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Average Equivalent Diameter of A Particulate Material

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ABSTRACT

In the field of mineral processing, it is important to determine the size of a particle. A method of defining an average diameter for a collection of particles is presented. The theoretical basis developed for the purpose is verified by a specially designed experimental technique.

Key words: mineral processing, particle size, equivalent diameter

1. INTRODUCTION

The study of particulate materials is of fundamental importance in a number of fields of human activity. This particularly applies to mineral processing where particle size of the feed of any processing plant is governed by the degree of liberation of useful minerals from the gangue minerals, together with the optimal recovery and plant performance parameters, which heavily depends on size distribution and shape of particles. Unduly large particle size will result in insufficient liberation, whereas unnecessarily small particle size will involve an increase in comminution cost at diminished efficiency and reduction in processing plant performance on account of problems associated with particle fineness and flocculation.

An assessment of the particle size is extremely important at every stage of a mineral processing activity. A variety of techniques are available for size analysis of a particulate such as screen analysis, classification by sedimentation and elutriation, centrifuging, microscopic measurements, electrical sensing and radiation scattering. Sieve analysis is performed on relatively coarse sizes and rigorous standardization of procedures is required in terms of feed size, operating time and shaking mechanism so as to get reproducible results. Wet sieving or air jet methods may be applied for relatively finer size fractions. Microscopy is a very time-consuming manual method of rigorously measuring visible dimensions of individual particles. Since individual particles rest on a flat surface in stable condition, only length and breadth are normally measured. The smaller dimension can only be less accurately measured focusing at the top of the plate and then on the top of the particle. The average size of each particle is, then, computed. Various semi-automatic and automatic techniques have also been developed for counting and sizing. Electrical sensing and radiation scattering techniques have also been extensively used in determining particle - size distribution.

Fluid permeability through a particulate medium has also been used to measure pore and particle size. The principles of fluid permeability are extensively employed in determining pore and particle size distribution of a particulate medium. A comprehensive literature review is presented by Terence Allen (1).

One of the techniques of pore size measurements is based on the principle that liquid filled pores will become gas permeable at a certain pressure, because liquid has to be displaced by the gas first. This opening pressure, the so called bubble point, depends on the surface tension of the liquid and the pore size diameter. Topas GmbH pore size Meters, PSM 160/165 have been developed on this principle (2).

Another interesting development applied to on-line particle size analyzer is PSI 500 (3). The measured particle size can be used for on-line process and quality control. Its measurement is based on laser diffraction technology. The measured size ranges from 1 to 600 microns. Various other

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methods to measure particle size are available in literature (4, 5, 6, 7).

The principle of the method herewith presented is based on permeability of air through the voids between the particles of a sample contained in a U-shaped glass tube. The larger the size of the voids, and consequently of particles, the greater the permeability. A falling mass of mercury in a vertical tube creates a drop of pressure at one end of the sample, which makes the air from the other end of the sample to permeate through it. The greater the average rate of fall of the mercury through a given height, the larger are the particles. The objective of the paper presented here is on how to determine the average equivalent diameter of a particulate material. The apparatus was specially designed for the purpose and a mathematical technique was developed. The method has the following characteristics and advantages:

1. The proposed method is simple and gives results quite comparable with standard sieves analysis. This is a clear advantage of the method over several other methods of determining particle size.
2. A wide range of particle sizes can be measured.
3. It reduces the cost and the time associated with particle size analysis compared with other technique such as the Brunauer, and Teller technique.
4. It appears that the method is not dependent on the physical characteristics of the particles such as shape and surface as in other methods.
5. To apply the method, two velocities for air flow at two lengths should be known. This is easy to find compared with other methods.
6. The value of the length L_2 compared to L_1 should such that $0.4\ L_1 > L_2 > 0.25\ L_1$. This is very simple to control compared with other methods.
7. The method enables determination of an average equivalent diameter of particles if the void ratio is known. This void ratio is a new equation taking into consideration the effect of velocity $\frac{V_2 - V_1}{\bar{V}}$ and length of sample tube $\frac{r}{L_1 - L_2}$.
8. The method is sensitive to humidity which is the usual effect in most other methods.
9. It requires many initial experiments for controlling the apparatus.
10. For very fine particles, other technique rather than sieve analysis technique is needed for the comparison of results of the method.
11. The proposed method of measurement is based on air permeability, which leaves the sample unaltered. This may not be the case if liquid permeability methods are used.

2. THEORY

The exact size for irregular particles can not be measured. However, the expression most often used to quote the size of a particle is the "equivalent diameter". This refers to the diameter of a sphere that would behave in the same manner as the particle under some specified operation. The equivalent diameter depends on the method of measurements (8, 9, 10, 11).

