
ASYMPTOTIC EVALUATION OF SCATTERING OF INHOMOGENEOUS PLANE WAVES BY A PERFECTLY ELECTRIC CONDUCTING HALF PLANE

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Abstract: In the present study, reflected and diffracted fields of an inhomogeneous plane wave obliquely incident on the surface of a perfectly electric conducting (PEC) half plane are evaluated for a two-dimensional case asymptotically and reexamined by considering the different aspects. Obtained results are plotted by Matlab numerically. Relationships between the complex angle of incidence and field intensities, and phase shifts in the fields are noted. Matlab plots are interpreted and compared to the theory for consistence.

Keywords: Evanescent wave, inhomogeneous wave, scattered field, reflected field, diffracted field

Homojen Olmayan Düzlem Dalgaların Mükemmel Elektrik İletken (MEİ) Yarım Düzlem Tarafından Saçımının Asimptotik Olarak Hesaplanması

Öz: Bu çalışmada mükemmel elektrik iletken (MEİ) yarım düzlemine eğik olarak gelen homojen olmayan (inhomojen) bir düzlem dalgaının yansıyan ve kırılan alanları iki boyutlu durum için asimptotik olarak hesaplanmış ve farklı yönlerden yeniden incelenmiştir. Elde edilen sonuçlar Matlab ile sayısal olarak çizdirilmiştir. Kompleks geliş açısıyla alan yoğunlukları ve alanlardaki faz kaymaları belirlenmiştir. Matlab çizimleri yorumlanmış ve uyumluluk için teoriyle karşılaştırılmıştır.

Anahtar Kelimeler: Evanesent dalga, İnhomojen dalga, Saçınan alan, Yansıyan alan, Kırılan alan

1. INTRODUCTION

Inhomogeneous waves, along with the Gaussian beams, are considered in the category of evanescent waves that have been under investigation for decades. Noteworthy feature of these waves is they attenuate in the direction perpendicular to that of propagation. Due to their rapid fading with respect to the distance, they are negligible in the far field but they are taken into account in the near field because of their contribution. Ronchi et al. (1961) examined the scattering of evanescent waves by using complex angles to express an inhomogeneous wave. Keller and Streifer (1971) worked on complex ray application for the Gaussian beams. Choudhary and Felsen (1973) studied on the asymptotic theory for inhomogeneous waves. Wang and Deschamps (1974) applied complex ray tracing to scattering problems. Shevernev (1976) analyzed the diffraction of an inhomogeneous plane wave by a wedge. Later, Felsen (1976) examined evanescent waves. Bertoni et al. (1978) studied on the shadowing of an inhomogeneous plane wave by a wedge. Deschamps et al. (1979) investigated the diffraction of an evanescent plane wave by a half plane. Kouyoumjian et al. (1996) analyzed the diffraction of an inhomogeneous plane wave.

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Manara et al. (1998) worked on the diffraction of an inhomogeneous plane wave by an impedance in lossy medium in their study where they asymptotically evaluated the rigorous integral representation of the field by considering the uniform geometrical theory of diffraction (UTD). Kouyoumjian et al. (2007) examined inhomogeneous electromagnetic plane wave diffraction by a perfectly electric conducting wedge at oblique incidence.

Umul (2007) worked on the uniform theory for the diffraction of evanescent plane waves, and he studied the diffraction of homogeneous and inhomogeneous plane waves by a planar junction between perfectly electric conducting (PEC) and impedance half planes (Umul, 2007). Diffraction of evanescent plane waves by a resistive half-plane (Umul, 2007) and Scattering of a line source by a cylindrical parabolic impedance surface (Umul, 2008), Scattering of a plane wave by a cylindrical parabolic perfectly electric conducting reflector (Kara, 2017), and scattering of inhomogeneous plane waves by a resistive half-screen (Umul, 2013) are also studied. In the present study we reexamine the scattering of an inhomogeneous plane wave for a two-dimensional case by a PEC half plane.

