjusted over a longer time interval. This implies that in a continuously growing economy $t(t)$ should never be at its equilibrium level. Nevertheless, this is an optimal policy to save on costs which cause a utility loss more than the utility loss due to pollution. Another point which is worth to mentioning is that this model assumes perfect foresight. Therefore, response of $t(t)$ on the changes in time paths of $C(t)$, P(t), and $\dot{P}(t)$ differ under expected and unexpected changes. In the above analysis, we assumed an unexpected changes in $C(t)$. If the change is expected, the response of $t(t)$ will depend on the time paths of partial derivatives appearing in (18). For instance, optimal policy is to raise $t(t)$ immediately, if $C(t)$ is expected to increase.

4. Steady State Equilibrium

A steady state equilibrium is characterized by the condition $\dot{P}(t)=0$ and $\dot{t}(t)=0$. When $\dot{P}(t)=0$, we get

$$(19) \qquad P^* = \frac{1}{d}F(C^*),$$

where P^* and C^* are the steady state values of $P(t)$ and $C(t)$, respectively. $\dot{t}(t)=0$ yields the following expression for the steady state equilibrium value of $t(t)$:

$$(20) \quad t^* = \frac{m^* G_P(P^*,0) - U_P(C^*,P^*)}{s+d},$$

where

$$m^* = \frac{(s+d)l^* - U_P\left(C^*,P^*\right)}{G_P\left(P^*,0\right)}.$$

Phase plane of (P,t) is shown in Figure 2. The equilibrium is a unique saddle point at (P^*,t^*), which is approached asymptotically along the stable branch of the saddle. The slopes of the singular curves are derived in the Appendix.

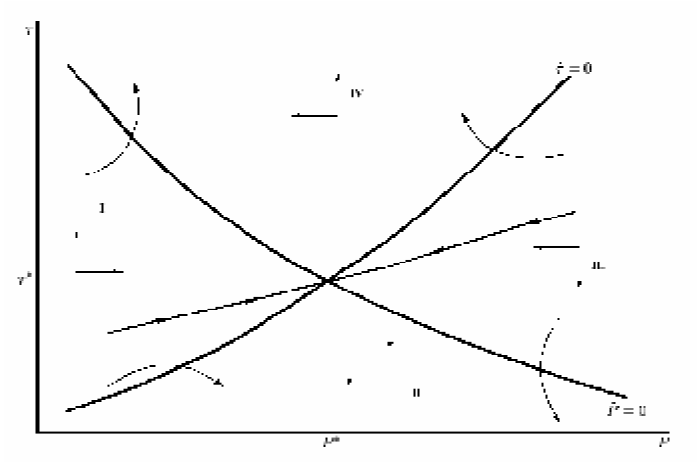

Figure 2: Phase Plane of Steady State Equilibrium

The saddle point equilibrium is a well known property of many economic models. One may wonder how come the initial conditions always guarantee convergence to equilibrium. In fact, various initial conditions will usually lead to unstable trajectories. However, unstable trajectories violates the transversality condition. Hence, imposition of the transversality condition is what guarantees a stable equilibrium. In the following section, we present an example that demonstrates the saddle point equilibrium property. In the example, imposition of the transversality condition leads to elimination of unstable solution due to positive eigenvalues.

5. An Example

The models appearing in the literature on adjustment costs always put the adjustment costs in the objective function additively. In dynamic firm theory, adjustment costs are assumed to be a loss in the revenue. Therefore, it appears in profit function in an additive way. The literature that introduces adjustment costs into pollution models also set up models with adjustment cost function additively appearing in the objective function. For instance, Beavis (1976) and Harford (1976) both minimize damage plus adjustment costs. In this way, they are able to put adjustment cost function additively in the objective. The reason for setting up a model with adjustment cost function additively appearing in the objective is the intractability of the resulting differential equations when adjustment costs do not appear additively in the objective. In our model, adjustment cost function appears as a nonlinear constraint. Therefore, resulting differential equations from the first order conditions of the dynamic optimality are intractable. For instance, a Cobb-Douglas utility function and a quadratic adjustment cost function results in a very complicated nonlinear differential equations system. In order to gain some more insight about the dynamic nature of the solution and make some numerical comparisons we need tractable differential equations. We tried a quadratic separable utility function along with a quadratic adjustment cost function in order to simplify the solution. However, resulting nonlinear differential equations were still hard to manage.

