A TWO DIMENSIONAL FINITE DIFFERENCE MODEL FOR PREDICTING THE TEMPERATURE PROFILES DURING MICROWAVE HEATING OF FOODS HAVING FINITE CYLINDER GEOMETRY

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Abstract

The development of accurate mathematical models for microwave heating of foods is necessary for the prediction of the possible hot and cold zones. A numerical model was developed to compute the temperature profiles ina finite cylinder during microwave heating, and these profiles were compared with experimental temperature profile data obtained from literature. Evaporation of moisture was not taken into account. Although the level of agreement between the experimental and predicted surface temperatures was very high (absolute mean error of 3 %) for potato finite cylinders, the predicted temperatures for the geometric center were significantly higher than the experimental values (absolute mean error of 29.7 %). Another observation to be noted was the dependency of temperature profiles on the number of nodes used in the computations. For one data set a mesh size was chosen such that the error between the finite difference predictions and experimental data was minimized. Then the same number of nodes was used for other data sets. By this means, the absolute mean errors between experimental and numerical temperatures for 2% agar gel cylinder (R = 0.035 m, H = 0.07 m) were 5.5 % and 7.5 % for center and near surface temperatures, respectively.

Key words: microwave, heating, two-dimensional model.

Introduction

In line with the growing trend of spending minimum time in meal preparation and the availability of cheap microwave ovens, microwave heating has become popular in middle class families. In microwave heating, heat is generated volumetrically in the foodstuff by direct absorption of the microwave energy. However, non-uniform heating of food product is often recognized as the major shortcoming in microwave heating [1]. Some of the complicating factors are uneven microwave distribution in the oven, dependence of power absorption to the load and uneven microwave energy absorption by different food constituents. The development of accurate mathematical models for microwave heating of foods will be helpful for the prediction of the possible hot and cold zones and optimization of the key parameters during microwave heating.

Generally two numerical techniques have been used: finite difference and finite element. Finite difference numerical models are simpler than the finite element models for solids having simple shape. Campanone and Zaritzky [2] have developed a finite difference model for unidirectional energy transfer and have applied this model satisfactorily to their own data and literature data. However, the initial temperatures of the samples, the ambient temperatures and heat transfer coefficients have not been reported. This makes the prediction of their data very difficult.

Finite element models are more suitable for foods with anomalous shape. Finite element models for predicting the temperature and moisture profiles for a variety of foods have been developed [1, 4]. Accurate predictions were generally observed. Romano, Marra and Tammara [5] have used the same equations with Lin, Anantheswaran and Puri [1] to develop a two dimensional finite element heat transfer model for finite cylinders and have investigated the effect of radius on center microwave absorption. They have confirmed that absorbing power effect is dominant in cylinders of smaller radius for a given attenuation factor. As the sample radius increased absorbed power was found to decrease at the center and increase at the surface Regardless of the numerical technique, microwave power absorption in the food has to be formulated. Maxwell equations describe the power absorption in a solid. Yang and

Gunasekaran [6] have compared the temperature distribution in 2 % agar gel cylinders based on Maxwell equations and Lambert's law during pulsed microwave heating. They state that the predictions based on Maxwell equations are statistically more accurate than those based on Lambert's law. For a normal, plane wave on a semi-infinite surface, Lambert's law may be written as [7]

$$P(x) = P_0 e^{-2\alpha x} \tag{1}$$

where P(x) is the microwave power at any distance x from the surface, P_0 is the surface microwave power and α is the attenuation constant. A penetration depth α_p may be defined as 1/(2α) where it is the distance at which $P(x)/P_0 = 1/e = 0.368$. i.e. for 2 % agar gel, α =38 m^{-1} [8], and the penetration depth α_p is 0.013 m. Oliveira and Franca [9] have compared the power distribution obtained by solving the Maxwell equation to that obtained by Lambert's law and they have concluded that for cylinders larger radii are required. For the Lambert's law to be applicable, they have proposed a relationship between the radius sample and the attenuation constant α as

$$R_{crit} = 7.03/\alpha - 0.0001 \tag{2}$$

where α is in cm^{-1} and R_{crit} is the critical radius above which the results predicted by Maxwell's equations can be approximated by Lambert's law predictions. If Eqn.(2) is used for the calculation of R_{crit} , for 2 % agar gel, R_{crit} is 18.5 cm. This value is much larger than the radii reported in literature to which Lambert's law have been satisfactorily applied to model the attenuation of microwave power [1, 2, 3, 4].

