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Optimal Investment in Hydroelectric Energy

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Abstract

Assuming that the total energy demand is satisfied by oil and hydroelectric energy, optimal investment in hydroelectric production capacity which is a function of the size of the dam and the water availability, which changes by seasons, is analyzed using optimal control theory. Investment cost in dam capacity depends not only on the purchase cost of additional capacity but also on the installation of that capacity. The optimal investment is to invest heavily in early stages by gradually decreasing it until the long-term equilibrium point is reached. The equilibrium point also oscillates but stable. However, since the dam size is usually large compared to the amplitude of the oscillation, the optimal path and the equilibrium level of investment can be considered to be functions of relevant parameters of the problem only.

Keywords: Hydroelectric, Optimal Control Investment

Optimal Hidroelektrik Enerji Yatırımları

Öz

Toplam enerjinin petrol ve hidroelektrik enerjiden karşılanacağı varsayımıyla hidroelektrik üretim kapasitesine yapılacak optimal yatırım konusu, üretim kapasitesinin baraj kapasitesi ve yeterli su seviyesine bağlı olduğu gözönüne alınarak, dinamik olarak incelenmiştir. Çalışmada Optimal Control Teorisi metodu kullanılmıştır. Ayrıca, yeni yatırım maliyetinin hem barajda kullanılan makina ve techizatın fiyatına hemde yeni yatırımın çalışır hale gelmesi için yapılan ilave harcamalara bağlı olacağı varsayılmıştır. Optimal yatırım planı başlangıçta yoğun yatırım yapılması ve yatırım tutarının, uzun vade denge tutarına ulaşıncaya kadar, yavaşlatılarak devam etmesi şeklindedir. Denge noktası istikrarlı ancak su seviyesine göre değişmektedir. Ancak baraj büyüklüğü su seviyesinin değişim seviyesine göre çok büyük olacağı düşünüldüğünde hem denge seviyesinin hemde optimal yatırımın zaman patikasının problemin ilgili parametrelerine bağlı olacaktır.

Anahtar Kelimeler: Hidroelektrik, Optimal Kontrol, Yatırım

1. Introduction

The crisis which started in the USA in 2008 have quickly spread over the world and it is still continuing even though a breakdown in world financial order have been averted by the concerted action of the governments of the United States and Europe. The real sectors in almost all major countries have been affected also. The financial crisis has affected the real sector also in almost all major economies of the world with the exception of China, India. A recent study by OECD study (Economic Outlook, 2018) estimates that the US will grow only by 2.4% in 2019 and 2% in 2020. Euro area OECD countries are expected to grow by 1.9% in 2018 and 1.4% in 2020.The total of OECD countries will grow by 2.1% in 2018 and 1.9% in 2020. Turkey has also been affected by the ongoing crisis. The growth rate of the economy is expected to be 2% in 2019 and 2.7% in 2020.Unemployment rates were 3.9%, 8.2%, and 5.3% in the USA, Eurozone OECD countries, and total OECD respectively in 2018. These rates are expected to be 3.5%, 7.2%, and 5% in 2020.

The major concern of all these countries was and still is to increase growth by fiscal and monetary measures to lower unemployment rates and to attain a somewhat sustainable growth rates within rapidly globalizing capital, goods and services markets (not labor) with marketing politics based on rapid consumption. Meanwhile the world's total population has increased from 3 billion to 7.5 billion between 1960 and 2017 (World Bank, 2012) and it is expected to reach over 9.7 billion by 2050 (UN, 2019). Moreover, increasing income of the population of most populous states of the world (China and India) is already and will continue to further accelerate the demand for goods (and services). However, production to meet the demand will use the scarce resources of the world especially oil, gas, and coal in accelerating rate. Table 1 below shows the demand for energy until the year 2035.