One way to simplify the mathematics involved in the solution of (15) is to follow the previous literature on adjustment costs and put adjustment cost function in the objective additively. In order to do so, we will assume that adjustment costs cause a direct utility loss. This is possible if adjustment costs and utility are measured in the same units. This assumption may be too strong and require a cardinal utility measure. However, other forms of utility functions are not more innocent and realistic than a cardinal utility function in the context of our analysis. Hence, we will assume away any objections and reformulate the model in the following way:

$$\max_{C(t)} \int_0^\infty \left[U\left(C(t),P(t)\right)-G\left(P(t),\dot{P}(t)\right)\right]e^{-st}\,dt,$$

(21) $$\text{s.t. } \dot{P}(t)=F\left(C(t)\right)-dP(t),$$

$$P(0)=P_0, P(\infty) \text{ free}, P(t)\geq 0, C(t)\geq 0.$$

In order to get tractable differential equations we will specify a separable quadratic utility function. That is,

(22) $$U=\frac{1}{2}aC(t)^2-\frac{1}{2}bG(t)^2.$$

The following adjustment cost function satisfies all conditions given in (11)-(13):

(23) $$G=\frac{1}{2}fP(t)^2+\frac{1}{2}y\dot{P}(t)^2.$$

The last function we need to specify is $F(C(t))$. Assuming each unit of $C(t)$ creates g percent pollution flow suffices for our purpose. Under this assumption pollution accumulates according to

(24) $$\dot{P}(t) = gC(t) - dP(t).$$

We can determine the conditions of dynamic optimality for the problem in (21) using the Pontryagin's maximum principle. These conditions for the specifications given in (22)-(24) are obtained in the Appendix. The resulting differential equations from the first order conditions for dynamic optimality are as follows:

(25) $$\dot{P}(t) = \frac{g^2}{g^2 y - a} l(t) + \frac{ad}{g^2 y - a} P(t),$$

(26) $$\dot{l}(t) = (s + d) l(t) + (b + f) P(t) + dy\dot{P}(t).$$

where $l(t)$ is the costate variable. Equations (25)-(26) form a homogenous simultaneous differential equations system. It is simple to obtain solution of this homogeneous linear differential equations system. Before proceeding to the solution we will put this system into a matrix form. Let us define the following matrices:

(27) $$\Pi = \begin{bmatrix} 1 & 0 \\ -dy & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} -ag^2/(g^2 y - a) & -g^2/(g^2 y - a) \\ -(b + f) & -(s + d) \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} P(t) \\ l(t) \end{bmatrix}.$$

Using these definitions, the system of differential equations in (25)-(26) can be expressed as

(28) $$\Pi\dot{\mathbf{x}} + \Gamma\mathbf{x} = \mathbf{0}.$$

The solution of (39) can be written as

(29) $$\mathbf{x}(t) = \mathbf{C}e^{xt},$$

where $\mathbf{C}$ is a 2×1 vector of constants to be determined and x is a scalar. To avoid a trivial solution it is necessary that

(30) $$|x\mathbf{\Pi} + \mathbf{\Gamma}| = \mathbf{0}.$$

The eigenvalues resulting from the solution of the characteristic equation (30) will yield the x_i we need for the solution in (29). The solution of (31) yields the following eigenvalues:

(31) $$x_1 = \frac{-q + \sqrt{q^2 - 4V}}{2(a - g^2 y)}, \qquad x_2 = \frac{q + \sqrt{q^2 - 4V}}{2(a - g^2 y)},$$

where

$$q = as - g^2(2d + s)y$$
$$V = (bg^2 - ad^2 - ads + g^2 f)(a - g^2 y)$$

A sufficient condition for all roots be real is $q^2 > -4V$. For simplicity, we assume that this condition is satisfied. It is simple to see that signs of x_1 and x_2 are ambiguous, but for most parameter values they have opposite signs. In Section 4, we showed that steady state equilibrium is a saddle point. This result assures that the solution has a saddle path equilibrium. To better illustrate this result assume that