Since Lambert's law have satisfactorily been applied to model the microwave heating of short cylinders having radii smaller than R_{crit}, it was considered worthwhile to develop a numerical model to compute the temperature profiles and to compare these profiles with experimental temperature profile data obtained from literature.

Development of the mathematical model

The unsteady state heat conduction equation for solids with varying thermo-physical properties for a finite cylinder can be modified for microwave heating by the addition of a microwave heat generation term.

$$\rho Cp(T)\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(rk(T)\frac{\partial T}{\partial r}) + \frac{\partial}{\partial z}(k(T)\frac{\partial T}{\partial z}) + \text{absorbed microwave power per volume} \quad (3)$$

Eqn. (1) has been solved for simultaneous heat and mass transfer in a cup cake baked in a convection oven [10, 11]. This model has been modified in the present study to account for microwave heat generation. Only heat transfer has been considered. The finite difference numerical model developed [10, 11] have been discussed in detail in these references and will not be repeated here. Only, the microwave energy term will be discussed.

An energy balance for microwave may be written as the following for any control volume shown in Figure (1).

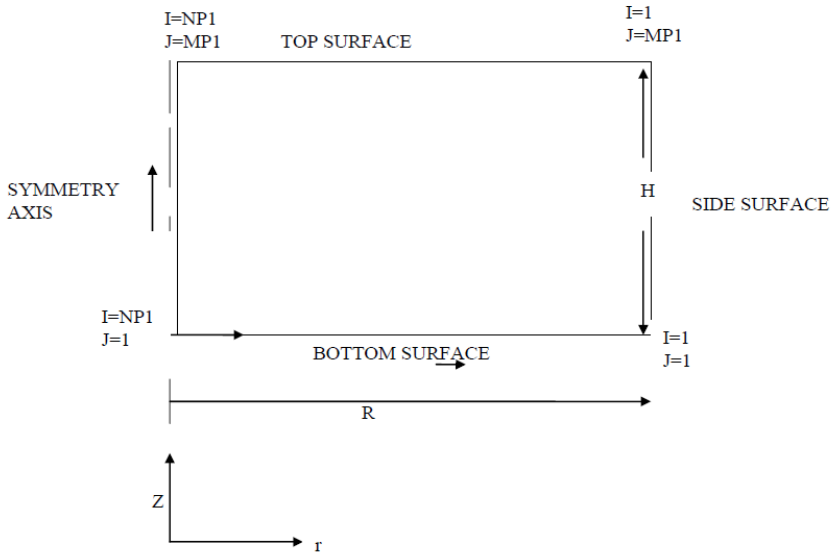

Figure 1. Schematic diagram of the grid system used

[Rate of accumulation of thermal energy due to microwave energy absorption]=[Rate of microwave energy in] – [Rate of microwave energy out] (4)

For a uniform flux distribution, microwave power flux at a surface will P_0/ATOT where ATOT is the total surface area of the cylinder. Microwave power entering the object at a surface will be the area of incidence times the power flux at the surface. Area factors have been defined to calculate the microwave energy at given location in the finite cylinder and to formulate the microwave power in the finite cylinder. Table (1) and (2) show the area factors for the radial and axial directions, respectively for the grid system shown in Figure (1). The attenuation of the surface power was expressed by Lambert's law and power entering any control element in r or z direction was calculated by multiplying the surface power by the area factor in the direction considered and the attenuation factor exp(-2α.distance).

Table 1. Area factors for the radial direction

Nodes	AFR(I,J)
$2 \le I \le N$; $2 \le J \le M$	2.*∏*(N-I+1.)*DR*DZ/ATOT
$2 \le I \le N$; J = 1 or MP1	2.*∏*(N-I+1.)*DR*DZ/2./ATOT
I = NP1; $2 \le J \le M$	∏*DR*DZ/ATOT
I = NP1; J = 1 or MP1	∏*DR*DZ/2./ATOT
I = 1; $2 \le J \le M$	2.*∏*N*DR*DZ/2./ATOT
I = 1; J = 1 or MP1	2.*∏*N*DR/2.*DZ/2./ATOT

Table 2. Area factors for the axial direction

Nodes	AFZ(I,J)
$2 \le I \le N$; $1 \le J \le MP1$	2.*∏*(N-I+1.)*DR*DR/ATOT
I = NP1; $1 \le J \le MP1$	∏*DR**2/4./ATOT
I = 1; $1 \le J \le MP1$	2*∏*N*DR*DR/2./ATOT