2. REFLECTION OF AN INHOMOGENEOUS PLANE WAVE BY A PEC HALF PLANE

Assuming that complex incident angle α is given as the combination of real and imaginary components as

$$\alpha = \varphi_r + j\varphi_i, \quad (1)$$

The expression of incident inhomogeneous plane with a complex angle α can be given as,

$$u_i = e^{jk_1\rho\cos(\varphi-\varphi_r)}e^{-k_2\rho\sin(\varphi-\varphi_r)}, \quad (2)$$

which can be rewritten as,

$$u_i = e^{jk_1(x\cos\varphi_r+y\sin\varphi_r)}e^{k_2(x\sin\varphi_r-y\cos\varphi_r)} \quad (3)$$

where

$$k_1 = kcosh\varphi_i, \quad (4)$$

$$k_2 = ksinh\varphi_i, \quad (5)$$

k is the wave number which is equal to $2\pi/\lambda$ where λ is the wavelength. Scattered field can be expressed as,

$$u_s = \frac{1}{4\pi} \iint_S (u_i \nabla G - G \nabla u_i) \cdot \vec{n} dS'. \quad (6)$$

If the surface is a perfect electric conductor, electric field along the surface becomes zero. In that case scattered field can be rewritten as,

$$u_s = -\frac{1}{2\pi} \iint_S \frac{\partial u_i}{\partial n} G dS' \quad (7)$$

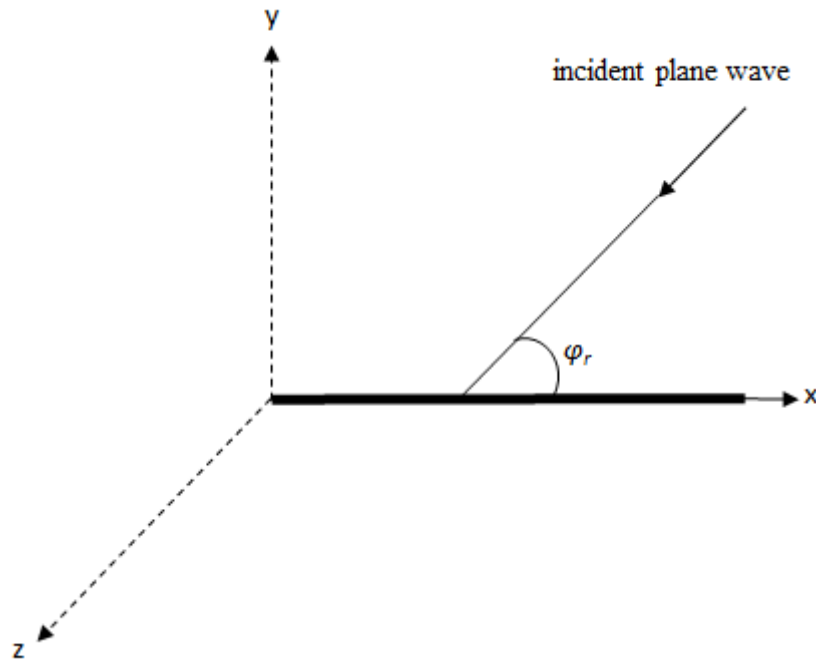


Figure 1:
Geometry of the problem

where n denotes the surface normal which is in y -direction for our geometry. Partial derivative of u_i with respect to y is obtained as,

$$\frac{\partial u_i}{\partial n} = \frac{\partial u_i}{\partial y} = (jk_1 \sin \varphi_r - k_2 \cos \varphi_r) e^{jk_1(x \cos \varphi_r + y \sin \varphi_r)} e^{k_2(x \sin \varphi_r - y \cos \varphi_r)}. \quad (8)$$

At $y=0$ plane,

$$\frac{\partial u_i}{\partial y} = jk \sin(\varphi_r + j\varphi_i) e^{jkx \cos(\varphi_r + j\varphi_i)} = jk \sin \alpha e^{jkx \cos \alpha} \quad (9)$$

$$u_s = \frac{-jk \sin \alpha}{2\pi} \iint_S e^{jkx' \cos \alpha} \frac{\exp(-jkR)}{R} dx' dz' \quad (10)$$

where

$$R = \sqrt{(x - x')^2 + y^2 + (z - z')^2} \quad (11)$$

For the solution of Eq.(14) let,

$$(x - x')^2 + y^2 = R_1^2 \quad (12)$$

and

$$z - z' = R_1 \sin \alpha. \quad (13)$$

Then the scattered field is obtained as

$$u_s = -\frac{k e^{j\frac{\pi}{4}} \sin \alpha}{\sqrt{2\pi}} \int_0^\infty e^{jkx' \cos \alpha} \frac{e^{-jkR_1}}{\sqrt{kR_1}} dx' \quad (14)$$