$b = f = s = y = 1,\ a = 2,\ g = 0.7$ and $d = 0.6$. Then, it is simple to show that

(32) $\quad x_1 \cong -0.54, \quad x_2 \cong 1.15.$

Since $x_2 > 0$ the equilibrium is a saddle path. We use the transversality conditions to force a stable equilibrium and write the solution as

(33) $\quad \mathbf{x}(t) = \mathbf{C}e^{x_1 t},$

where **C** is a 2×1 vector of arbitrary constants. To illustrate the dynamic properties of the solution we compute the time paths of *P*(*t*) and $P^*(t)$ for the solution in (33). In order to compute time paths of *P*(*t*) and $P^*(t)$, we need to know the time path of *C*(*t*). Assume that *C*(*t*)=*t* for *t*=1,2,..,19,21,..,50. We set *C*(20)=40, creating a 20 unit unexpected shock in *C*(*t*) at time 20. The time path of *C*(*t*) under this specification is plotted in Figure 3. This shock in *C*(*t*) takes the system out of equilibrium. Starting from the disequilibrium position at time 20 we will be able to derive the time path of *P*(*t*).

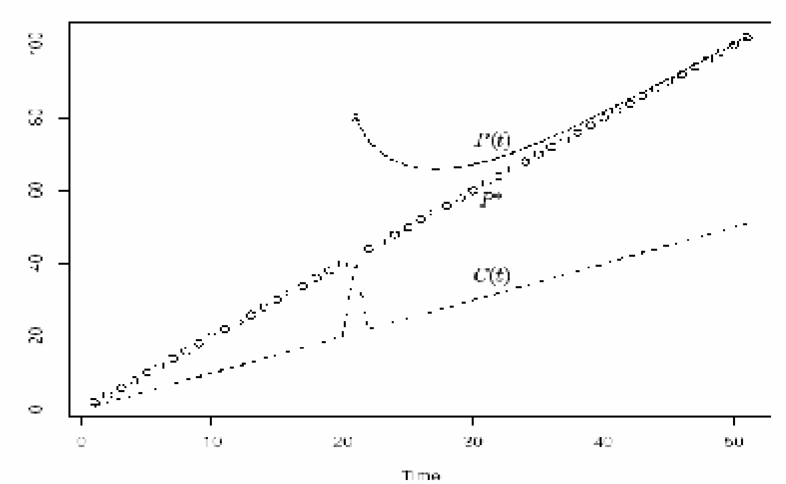

Figure 3: Response to an Unexpected Shock in Consumption

In Figure 3, we plot the time path of $P^*(t)$ obtained from the steady state solution. Note that the shock in *C*(*t*) creates a corresponding jump in *P**(*t*). Further, as *C*(*t*) falls down to its normal trend *P**(*t*) also falls down to its normal trend. However, *P*(t) will not fall immediately and diverge from *P*(*t*) for a long time. It takes about 30 periods for *P*(*t*) to converge to *P**(*t*). The time path of *P*(*t*) is also plotted in Figure 3.

In order to illustrate the effect of adjustment costs we simulate a simple economy with two cases. In the first case, we assume that there are no adjustment costs, $y = 0$. In the second case, an economy with adjustment costs is simulated with $y = 1$. Both models are calibrated using values for other parameters given as above. The economy with adjustment costs will have the roots given in (32). The economy without adjustment costs will have the following roots:

(34) $x_1 \cong -0.35, \quad x_2 \cong 1.35.$

We again force a stable solution for both cases. We compare results against the following values of steady state equilibrium:

$C^* = 438, \quad P^* = 184, \quad t^* = 0.22$

We assume that *C* and *P* are measured in thousands and t is measured as percentage.