2.1. Calculation of Average Equivalent Diameter of Particles

The cross-section across a particular sample of particles contained in a cylindrical tube (Fig 1) will have a certain ration of the area of voids to the area of the solid particles. There is a relationship between the ratio of voids and the size of these particles. A method is developed to calculate average equivalent diameter of particles depending on the voids ratio at any cross section of sample tube. This may be called Nahir's Method. The method requires that the maximum particle size must not be greater than radius of sample tube.

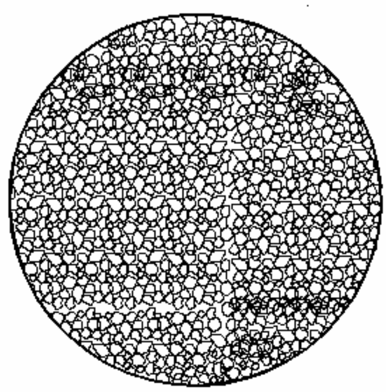

Figure 1. A typical cross section of a sample of particles contained in a tube

2.2. General Concept of Nahir's Method

Consider two spherical particles with centers in the common cross-section of a cylindrical tube. If the diameter of each particle is one half the diameter of the tube, each will fit in a quarter of the cross-section. If the size of these two particles is equal but is gradually reduced, with their common contact point remaining on y-axis, the centers of the particles will trace a curve which will be confined to within the respective quarters (Fig. 2).

There appears to be a relationship between void ratios and the relative geometry of the particle size distribution from which equivalent size of the particles may be determined. The total area of these voids is in direct proportion with the size of these particles, whenever the size of these particles decreases, the total area of voids will likewise decrease. This shows that there exists a relationship between the ratio of voids and the size of the particles. Through the study of this relationship, a method can be evolved by which average equivalent diameter of particles can be determined.

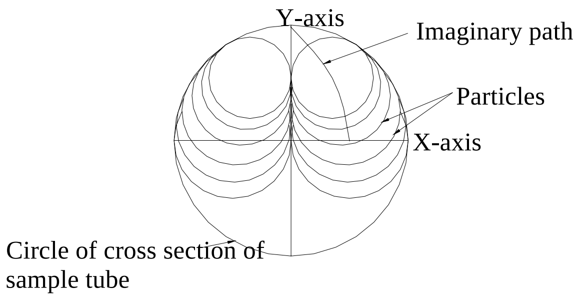

Figure 2. Trace of centers of particles of successively decreasing size.

It may be noticed that the imaginary path (for every particle) is always within the range of one quarter of the circle of cross section of sample tube.

Through this quarter and angle 90^o, an equivalent diameter can be found of these particles depending on voids ratio. This equivalent diameter will represent average equivalent diameter of all the particles at the cross section of sample tube.

2.3. Method Statement

Referring to (Fig 3) a set of five angles θ_1, θ_2, θ_3. θ_4 and θ_5 are shown for 2, 3, 4, and 7 spherical particles of equal size, closely packed in a circular cross-section of a tube. Angle 1 is between y-axis and a line joining the center of the tube with the point of intersection of a diameter of first particle parallel to y-axis. Similarly other angles θ_2, θ_3. θ_4 and θ_5 have a common definition for various number of particles.