In the asymptotic evaluation of scattered field, reflected field can be found by the method of stationary phase point. For this method phase function is written as,

$$g = x' \cos \alpha - R_1 \quad (15)$$

$$\frac{dg}{dx'} = \cos \alpha - \frac{dR_1}{dx'} \quad (16)$$

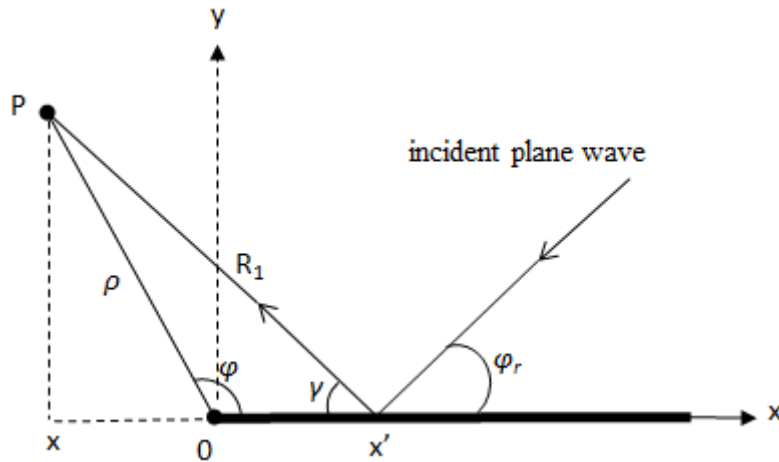


Figure 2:
Geometry for the evaluation of the phase function and reflected field

From Fig. 2 it can be written that,

$$\frac{dg}{dx'} = \cos \alpha - \frac{-2(x-x')}{2R_1} \quad (17)$$

$$\cos \gamma = \frac{-(x-x')}{R_1} \quad (18)$$

$$dg/dx' = \cos\alpha - \cos\gamma \tag{19}$$

Since the first derivative of the phase function at the stationary phase point is zero, we write

$$\gamma_s = \alpha_s . \tag{20}$$

Its second derivative is found as,

$$d^2g/dx'^2 = -\sin^2\gamma/R_1 . \tag{21}$$

By employing Taylor expansion, phase function is concluded as,

$$g = \rho\cos(\alpha + \varphi) + \frac{(x' - x_s)^2 \sin^2\alpha}{2R_1} \tag{22}$$

Reflected field is written as

$$u_r = -\frac{e^{j\frac{\pi}{4}} e^{jk\rho\cos(\alpha+\varphi)} k\sin\alpha}{\sqrt{2\pi kR_s}} \int_{-\infty}^{\infty} e^{\frac{-jk(x' - x_s)\sin^2\alpha}{2R_s}} dx', \tag{23}$$

$$u_r = -e^{jk\rho\cos(\alpha+\varphi)}, \tag{24}$$

or

$$u_r = -e^{-jk_1\rho\cos(\varphi+\varphi_r)} e^{k_2\rho\sin(\varphi+\varphi_r)}, \tag{25}$$

where the first exponential term represents the plane wave, and the second term denotes the amplitude of the plane wave. The following figures, Fig. 3 through Fig. 6, illustrate the reflection behavior of the inhomogeneous wave for the geometry given in Fig. 2. For the Matlab plots, the wavelengths λ is taken as 0.1 meter and the observation distance $\rho=\lambda$ is considered