In order compare the properties of these two hypothetical economies we take each economy out of equilibrium and compare how C(*t*), *P*(*t*), and $t(t)$ approach the equilibrium. For this, we set $C^* = 656$. The values of P^* and t^* consistent with this value of C^* are 275 and 0.51, respectively. The time paths of each economy under these configurations are calculated from the equations in (A17)-(A19). These time paths are given in Table 1.

Table 1: Effect of Adjustment Costs in a Simulated Economy

Time	*C*(*t*)	*P*(*t*)	$t(t)$	*C*(*t*)	*P*(*t*)	$t(t)$
	y=0 (no adjustment costs)			y>0 (with adjustment costs)		
0	656.00	275.00	0.55	656.00	275.00	0.55
1	565.04	237.03	0.41	591.62	248.13	0.45
2	512.03	214.90	0.33	546.26	229.19	0.38
3	481.14	202.01	0.29	514.29	215.84	0.34
4	463.14	194.49	0.26	491.76	206.44	0.30
5	452.65	190.12	0.24	475.88	199.81	0.28
6	446.54	187.56	0.23	464.70	195.14	0.26
7	442.98	186.08	0.23	456.81	191.85	0.25
8	440.90	185.21	0.22	451.26	189.53	0.24
9	439.69	184.71	0.22	447.34	187.90	0.23
10	438.98	184.41	0.22	444.58	186.75	0.23

Comparison of values $t(t)$ for the economies with adjustment costs and without adjustment costs very clearly reveals the prediction of our model about the adjustment of $t(t)$. When there are adjustment costs $t(t)$ is adjusted much slowly. When there are no adjustment costs $t(t)$ approaches the equilibrium two times faster than $t(t)$ under adjustment costs. As predicted by the model, slow adjustment of $t(t)$ leads slow adjustments of consumption and pollution level. Therefore, along the adjustment path consumption and pollution level will be higher when there adjustment costs.

6. A Pigouvian Pollution Tax

Baumol (1972) proposes a tax adjustment scheme leading to the Pareto optimal solution. Baumol's proposition is to set a Pigouvian tax which is equal to current marginal damage. This tax rate may or may not be equal to t^*. We can analyze Baumol's proposition using Figure 4. Assume that current pollution level is P_a. Under Baumol's proposition government would set a tax equal to t_a. At this tax rate $\dot{P}<0$ and the marginal damage falls. This leads government to set a lover tax rate. If pollution level falls to P_b, then government should set the new tax rate to t_b. When the tax rate is set at t_b the marginal damage will further fall leading to a further decrease in the tax rate. This process will eventually converge to (P^*,t^*). The process starts at point *a* and continues along the $\dot{t}=0$ curve. We should point out that it may take a long time to locate the optimal tax rate. There is a practical difficulty for the implementation of this tax adjustment scheme. That is, government may not be able to calculate the current marginal damages. In that case, there is no trajectory leading to (P^*,t^*) and t^* can never be located but chance.