Table 1. World Total Energy Consumption by Fuel and Region (Quadrillion Btu-British Thermal Unit)

Region/Country	2005	2006	2007	2008	2010	2015	2020	2025	2030	2035
Total OECD										
Liquids	100,4	99,6	99,3	96,5	91,6	94,1	95,6	96,7	97,9	99,7
Natural Gas	53,6	54,1	55,7	56,3	56,5	59,2	61,4	63,6	67,1	70,6
Coal	46,7	46,8	47,8	46,8	43,5	42,6	43,1	44,6	45,3	46,7
Nuclear	23,30	23,40	22,60	22,60	22,70	25,20	26,70	27,80	29,10	29,80
Other	20,00	20,40	20,70	22,10	23,70	29,30	33,60	37,10	39,40	41,40
Total	**243,9**	**244,3**	**246,1**	**244,3**	**238**	**250,4**	**260,6**	**269,8**	**278,7**	**288,2**
Total Non-OECD										
Liquids	70,4	72,1	73,4	76,4	81,6	93,1	100,1	110,3	118,7	125,5
Natural Gas	51,5	53,4	55,2	58	60,1	68,1	76,6	85,9	95,2	104,1
Coal	75,6	80,4	85,6	92,2	105,9	114,7	121,4	135,1	149,4	162,5
Nuclear	4,20	4,40	4,50	4,60	5,00	7,90	12,20	15,80	18,30	21,40
Other	25,50	26,80	27,80	29,20	31,50	39,30	48,60	54,60	61,20	68,10
Total	**227,20**	**237,00**	**246,50**	**260,50**	**284,10**	**323,10**	**358,90**	**401,70**	**442,80**	**481,60**
Total World										
Liquids	170,80	171,70	172,70	173,00	173,20	187,20	195,80	207,00	216,60	225,20
Natural Gas	105,00	107,50	110,90	114,30	116,70	127,30	138,00	149,40	162,30	174,70
Coal	122,30	127,20	133,30	139,00	149,40	157,30	164,60	179,70	194,70	209,10
Nuclear	27,50	27,80	27,10	27,20	27,60	33,20	38,90	43,70	47,40	51,20
Other	45,40	47,10	48,50	51,30	55,20	68,50	82,20	91,70	100,60	109,50
Total	**471,10**	**481,30**	**492,60**	**504,70**	**522,00**	**573,50**	**619,50**	**671,50**	**721,50**	**769,80**

Source: International Energy Outlook, 2011

Notice that the energy demand will increase by 24% from 619.5 quadrillion Btu in 2020 to 769.8 in 2035 while the share of other (hydroelectric and other renewable) increase by 33 % from 82.2 to 109.5 Quadrillion Btu based on these estimates.

According to this table increasing demand will largely be met by nonrenewable sources. However, share of renewable sources will increase from 9.5% in 2005 to 14% in 2035 more than doubling the amount from 45 QBtu to109 QBtu.

Currently oil, gas, and coal (carbon-based energy sources) constitute almost 80% of the total consumption while renewable energy (hydroelectric, sun, wind) constitutes only 13% of the total. However, oil and gas are expected to be depleted in 65, and 50-75 years respectively (Meadows, 2004). Coal sources are abundant, but its use will be limited by the atmospheric sink for carbon dioxide (Meadows, 2004). This implies that the world needs to find ways to better utilize the renewable energy sources. Hydroelectric energy is one of them and it is the subject of this paper.

Hydroelectric consumption (in equivalent million tons of oil) increased from 697 in 2007 to 918 in 2017(BP Statistical review, 2018) implying an increase of 2.7% per year. The importance of this energy source for OECD countries is evident from the fact that its use increased from 289(in million tons of equivalent oil) in 2007 to 314 in 2017, from 407 in 2007 to 603 in 2017 for non-OECD countries. The total use of this energy source increased from 696 in 2007 to 918 in 2017 implying a rate of increase of 3 % per year. From Table 1, the increase of total energy use was calculated to be about 1.9% per year between 2005 and 2015. Hydroelectric energy remains a significant source of energy since its potential is barely utilized as is evident from table 2 below.