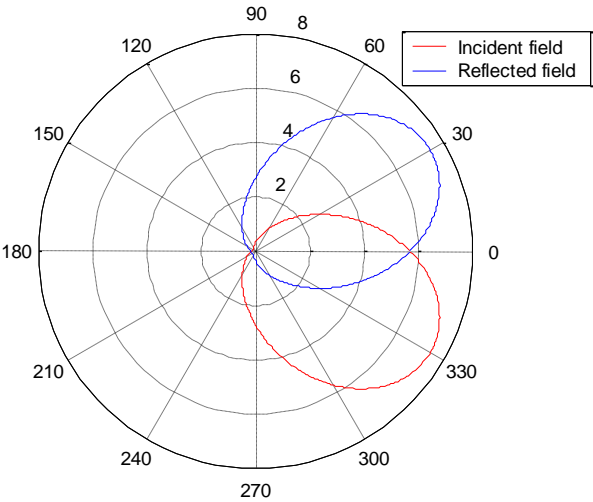


Figure 3:
 Incident and reflected field intensities, $\alpha = \frac{\pi}{3} + j \frac{\pi}{10}$

In Fig. 3 incident angle is 60 degrees which is the real component of the incident angle denoted by ϕ_r . It is in the first quadrant of the unit circle due to the positive imaginary component ϕ_i of the incident angle α . This is the propagation direction of the plane wave. Incident electric field is directed perpendicular to this angle which is in the direction of 330 degrees. Reflected wave in the direction of 120 degrees and reflected field will be in 30-degree direction which is perpendicular to the direction of the reflected wave.

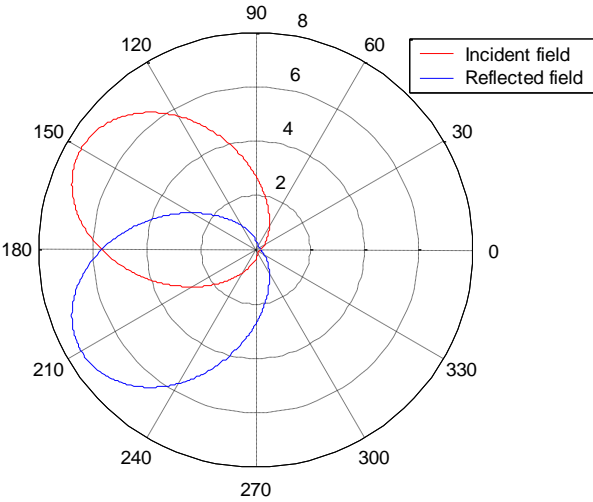


Figure 4:
 Incident and reflected field intensities, $\alpha = \frac{\pi}{3} - j \frac{\pi}{10}$

Fig. 4 shows the incident and reflected fields of the inhomogeneous plane wave with incident angle α which is complex conjugate of the one used for Fig. 3. In this case propagation direction is in -60 degrees, and incident electric field is in 210 -degree direction. As a result, reflected field lies in the 150 degree direction.

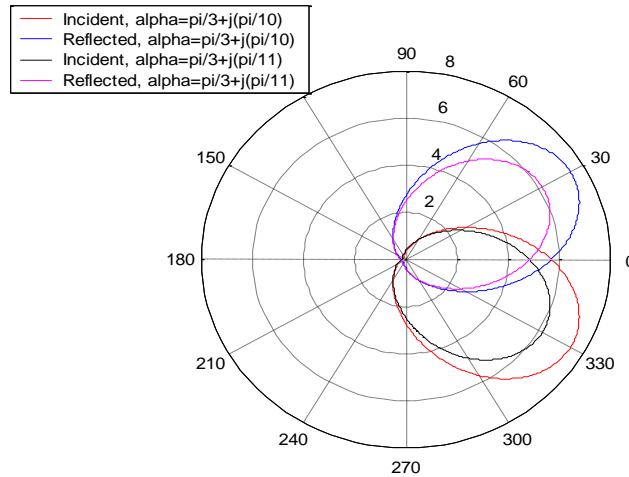


Figure 5:
Intensity variations of incident and reflected fields with respect to positive φ_i

In Fig. 5 the variation of field intensities with respect to imaginary parts of the incident angle α are given. It can be observed that field intensities and φ_i values are directly proportional. Thus, imaginary component of complex incident angle determine the amplitude of the incident or reflected field. In Fig. 5, real part of the incident angle is 60° , and the direction perpendicular to this angle is -330° which is the field direction. In this direction, amplitude of the field is attenuates by the decrease in φ_i . In Fig. 6, this time the field direction perpendicular to 60° is 150° . Attenuation of the field can be implemented by decreasing φ_i again. In either case, attenuation occurs in the way that is perpendicular to the propagation direction. In a similar manner, this phenomenon takes place for the reflected field as well.