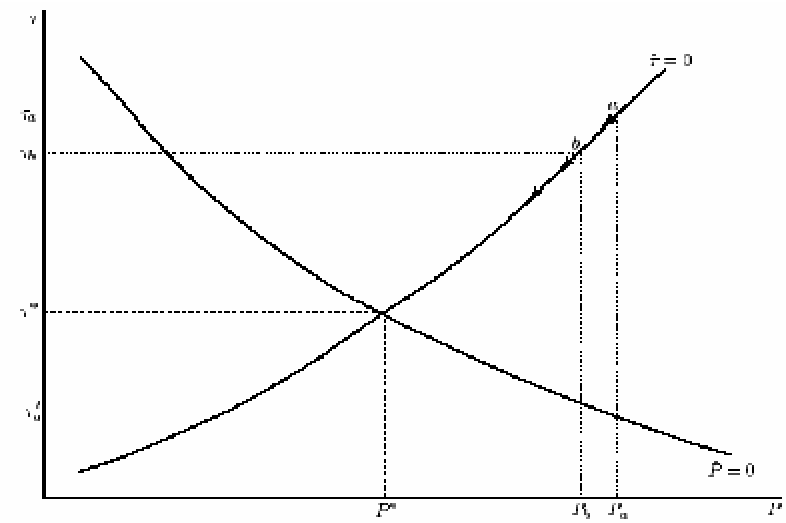

Figure 4: Baumol's Adjustment Scheme for a Pigouvian Tax

Suppose that government knows neither P^* nor the current marginal damage. Is there a way to obtain a better outcome than the one prevailing under competitive markets? If we follow the Baumol's suggestion of setting pollution standards, which may not be the same as the Pareto optimal level of pollution, this would be possible. Suppose that government sets a pollution standard of P_a. A tax rate of t'_a would satisfy $\dot{P}=0$. Therefore, a pollution level of P_a would be sustained even if it is not the Pareto optimal steady state level. Although P_a is not the Pareto optimal pollution level, it is still better than a pollution level that exceeds P_a under no government intervention.

7. Conclusion and Summary

Recent years witnessed a growing awareness of many countries about the effects of environmental pollution on the human life. Many countries have been making legal regulations to control pollution. These regulations usually set certain standards including the restriction of certain methods of production or consumption of some goods. We believe that certain intertemporal aspects of pollution require economic regulations through centralized control. A most important and efficient measure is the taxation of certain goods. In a static environment taxation is a simple matter. However, dynamic aspects of pollution complicate the taxation. Furthermore, taxation may have significant economic costs.

In this study, we point out that taxation will usually involve significant adjustment costs. In addition, pollution reduction itself will have capital investment with large adjustment costs. We showed that adjustment costs lead to a different optimal tax rate. Particularly, the Pigouvian tax rate obtained from a dynamic model differs from the one obtained from a static model due to two factors. First, adjustment costs alter the time path of optimal tax when the pollution level is not at the steady states. Second, optimal tax rate will be different both on and out of the steady states to the extend that future utility is discounted. Assuming decay rate is one and no taxation costs, a dynamic model implies that the optimal Pigouvian tax rate should be lower than the one implied by a static model. Furthermore, once the cost of pollution taxation is introduced, tax rate will increase, and relative magnitude will depend on these two effects. If the marginal damages can be determined, government can reach the Pareto optimal allocation by adjusting the tax rate even if the optimal tax rate is not known. If the marginal damages cannot be determined and the Pareto optimal allocations are not known, government can set pollution standards for a better outcome.

APPENDIX

In order to solve the maximization problem given in (15) we use the optimal control theory. We translate the problem into the optimal control format by forming the following current value Hamiltonian:

(A1) $$H = U\big(C(t), P(t)\big) + l(t)\big[F\big(C(t)\big) - dP(t)\big].$$

This Hamiltonian takes care of only the constraint imposed through the state equation. In order to incorporate the remaining constraints into the problem we form the following Lagrangian:

(A2) $$L = H + m(t)\Big[Y^0 - C(t) - G\Big(P(t), F\big(C(t)\big) - dP(t)\Big)\Big] + p(t)\big[F\big(C(t)\big) - dP(t)\big] + w(t)C(t).$$

(A2) can be maximized using the Pontryagin's maximum principle. The first order necessary conditions for dynamic optimality are given by

(A3) $$\begin{aligned} L_C &= U_C\big(C(t), P(t)\big) + l(t)F'\big(C(t)\big) \\ &- m(t)\big[1 + G_{\dot{P}}\big(P(t), \dot{P}(t)\big)F'\big(C(t)\big)\big] + p(t)F'\big(C(t)\big) + w(t) = 0, \end{aligned}$$

(A4) $$\begin{aligned} L_P &= -\dot{l}(t) + (s + d)\,l(t) - U_P\big(C(t), P(t)\big) \\ &- m(t)\Big[dG_{\dot{P}}\big(P(t), \dot{P}(t)\big) - G_P\big(P(t), \dot{P}(t)\big)\Big] + p(t)d = 0, \end{aligned}$$

(A5) $$L_l = \dot{P}(t) = F\big(C(t)\big) - dP(t),$$

(A6) $m(t) \geq 0, \quad m(t)\left[Y^0 - C(t) - G\big(P(t), F\big(C(t)\big) - dP(t)\big)\right] = 0,$