Table 2. Hydropower Generation and Potential by Region (Gigawatt Hour)

Region	Gross Theoretical hydropower potential (GWh/year)	Technically feasible hydropoer potential (GWh/year)	Economically feasible hydropower potential (GWh/year)	Installed hydrocapacity (Mw)	Hydro Gen in 2008 or average/most recent (GWh/year)	Hydro capacity under construction (MW)
Africa Total	>2.385.211	>1.161.467	>773.996	22.304	102.107	>7.961
Asia Total	16.990.783	5.785.657	3.553.627	299.182	1.107.055	>120.904
Australia Total	654.177	185.012	88.701	13.626	41.886	182
Europe Total	5.380.005	2.885.587	1.772.478	246.491	771.408	>18.812
North&Cent. America Total	>7.417.847	1.979.778	1.024.406	167.105	688.873	>5.362
South America Total	>5.778.880	2.606.408	1.558.523	>138.644	641.216	>18.619
World Total	**>38.606.913**	**>14.504.209**	**>8.771.502**	**>887.352**	**3.352.546**	**>171.840**

Source: World Atlas, 2009

As observed from Table 2, the additional total economically feasible hydropower potential is (8.771.502MToe-Million Tons of oil equivalent)) whereas the installed capacity is only about the tenth (887.352) of this capacity as sh. This implies that it is possible to increase, very significantly, the energy provided by hydro power.

Furthermore (IEA 2011) states;

- Hydropower is a widely used resource around the world. There is potential in about 150 countries.
- It is a proven and well-advanced technology (more 90% energy conversion efficiency) with more than half a century of experience.
- It has a long plant life (40-50 years) with lowest production costs and with the possibility of extending (double) the life of the plant.
- Its fuel (water) is renewable.

However, investment in hydro energy (dams) require very large investment and operational costs. Thus, the level of optimal investment, through time, which will maximize the profit of the investor is important and that is the problem which will be addressed in this paper.

The rest of the paper consists of three sections. In the first section the mathematical model, the necessary conditions for optimality are presented. In the seconds section the solution of the problem is characterized. The last sections consist of implications of the solution and suggestions for further research on the subject.

2. The Model and the Solution

The analysis is done assuming energy sector is controlled by a central planner (the government) and energy is sold at a given price at any given time. It is also assumed that energy is not stored, carbon-based energy price is exogenously determined, and the cost of operation of hydro -energy source (dam) is negligible.

Renewable energy resource is assumed to be the hydro-energy only and seasonably varying since energy produced from hydro-power plants is dependent on the precipitation rate and weather conditions. The actual production trend, for example in Turkey, by month for two years (2011-2012), is given in Figure 1. Below to show that production is actually cyclical. It is safe to assume that this cyclical character is also valid in most countries. An optimal control theoretic model will be employed since investment in capacity is made to meet the current and the expected demand in the future and that the capacity of production of fixed nature depreciates through time taking into consideration that the water level affects the production.

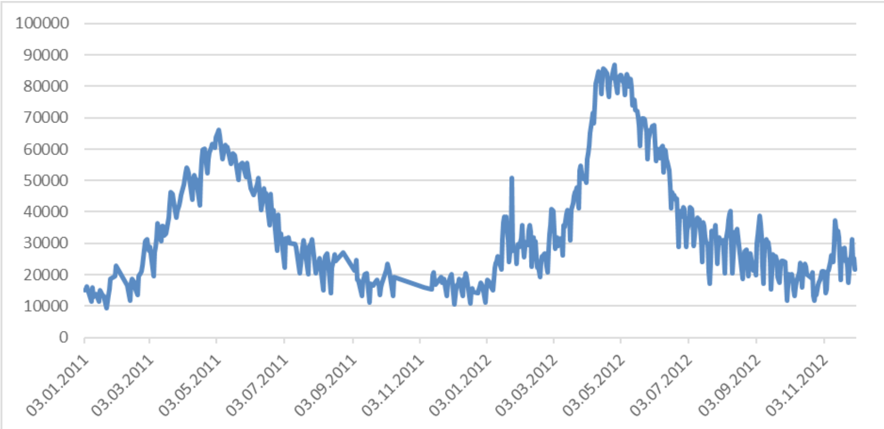

Figure 1. Turkey Hydroelectric Production Trend Between January 2011 and December.2012