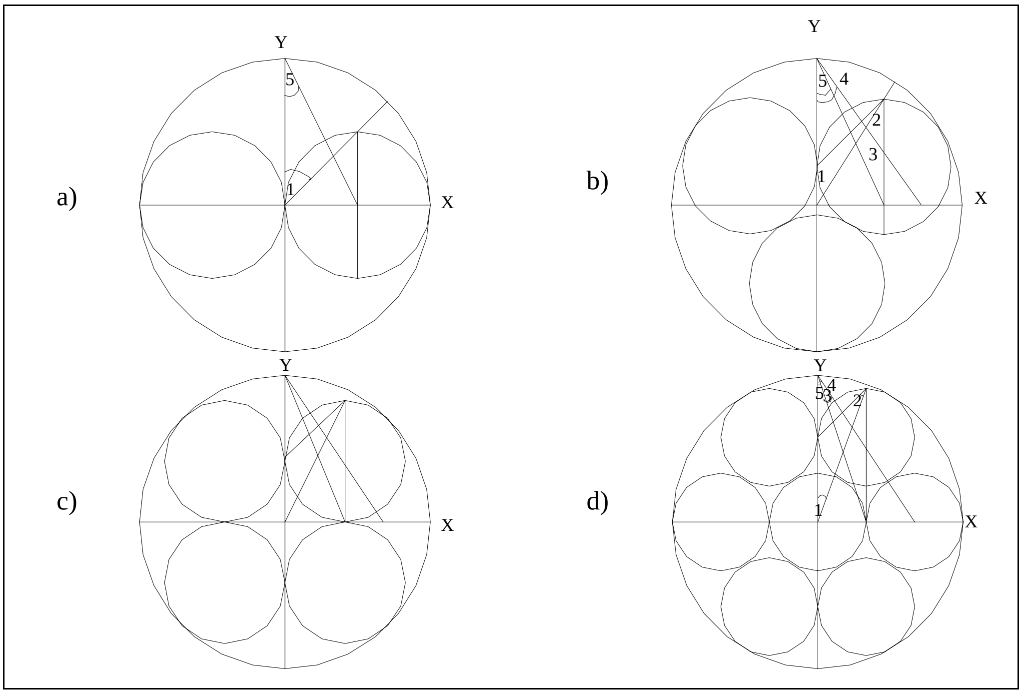

Figure 3. Cross section of the samples

By geometry, it may be found that:

1. Void ratio, VR = Area of voids / cross sectional area of sample tube.
2. $\theta_1 = 90^o * VR.$
3. $\theta_2 = 45^o - \theta_1$
4. $\theta_3 = (1 - VR) * \theta_2$
5. $\theta_4 = \tan^{-1}(1 - VR)$
6. $\theta_5 = \theta_4 - \theta_3$
7. Average equivalent diameter of a particle $= \tan\theta_5 * D$

Where D is the diameter of the sample tube.

2.4. Arithmetic proof of the Method

Two examples are chosen, one with 2 particles (Fig. 3a) and the other with 7 (Fig. 3d). At diameter of the sample tube = 6 cm and average equivalent diameter of particles = 3 cm.

Voids area = cross section area of tube- total cross sectional area of particles

$= 28.27 - 14.14 = 14.14\ cm^2$

$VR = 14.14 / 28.27 = 0.5$

$\theta_1 = 90^o * 0.5 = 45^o$

$\theta_2 = 45^o - 45^o = 0$

$\theta_3 = (1 - 0.5) * 0 = 0$

$\theta_4 = \tan^{-1} 0.5 = 26.56^o$

$\theta_5 = \theta_4 - \theta_3 = 26.56^o$

Average equivalent diameter of the particle = Tan θ_5 * D

= 0.5*6
= 3 cm

For the second example, with the diameter of the sample tube = 6 cm and average equivalent diameter of the particles = 2 cm.

Voids area = Cross sectional area of the tube - total area of cross section of all particles in the section

$$= 28.27 – 21.99 = 6.28\ cm^2$$

$$VR = 6.28/ 28.27 = 0.222$$

$$\theta_1 = 90^o * 0.22 = 20^o$$

$$\theta_2 = 45^o – 20^o = 25^o$$

$$\theta_3 = (1 – 0.22) * 25^o = 19.44^o$$

$$\theta_4 = Tan^{-1} (1 – 0.22) = 37.87^o$$

$$\theta_5 = \theta_4 = \theta_3 = 18.43^o$$

Average equivalent diameter of the particles = 2.00 cm.

Thus, this method enables the determination of average equivalent diameter of particles if the void ratio is known. However, it may be noticed that the following assumptions are made in the development of this method:

1. The particles are spherical.
2. All the particles are of the same size.
3. The particles are closely packed.

2.5. Estimating Voids Ratio

If an air current is made to flow through voids between solid particles contained in a tube over a certain length, the velocity of air flow will depend on particle size.

The voids between the particles are typical in their shape and size so it can be assumed that the average of the passage of air is equal to average of separate passages through individual tubes formed by inter connected voids. The voids between these particles are not identical and the passage of air through these particles is affected by many strangulations and some of such tubelets may be totally blocked.

Thus, the geometrical form and the way of passage are very complicated, and the velocity of passage differs from one point to another. For this reason, the passage through a complicated medium is generally described as macroscopic flow velocity vector, which is represented by average of microscopic velocities through these particles (12).

By means of this vector the area "A" and voids ratio "VR" at any section of the sample can be calculated by using the following equations:

Q = AV (Equations of Continuity)
A = Q / V
VR = A / cross section of sample tube.

Where Q represents known air flow quantity, A is total voids area through which the air passes at any section of the sample and V represents the average of macroscopic velocities through the particles.

Due to the difficulty in finding the average of the microscopic velocities V through complicated medium such as particles sample and after many laboratory tests, the following equation was developed which can be used to calculate voids ratio at any cross section of the sample.

$$\boxed{VR = \left[\begin{matrix} Sin^{-1} \left\{ (V_1 L_1 + V_2 L_2) / \bar{V} (L_1 + L_2) \right\}^{0.5} - \\ (V_2 - V_1)^{0.5} / \bar{V} . Sin^{-1} \left\{ r / (L_1 - L_2) \right\}^{0.5} \end{matrix} \right] / 180^O} \quad (1)$$

Where V_1 and V_2 represent air velocity which are represented by the velocity of the mercury on the test apparatus at two lengths of the sample, L_1 and L_2 , $(\bar{V})$ is the free air velocity when the sample tube is empty and r is the radius of sample tube.