For the bottom half of the finite cylinder shown in Figure (1), for the inner nodes, the microwave energy balance in finite difference form is

$$V_i \rho Cp(T)\frac{T'(I,J)-T(I,J)}{\Delta t} =$$
$$P_0\left(AFRIM12(I,J)\exp^{-2\alpha(I-1.5)DR} - AFRIP12(I,J)\exp^{-2\alpha(I-0.5)DR} + \right.$$
$$\left. AFZJM12(I,J)\exp^{-2\alpha(J-1.5)DZ} - AFZJP12(I,J)\exp^{-2\alpha(J-0.5)DZ}\right) \quad (5)$$

where V_i is the volume of the element considered. AFRIM12(I,J) and AFRIP12 (I,J) are the average area factors in the radial direction between nodes (I-1,J) and (I,J); and nodes (I+1,J) and (I,J), respectively.

For the top half of the finite cylinder shown in Figure (1), for the inner nodes, the microwave energy balance in finite difference form is

$$V_{ij} \rho Cp(T)\frac{T'(I,J)-T(I,J)}{\Delta t} =$$
$$P_0\left(AFRIM12(I,J)\exp^{-2\alpha(I-1.5)DR} - AFRIP12(I,J)\exp^{-2\alpha(I-0.5)DR} + \right.$$
$$\left. AFZJM12(I,J)\exp^{-2\alpha(M-J+1.5)DZ} - AFZJP12(I,J)\exp^{-2\alpha(M-J+0.5)DZ}\right) \quad (6)$$

where $T'(I,J)$ is the unknown temperature at the end of time step Δt.

The following boundary conditions were used for the microwave power terms:

1) The attenuation at the top, bottom and side surfaces is zero.

2) The axis of the cylinder (I = NP1, J=1, MP1) is adiabatic to the microwave power reaching the axis from the radial direction. All the microwave power entering the control volume will be absorbed in the control volume.

3) The cylinder is symmetric in the z direction at H/2 (I=1, NP, J = M/2 + 1). So, this axis will be adiabatic to microwave power reaching this location in the z direction from bottom and top surfaces.

Solution of the system of equations as shown in Eqns (3) and (4) as such is tedious. For a 20x20 grid system, 400 simultaneous equations have to be solved at one time step. The implicit alternating direction method [12] overcomes this difficulty by solving the systems of equations implicitly in one direction for a time step of $\Delta t/2$ and using these computed temperatures as input for the next time step of $\Delta t/2$ which is

solved implicitly this time in the alternating direction, thereby reducing the number of equations to be solved for a time step to 40.

A 20x20 grid system was used for the solution of Eqn. (3). The implicit alternating direction method was used. The time step was taken as 1 second. The system of equations obtained was solved by a FORTRAN 95 computer program. Before using the prepared computer program in microwave calculations, the predictions of the model were compared with the predictions of analytical solutions for finite cylinder [13]. Excellent agreement was obtained between analytical and numerical results.

Materials and Methods

The experimental data used for the verification of the numerical model were obtained from literature. Only reproducible experimental data were used. Table 3 shows the experimental test conditions for microwave heating of potato and 2 % agar gel finite cylinders. The input thermo-physical data for the numerical model are shown in Table 4.

Table 3. Experimental conditions for microwave heating of finite cylinders

Material	R,, m	H, m	T_i, °C	T_a, °C	h, W/m^{2}K	P_0, W	Source
Potato	0.025	0.04	22	25*	17.65	355.3	[4]
2 % agar gel	0.035	0.07	4	25*	42	225	[14]
2 % agar gel	0.040	0.07	4	25*	42	234	[14]

* assumed

Table 4. Thermophysical input data to the numerical model

Material	k, W/mK	ρ, kg/m^3	Cp, J/kgK	α, m^{-1}	Source
Potato	0.648	1067	3630	64.56 – 1.366T + 0.0252T^2 – 0.0001113T^3	[2], [4]
2 % agar gel	0.5	1000	4184	38	[2], [8]