Source: TEIAS Publications, 2012(Y Axis: Megawatts, X Axis: Dates)

As can be observed from this figure production roughly follows a sinusoidal trend. So, seasonality will be expressed as a sinus square function in our model as in Moser et al. (2012).

The planner (the government) is assumed to maximize its profits over an infinite horizon by optimally investing in hydro energy to meet the demand for energy by producing hydro energy and by buying carbon-based energy at predetermined prices.

Mathematically;

$$\max_{I(t)\geq 0, p(t)\geq 0} \int_0^\infty e^{-rt}\left(p(t)E(p(t)) - I(t)(b+cI(t)) - p_F E_F(t)\right)dt$$

$$K'(t) = I(t) - \delta K(t) \tag{1}$$

$$E_F(t) + E_H(K(t)) = E(t) \tag{2}$$

$$E_H(K,t) = (\upsilon \sin^2(kt\pi) + \tau)K^\alpha(t)\sigma \tag{3}$$

$$E_F(t), I(t), K(t) \geq 0$$

Where;

t=time

r=constant discount rate

p (t): price of energy (electricity)

E (p (t)): Energy demand at time t

I (t) =level of new investment in hydro electricity

b: unit cost of investment

c: adjustment cost parameter (the term bI represents the purchase cost of the required machinery-land while bI^2 represents the adjustment-installation cost of machinery. The squared term indicates that higher the level of investment higher will be the installation costs)

p_F :unit cost of carbon -based energy

E_F: Amount of imported carbon -based energy

K (t): The level of total investment in hydro energy which increases by new investment and depreciates at an exponential rate of δ (equation 1).

K' (t) =Time derivative of K (t)

K (0) =K_0, the initial level of hydro energy investment

E_H(K, t): Energy supplied by hydro facilities K varying sinusoidal because of seasonal availability of water supply. We will assume that the periodicity k=1.

$\propto \square 1$: output elasticity of capital for hydro energy

k: period length which will be assumed to be 1

Equation (2) signifies the equality of supply and demand for energy. The total demand of energy is met by hydro energy and the imported energy.

The integrand above shows the profit of the planner at time t taking into consideration the investment costs, adjustment costs, and the payment for carbon -based energy purchased at the prevailing world prices.

This is an optimal control theory problem and Pontryagin et.al (1962), Kamien and Schwartz (1991) and Sethi (2000) are only three of the many books written on the topic of optimal control theory and its applications. The necessary and sufficient conditions used below are clearly set out in these books.

Using equation (2) in the integrand and rearranging, and integrating out the known function of time (E (t)) yields:

$$\max_{I(t)\geq 0,\, p(t)\geq 0} \int_0^{\infty} e^{-rt}\left(E(p(t))(p(t)-p_F)+p_F(t)E_H(K,t)-I(t)(b+cI(t))\right)dt \tag{4}$$

In addition to equations 1-3.

The Current Value Hamiltonian for this problem is:

$$\text{H}= E(p(t))(p(t)-p_F)+p_F(t)E_H(K,t)-I(t)(b+cI(t))+\lambda(I(t)-\delta K(t)) \tag{5}$$

Where l(t) is the marginal valuation of a unit of investment in capacity (K). It reflects the value of a unit of new investment capacity from the time of investment to infinity.