(A7) $p(t) \geq 0, \quad p(t)P(t) = 0, \quad p(t)\dot{P}(t) = 0, \quad P(t) \geq 0,$

(A8) $w(t) \geq 0, \quad w(t)C(t) = 0, \quad C(t) \geq 0,$

(A9) $\lim_{t\to\infty} e^{-st}\, l(t) \geq 0, \quad \lim_{t\to\infty} e^{-st}\, l(t)P(t) = 0.$

It is easy to show that p(t)=w(t)=0 for any level of positive consumption. We assume that C(t)>0 since zero consumption is not Pareto optimal. Therefore, rearranging (A3) and (A4) we get the equations (16) and (17) in the text.

To derive the slopes of $\dot{P}(t) = 0$ and $\dot{l}(t) = 0$ locu we solve (A3)-(A5) for C(t) by eliminating $\dot{P}(t)$ and m(t). This yields

(A10) $C(t) = C\big(P(t), l(t)\big).$

Substituting (A10) into (19) and (20) and using t(t)=-l(t), we derive the following slopes for the singular curves in Figure 2:

(A11) $$\left.\frac{dt}{dP}\right|_{\dot{P}=0} = \frac{d - F'C_P}{F'C_t} < 0,$$

(A12) $$\left.\frac{dt}{dP}\right|_{\dot{l}=0} = \frac{-\left[U_{CC}U_{PP} - U_{CP}^2 - t^*U_{PP}F'' - \left(U_{CC} - t^*F''\right)G_{PP}\right]}{\left(U_{CC} - t^*F''\right)\left(s + d + U_{CP}C_t\right)} > 0,$$

where

(A13) $$C_t = \frac{F'}{U_{CC} - t^*F''} < 0,$$

(A14) $$C_P = \frac{-U_{CP}}{U_{CC} - t^*F''} < 0.$$

The maximization problem given in (21) can be solved in the same way. In order to put the problem in the format of the optimal control theory, let $V(t) = \dot{P}(t)$. Using this definition and the specifications given in (22)-(24), the current value Hamitonian can be set up as

(A15) $$H = \left[\frac{1}{2}aC(t)^2 - \frac{1}{2}bP(t)^2\right] - \left[\frac{1}{2}fP(t)^2 + \frac{1}{2}y\,\big(gC(t) - dP(t)\big)^2\right] + l(t)\big[gC(t) - dP(t)\big].$$

As noted before, we need to care of nonnegativity constraints in addition to the constraint imposed by the state equation. The following Lagrangian incorporates the nonnegativity constraints:

(A16) $L = H + +p(t)\big[gC(t) - dP(t)\big] + w(t)C(t).$

The Pontryagin's maximum principle yields the following first order necessary conditions for dynamic optimality:

(A17) $L_C = g\,l(t) + aC(t) + dp(t) + w(t) - gy\dot{P}(t) = 0,$

(A18) $L_P = -\dot{l}(t) + (s + d)\,l(t) + (b + f)P(t) + dyP(t) + p(t)d = 0,$

(A19) $L_l = \dot{P}(t) = gC(t) - dP(t),$

(A20) $p(t) \geq 0, \quad p(t)P(t) = 0, \quad p(t)\dot{P}(t) = 0, \quad P(t) \geq 0,$

(A21) $w(t) \geq 0, w(t)C(t) = 0, C(t) \geq 0,$

(A22) $\lim_{t\to\infty} e^{-st}\, l(t) \geq 0, \lim_{t\to\infty} e^{-st}\, l(t)P(t) = 0.$

As we did before, we will assume that $C(t)>0$, since $C(t)=0$ is not Pareto optimal. A positive consumption implies that $p(t)=w(t)=0$. Using this last result and substituting (A17) into (A19), we obtain the following differential equations:

(A23) $\dot{P}(t) = \frac{g^2}{g^2 y - a}\, l(t) + \frac{ad}{g^2 y - a} P(t),$

(A24) $\dot{l}(t) = (s + d)\, l(t) + (b + f)P(t) + dy\dot{P}(t).$

These there differential equations form the system of differential equations given in (25)-(27) in the text.

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