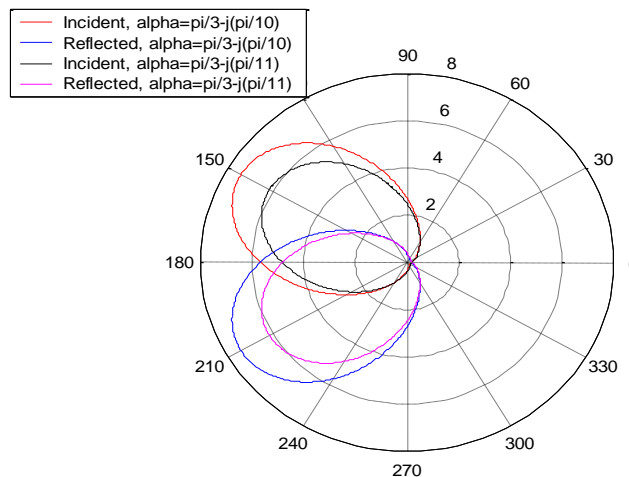


Figure 6:
Intensity variations of incident and reflected fields with respect to negative φ_i

3. DIFFRACTION OF AN INHOMUGENEOUS PLANE WAVE BY A PEC HALF PLANE

For the scattered field expression given in Eq. (26), diffracted field can be evaluated by the edge point method given by Eq. (27).

$$u_s = -\frac{k}{\sqrt{2\pi}} e^{j\frac{\pi}{4}} \sin\alpha \int_0^\infty \frac{e^{jkx' \cos\alpha} e^{-jkR_1}}{\sqrt{kR_1}} dx' , \quad (26)$$

$$u_d \cong \left(\frac{1}{jk}\right) \frac{f(x_e) e^{jk g(x_e)}}{g'(x_e)} , \quad (27)$$

where the amplitude function $f(x)$ is,

$$f(x) = \frac{-k e^{j\frac{\pi}{4}} \sin\alpha}{\sqrt{2\pi} \sqrt{kR_1}} . \quad (28)$$

From Fig. 2 it can be seen that, at the edge point,

$$x' = x_e , R_1 = \rho \text{ and } g(x_e) = -\rho . \quad (29)$$

Since,

$$g(x) = x' \cos\alpha - R , \quad (30)$$

$$\frac{dg}{dx'} = g'(x) = \cos\alpha - \frac{dR}{dx'} = \cos\alpha - \cos\gamma . \quad (31)$$

At $x' = 0$, $\gamma = \pi - \varphi$ and

$$g'(x_e) = 2 \cos\left(\frac{\alpha - \varphi}{2}\right) \cos\left(\frac{\alpha + \varphi}{2}\right) , \quad (32)$$

Finally diffracted field becomes,

$$u_d = \frac{-1}{2\sqrt{2\pi k\rho}} \frac{\sin\alpha e^{-j\frac{\pi}{4}} e^{-jk\rho}}{\cos\left(\frac{\alpha - \varphi}{2}\right) \cos\left(\frac{\alpha + \varphi}{2}\right)} \quad (33)$$

Which can be written as,

$$u_d = (2\sqrt{2\pi k\rho})^{-1} e^{-j\frac{\pi}{4}} \left(\tan\left(\frac{\alpha - \varphi}{2}\right) + \tan\left(\frac{\alpha + \varphi}{2}\right) \right) \quad (34)$$