The necessary conditions for the solution are:

$$H_I=-(b+2cI)+\lambda\leq 0,\ \ \text{H}_I I=0 \tag{6}$$

$H_p = E'(p)(p-p_F)+E(p)=0$ which determines the optimal price of energy since p_F is a given constant or a given function of time. This simplifies the solution because this result does not impact the rest of the solution of the problem.

$$\lambda' = \lambda(r+\delta) - p_F((\upsilon\sin^2(kt\pi)+\tau)\sigma\,\alpha K(t)^{\alpha-1} \tag{7}$$

In addition to equation (1) and the transversality condition;

$$\lim_{t\to\infty} e^{-rt}\lambda(t) = 0 \tag{8}$$

Then, using equations (6), (7), and (8), we get the following system of first order linear differential equation:

$$I' = I(r+\delta) + b(r+\delta)/2c - p_F(\upsilon\sin^2(kt\pi)+\tau)\sigma\alpha K^{\alpha-1}/2c \tag{9}$$

And rewriting equation (1);

$$K'(t) = I(t) - \delta K(t) \tag{10}$$

From equation (10), λ (t) can also be written as:

$$\lambda(t) = \int_t^\infty e^{-(r+\delta)(s-t)}(p_F(s)\ (\upsilon\sin^2(ks\pi)+\tau)\sigma\alpha K^{\alpha-1})\mathrm{ds} \tag{11}$$

Which implies that λ (t) is equal to the present value of marginal investment in hydro energy taking into effect the depreciation of investment.

The system defined by equations (9) and (10) cannot be solved explicitly (This system is easily solvable if ∝ was 1 as was assumed by Moser et.al. which is not very realistic). Hence, we will use phase diagrammatic analysis to describe the optimal investment strategy of investment.

The locus of points where I'=0 is yields a relationship between I and K in (I, K) space and it is represented by I'=0 curve in the phase diagram (Figure 1) below. However, this curve oscillates because of the presence of sine square function. 'I' will increase above this curve while it decreases below it. The locus of points where K'=0 which implies a relationship between I and K in (I, K) space is indicated by K'=0 curves'' decreases to the right of this curve while it increases to the left. These dynamics are indicated by directional arrows in the phase diagram. These loci divide the positive quadrant of (I, K) space into four quadrants which are indicated by roman numerals. The intersection of I'=0 and K'=0 is the Steady State point indicated on Figure 3 as point S and Y. Notice that Steady State point oscillates between points S and Y since I'=0 locus oscillates. The point K_0 is a point where I'=0 at t=0 while K_1 is where I'=0 when sine squared function reaches its maximum. The double arrow (↔) in Graph 1 indicates the oscillation of I'=0 locus.

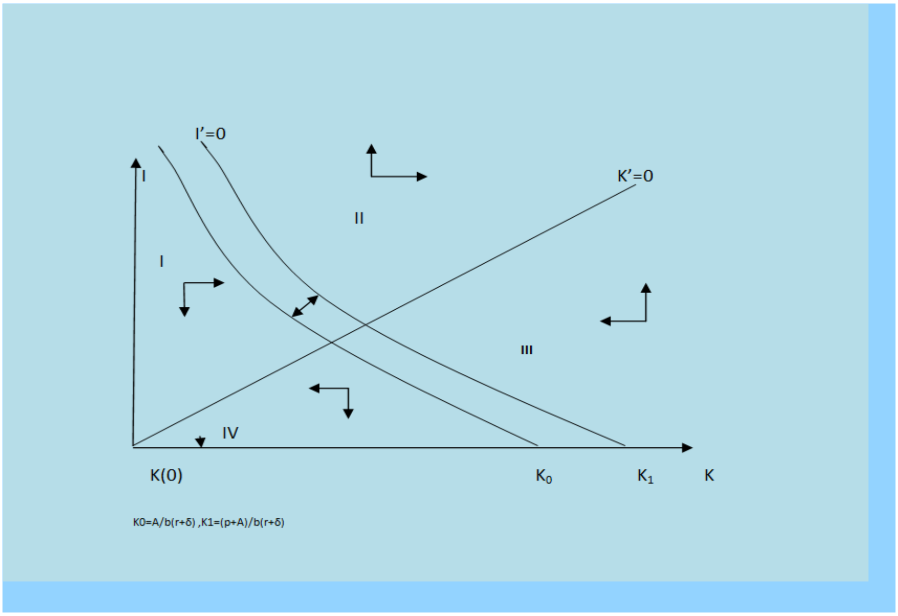

Figure 2. The Phase Diagram of The System (9) and (10).