Results and Discussion

Comparison of the predictions of the present numerical model with the experimental data of Zhou, Anantheswaran, and Yeh [4] for potato finite cylinders is shown in Figure 2. Since the evaporation of moisture in potato samples was not considered in the

present model, comparison with experimental data was carried out at the early stages of microwave heating; before the predicted temperature at any point within the cylinder reached 100 ^{o}C. All the temperatures presented are for the symmetry plane in the z direction (J = M/2 + 1). A 20x20 grid system was used. As may be observed from Figure 2, the level of agreement between the experimental and predicted surface temperatures are very high. However, the predicted temperatures for the geometric center were significantly higher than the experimental values at the early stages of microwave heating. The experimental and numerical temperatures approached each other after 40 seconds of heating. It must be pointed out that the predicted temperature at the geometric center (I = NP1, J = M/2 +1) was appreciably higher than the temperatures in the surrounding nodes. After 10 seconds of heating, the experimental temperature reported by Zhou, Puri, Anantheswaran and Yeh [4] at the geometric center was 41 ^{o}C, whereas the predicted temperature was 68.1 ^{o}C. The predicted temperatures at the four nodes surrounding the geometric center were 46.7, 47.4, 46.7 and 47.4 ^{o}C. In other words, a slight error in thermocouple placement can lead to large error in predicted temperatures. The absolute mean errors between experimental and numerical temperatures were 29.7 % and 3.0 % for center and near surface temperatures, respectively.

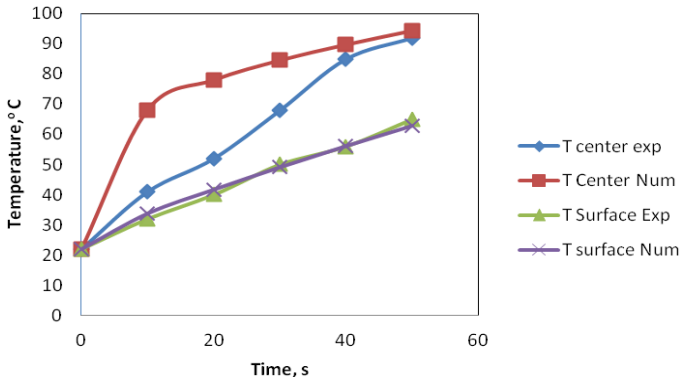

Figure 2. Comparison of the predictions of the present numerical model with the experimental data of Zhou, Puri, Anantheswaran and Yeh, [4] (M = 20, N = 20)

One other observation to be noted was the dependency of temperature profiles on the number of nodes used in the computations. As the number of node increased in the r direction, the geometric center temperature increased. This was attributed to the decrease of the volume of the control elements as the number of nodes increased which resulted in the concentration of microwave energy at the geometric center. Decreasing the number of nodes had the opposite effect. When the number of nodes in the axial direction was increased, the predicted geometric center temperatures were observed to decrease. This was attributed to the decrease in the area factors in the z direction which results in decreased microwave absorption at the center. To quantify this effect the numerical model was run with a coarser mesh in the r direction. The experimental data of Zhou, Puri , Anantheswaran and Yeh [4] were predicted by a coarser mesh in the r direction (N = 10, M = 20) and a finer mesh in the z direction (N=20, M = 30). Using a coarser mesh in the radial direction has resulted in the improvement of the agreement between the numerical and experimental center temperatures. However, this has been at the expense of the loss of accuracy in surface temperature predictions. The absolute mean errors between experimental and numerical temperatures were 8.2 % and 11.9 % for center and near surface temperatures, respectively. When a finer mesh in the z direction was used, the absolute mean errors between experimental and numerical temperatures were 16.5 % and 5.2 % for center and near surface temperatures, respectively.

What may be proposed to overcome this difficulty is to choose a mesh size that will be minimizing the error between the finite difference predictions and experimental data for one data set and then use the same number of nodes for other data sets from the same source. A similar approach has been used by Lin, Anantheswaran and Puri, [1] in determining the element size for their finite element model.

Figure 3 shows the predictions of the present model for 2 % agar gel finite cylinders (R = 0.035 m, H = 0.07 m) and their comparison with the experimental data of Gunesakaran and Yang [14]. All the temperatures presented are for the symmetry plane in the z direction (J = M/2 + 1). A 20x20 mesh was used for this data set also. As may

be observed from Figure 4, the level of agreement between the numerical predictions and experimental data was satisfactory. The absolute mean errors between experimental and numerical temperatures were 5.5 % and 7.5 % for center and near surface temperatures, respectively.