Ud can be divided into incident-diffracted and reflected-diffracted parts as

$$u_{id} = \sin\left(\frac{\alpha - \varphi}{2}\right) e^{-jk\rho \sin(\alpha - \varphi)} \text{sign}(p) F[|p|] , \quad (35)$$

and

$$u_{rd} = \sin\left(\frac{\alpha + \varphi}{2}\right) e^{-jk\rho \sin(\alpha + \varphi)} \text{sign}(q) F[|q|] , \quad (36)$$

where the detour parameters are given as,

$$p = -\sqrt{2k\rho}\cos\left(\frac{\alpha-\varphi}{2}\right), \quad (37)$$

and

$$q = -\sqrt{2k\rho}\cos\left(\frac{\alpha+\varphi}{2}\right). \quad (38)$$

For complex parameters $\text{sign}(x)F[\text{abs}(x)]$ does not yield correct results. In that case complex detour parameter decomposition [12] is employed. Letting,

$$p = t_1 + jw_1 = \sqrt{k\rho}\cos\left(\frac{\alpha-\varphi}{2}\right), \quad (39)$$

it can be written that,

$$p = \sqrt{k\rho}\cos\left(\frac{\varphi_r-\varphi}{2} + j\frac{\varphi_i}{2}\right),$$

and

(40)

$$p = \sqrt{k\rho}\left[\cos\left(\frac{\varphi_r-\varphi}{2}\right)\cos\left(j\frac{\varphi_i}{2}\right) - \sin\left(\frac{\varphi_r-\varphi}{2}\right)\sin\left(j\frac{\varphi_i}{2}\right)\right]. \quad (41)$$

Using the identities of,

$$\cos\left(j\frac{\varphi_i}{2}\right) = \cosh\left(\frac{\varphi_i}{2}\right), \quad (42)$$

and

$$\sin(j\varphi_i/2) = j\sinh\left(\frac{\varphi_i}{2}\right), \quad (43)$$

we write

$$p = \sqrt{k\rho}\left[\cos\left(\frac{\varphi_r-\varphi}{2}\right)\cosh\left(\frac{\varphi_i}{2}\right) - j\sin\left(\frac{\varphi_r-\varphi}{2}\right)\sinh\left(\frac{\varphi_i}{2}\right)\right], \quad (44)$$

where,

$$t_1 = \sqrt{k\rho}\cos\left(\frac{\varphi_r-\varphi}{2}\right)\cosh\left(\frac{\varphi_i}{2}\right), \quad (45)$$

and

$$w_1 = \sqrt{k\rho}\sin\left(\frac{\varphi_r-\varphi}{2}\right)\sinh\left(\frac{\varphi_i}{2}\right). \quad (46)$$

Letting,

$$\beta_1 = t_1 - w_1 \quad (47)$$

and

$$a_1 = w_1\sqrt{2}e^{j\frac{\pi}{4}}, \quad (48)$$

We write,

$$\text{sign}(p)F[|p|] = \text{sign}(\beta_1)F[|\beta_1| + a_1\text{sign}(\beta_1)]. \quad (49)$$

In a similar manner by letting,

$$q = t_2 + jw_2 = \sqrt{k\rho}\cos\left(\frac{\alpha+\varphi}{2}\right), \quad (50)$$

$$\beta_2 = t_2 - w_2 \text{ and } a_2 = w_2\sqrt{2} e^{j\frac{\pi}{4}}, \quad (51)$$

and using the same steps it is found that,

$$\text{sign}(q)F[|q|] = \text{sign}(\beta_2)F[|\beta_2| + a_2\text{sign}(\beta_2)]. \quad (52)$$

Finally incident diffracted and reflected diffracted fields are respectively obtained as,

$$u_{id} = \sin\left(\frac{\alpha-\varphi}{2}\right) e^{-jk\rho\sin(\alpha-\varphi)} \text{sign}(\beta_2)F[|\beta_1| + a_2\text{sign}(\beta_1)], \quad (53)$$

and

$$u_{rd} = \sin\left(\frac{\alpha+\varphi}{2}\right) e^{-jk\rho\sin(\alpha+\varphi)} \text{sign}(\beta_2)F[|\beta_2| + a_2\text{sign}(\beta_2)]. \quad (54)$$

The following Matlab plots depict the incident diffracted and reflected diffracted field variations according to some parameters. The wavelengths λ is taken as 0.1 meter and the observation distance $\rho=\lambda$ is considered.