The necessary conditions are also sufficient because of the concavity of the Hamiltonian in both the state and the control variable.

It is clear from equations (9) and (10) that the solution (I (t)) will be a nonlinear function of time. It will have a high value at the beginning of the planning horizon and decrease smoothly to its long-term equilibrium level at time increases.

We will describe the behavior of the optimal investment path of investment (I (t)) more precisely in the next section.

2.1. Characterization of the Optimal Solution Using Phase Analysis

The initial investment K (0) = K_0 is an important parameter in characterization of the solution. If K_0 is small as shown in the phase diagram, it is optimal to start at point in quadrant I, investing until the Steady State is reached and then invest just enough to stay on the oscillating Steady State. Steady State is the point where the system of differential equations (Equations 9 and 10) reaches an equilibrium which is dented as the point (K_s,I_s) in Figure 3 below. Starting in quadrant IV will diverge the system to the origin hence a no optimal solution exists. Starting in Quadrant II will also imply a divergent system. High initial level of investment will (Quadrant III) lead to the steady State only if we start with a low level of investment decreasing the capital stock to the Steady State and then invest just enough to stay at the oscillating Steady State.

It will be shown in Figure 3 that the starting points like A and C are not possible (we will do so only for the point A. The same reasoning can be applied to point C also). The double arrow line shows the oscillation of I'=0 locus. Assuming it is optimal to start at a point A, the system will move towards northeastern direction implied by the directional arrows. However, the locus of points where I’=0 will also move in the same direction because of the sine squared term which is always positive. Therefore the, if I’=0 locus moves faster than the system, the system may fall into the quadrant I indicated by the point B in the following Figure 3. Moreover, when I’=0 reaches its maximum in the northeasterly direction (when sine squared function reaches its maximum), it will reverse direction possibly putting the system in quadrant II again. This may continue until the oscillating Steady State is reached. We will show below that this is not possible.

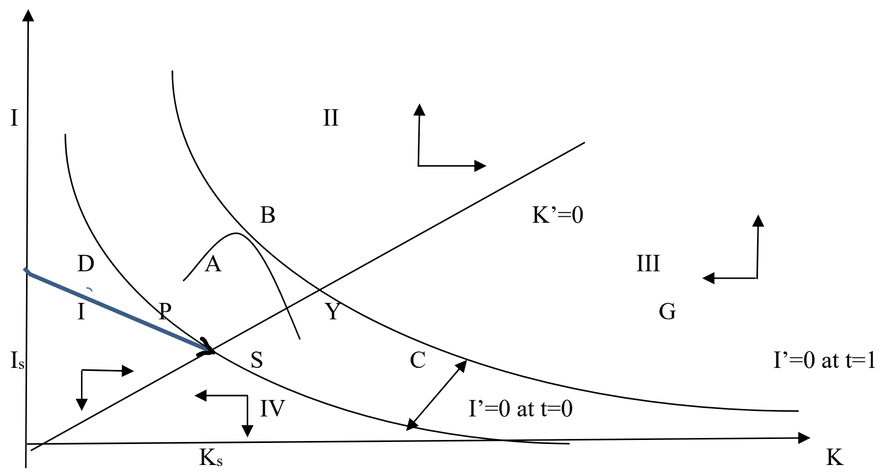

Figure 3. Phase Diagram at t=0 and t=1

At point B we have;

$$I' = 0 = I(r+\sigma) + b(r+\sigma)/2\mathrm{c} - p_F(\nu \sin\hat{}\ 2(\pi \mathrm{t}) + \tau)\sigma\alpha \mathrm{K}^{\alpha-1}/2c \qquad (12)$$