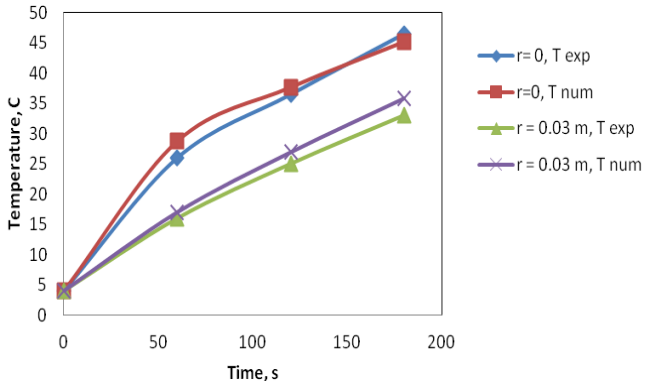

Figure 3. Comparison of the predictions of the present numerical model with the experimental data of Gunesakaran and Yang [14] (M=20, N=20, R=0.035 m, H=0.04 m)

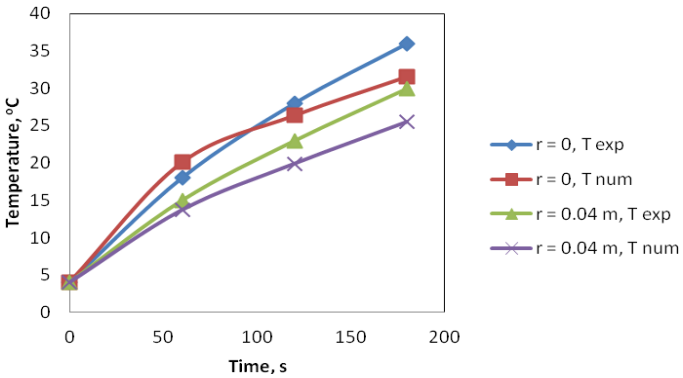

Figure 4. Comparison of the predictions of the present numerical model with the experimental data of Gunesakaran and Yang [14] (M=20, N=20, R=0.04 m, H=0.04 m)

Figure 4 shows the predictions of the present model for 2 % agar gel finite cylinders (R = 0.04 m, H = 0.07 m) and their comparison with the experimental data of Gunesakaran and Yang [14]. The level of agreement between the model predictions and experimental data is not as high as that observed from Figure 3. Nevertheless, the absolute mean errors between experimental and numerical temperatures were 9.9 % and 12.4 % for center and surface temperatures, respectively.

Conclusion

A numerical model was developed to compute the temperature profiles in a finite cylinder during microwave heating, and these profiles were compared with experimental temperature profile data obtained from literature. Although the level of agreement between the experimental and predicted surface temperatures was high, the predicted temperatures for the geometric center were significantly higher than the experimental values. Since the agreement between the predicted temperatures at the four nodes surrounding the geometric center and the experimental data given for center was much higher, it was concluded that a slight error in thermocouple placement during experiments can lead to large error for center temperatures. One other observation to be noted was the dependency of temperature profiles on the number of nodes used in the computations. As the number of node increased in the r direction and the number of nodes in the axial direction was decreased, the geometric center temperature increased. This was attributed to the decrease of the volume of the control elements as the number of nodes in radial direction increased which resulted in the concentration of microwave energy at the geometric center. On the other hand, the decrease in the area factors in the z direction which results in decreased microwave absorption at the center. To overcome the difficulty in mesh sizing, such a mesh size was chosen that the error between the finite difference predictions and experimental data for one data set was minimized, and then the same number of nodes was used for other data sets from the same source. By this method, the level of agreement between the numerical predictions and experimental data was satisfactory.

Nomenclature

AFR	area factor in the r direction (Table 1)
AFZ	area factor in the z direction (Table 2)
AFRIM12	average area factor between nodes (I-1,J) and (I,J)
AFRIP12	average area factor between nodes (I+1,J) and (I,J)
AFZJIM12	average area factor between nodes (I,J-1) and (I,J)
AFZIP12	average area factor between nodes (I1,J+1) and (I,J)
ATOT	the total surface area of cylinder (m^2)
Cp	specific heat capacity (J/kgK)
DR	distance between two nodes in the r direction, R/N (m)
DZ	distance between two nodes in thezr direction, H/M (m)
H	surface heat transfer coefficient (W/m^2K)
H	height of cylinder (m)
I	the i^{th} node
J	the J^{th} node
k	thermal conductivity (W/mK)
M	number of intervals in z direction
MP1	M+1, the top surface
N	number of intervals in r direction
NP1	N+1, the radial axis of symmetry
r	distance in radial direction (m)
R	the radius of the cylinder (m)
P_0	normal surface power (W)
T	temperature (°C)
t	time (s)
V	volume (m^3)
z	distance in upward direction (m)

Subscripts

a	ambient

ij	any node
ini	initial
r, z	the r and z directions

Greek symbols

α	attenuation factor (1/m)
Δt	the time step (s)
ρ	density (kg/m^3)

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