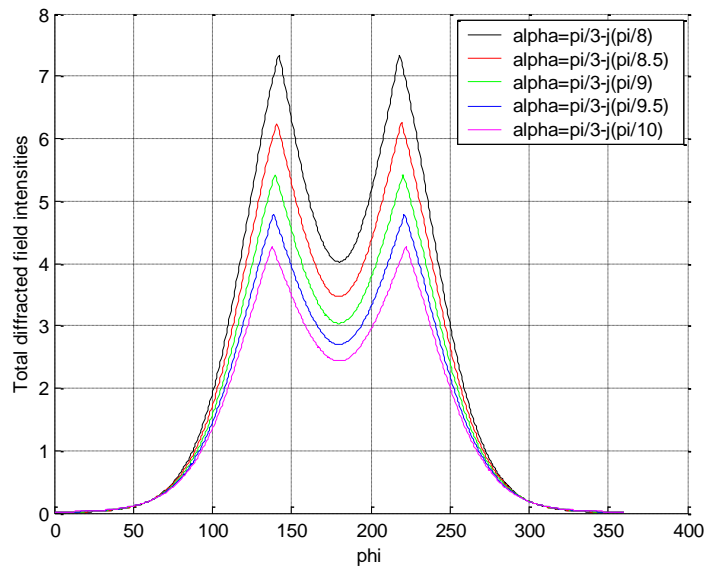


Figure 7:
Total diffracted field variations for some φ_i values

In Fig. 7, it is observed that field intensity and the imaginary component φ_i of the incident angle α are directly proportional. Peak points of the diffracted field shift due to the change in φ_i .

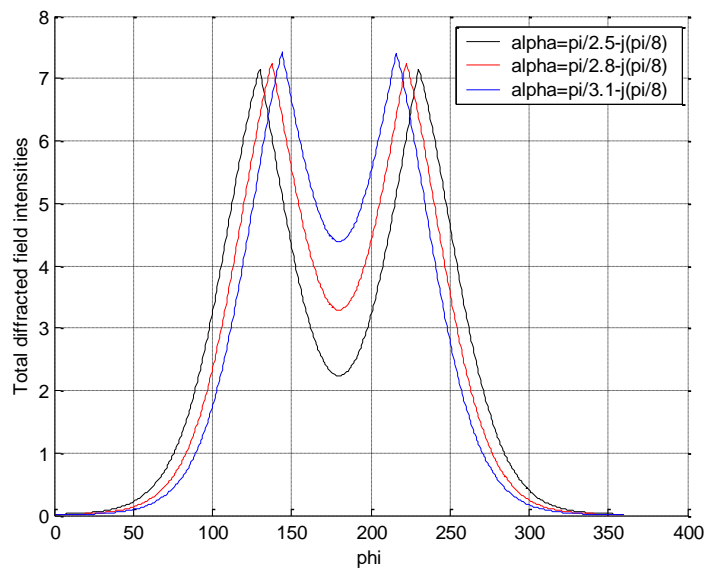


Figure 8:
Phase shifts in total diffracted fields with respect to φ_r

Fig. 8 shows the phase shift variations of the diffracted field with respect to φ_r where φ_i is kept constant. It can be seen that any change in φ_r yields the same amount of phase change in the diffracted field.

4. CONCLUSION

In this study firstly, for the asymptotic evaluation scattering integral, reflection behavior of an inhomogeneous plane wave by a perfectly electric conducting half plane is investigated by the method of stationary phase point. As a second step diffracted field expression is obtained through the edge point method. To obtain correct results for the uniform fields, obtained field expressions are transformed into another form eliminating incorrect results. Matlab plots of the derived fields are obtained for some parameters, and the results are interpreted. It is concluded that intensity of the reflected fields and imaginary component of the incident angle are directly proportional, but for the diffracted fields they are inversely proportional. Also it is observed that any phase change in the real component of the incident angle resulted in the same amount of phase shift in the reflected and diffracted fields.

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