And, dividing equations (7) and (8) we have:

$$dI/dK = (I(r+\sigma) + b(r+\sigma)/2\mathrm{c} - p_F(\nu \sin\hat{}\ 2(\pi \mathrm{t}) + \tau)\sigma\alpha \mathrm{K}^{\alpha-1}/2c)/(I - \delta K) = 0 \qquad (13)$$

However, at the same point, for the I'=0 locus at point B, we must have;

$$dI / dK = (I(r+\sigma) + b(r+\sigma)/2c - p_F(\nu \sin\hat{}\, 2(\pi t) + \tau)\sigma\alpha K^{\alpha-1}/2c)/(I - \delta K) < 0 \quad (14)$$

Equations (13) and (14) indicates a contradiction. Therefore, it is not optimal to start a point like A and reach a point like B.

Therefore, it is only optimal to start at a point like D for small starting levels of capital, and to reach to the Steady State point K_s and oscillate between K_s and point Y on the K'=0 locus. A similar analysis can be made if the starting level of investment (K_0) is at point C. This behavior can be exhibited as in the following figure.

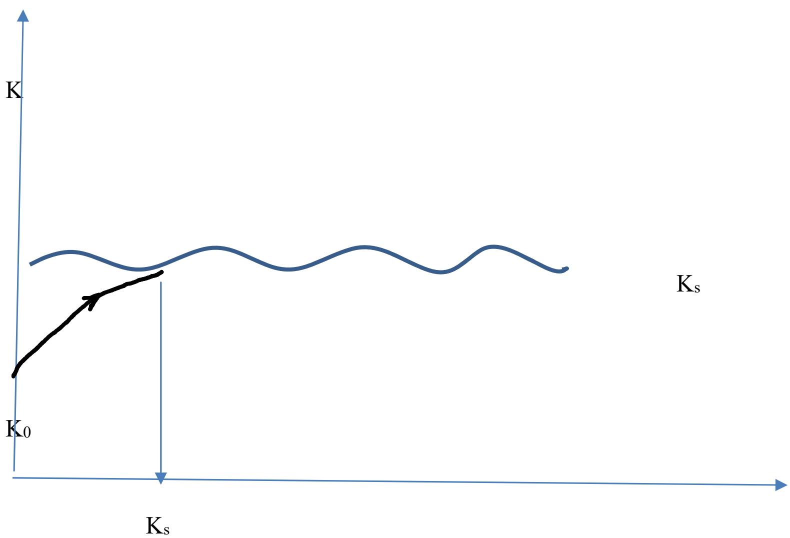

Figure 4. Behavior of Optimal Path of Investment

It looks as if the investment strategy is simple in that there should be heavy new investment in the capacity at early periods (See the path of

I (t) in Figure 3) gradually decreasing it to its Steady State value with an objective of increasing the total capacity (K (t)) to its Steady state value, Ks which will oscillate sinosiodally as shown above.

The implications of the solution of the mathematical model are:

- Invest in hydroelectric energy to increase (decrease if the initial investment is larger than the desire long term level) it to the desired long -term level and then invest just enough to keep the investment at this level adjusting it continually to the changing water levels.
- Higher price of imported energy will lead to higher investment in hydro energy with higher long-term levels of investment.
- The demand for energy can be assumed to be dependent not only on price but time also.
- Changing climate will affect the rainfall. This may have profound implications on the amount and frequency of rainfall. This will directly impact the investment strategy in hydroelectric energy capacity.

3. Conclusions and Suggestions for Further Research

The solution of the model is simple. The strategy that the solution offers is that the investor should gradually increase the production capacity to the desired long term level which is sinusoidal. The simplicity of the strategy of investment is a positive aspect for more investment in this underutilized energy resource .A more complex model can be studied by making some parameters like the price of carbon based sources as time dependent.

This study can be expanded further to include solar, wind, and nuclear energy in addition to carbon based energy. This more complex model will be more realistic but more difficult to solve since the model will then have four state variables and five control variables.

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