To apply this relationship the two velocities V_1 and V_2 for air flow at two lengths L_1 and L_2 should be known. The value of the length L_2 compared to L_1 should such that:

$$0.4\ L_1 > L_2 > 0.25\ L_1$$

3. EXPERIMENTAL STUDY

An apparatus (Fig.4) was designed to measure flow velocity through a sample of granular medium contained in a U – tube. Mercury is allowed to move down under gravity by opening a valve. The velocity of movement of mercury is determined by noticing the time it takes to move along a graduated tube. The downward movement of mercury sucks air through the sample. The volume of mercury moving down the graduated tube represents the volume of air passing through the sample.

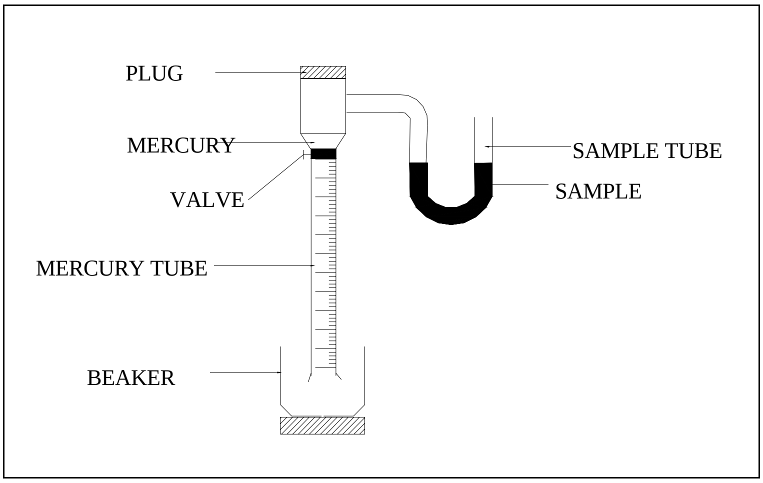

Figure 4. Apparatus for measuring the velocity of air flow

The significant Specifications of the Apparatus are as follows:

1. Mercury tube length = 75 cm.
2. Sample tube length = 30 cm.
3. Each mercury and sample tube internal diameter = 4.12 mm.

A number of experiments were performed on the apparatus to determine average equivalent diameter of particles.

The steps of the experiments were as follows:

1. Shut the valve and put a quantity of mercury through the upper opening of the mercury tube. Then, shut the upper opening by a plug.
2. Make sure that the sample tube is empty and then open the valve. By using a stop watch, measure the time during which the mercury goes from one point to another when it falls down through the tube. This enables to measure mercury velocity, which represent the velocity of the free flow of the air $\overline{V}$ according to the following equation:
   V = distance / time (cm /sec)
   Where V is the velocity of mercury.
3. Repeat the first step and put the same quantity of mercury.
4. Put a quantity of the sample at length L_1 in the sample tube and then open the valve, and measure the time taken for the movement of mercury through a certain length – calculate V_1.
5. Repeat the fourth step at sample length L_2 to find V_2.
6. By finding $\overline{V}$,V_1 and V_2 at two lengths for the sample L_1 and L_2 we can calculate the average equivalent diameter for particles according to the following equations:

$$VR = \left[\begin{array}{l} Sin^{-1}\left\{(V_1L_1 + V_2L_2)/\overline{V}(L_1+L_2)\right\}^{0.5} - \\ (V_2 - V_1)^{0.5}/\overline{V}.Sin^{-1}\left\{r/(L_1-L_2)\right\}^{0.5} \end{array} \right] /180^{O} \quad (2)$$

$\theta_1 = 90^o * VR.$
$\theta_2 = 45^o - \theta_1$
$\theta_3 = (1 - VR) * \theta_2$
$\theta_4 = \tan^{-1}(1 - VR)$
$\theta_5 = \theta_4 - \theta_3$

Average equivalent diameter of particles = Tan θ_5* D
Where D is the diameter of the sample tube.

4. DISCUSSION AND CONCLUSION

After sizing silica sand by sieves, six samples were selected as shown in Table 1. Each of these samples was tested on the apparatus so as to calculate average equivalent diameter of particles.

Table 1. Fractions of Silica Sand used for the Experiments

The samples	Sieve size (mm)
First sample	- 1 + 0.71
Second sample	- 0.71 + 0.5
Third sample	- 0.5 + 0.35
Fourth sample	- 0.35 + 0.25
Fifth sample	- 0.25 + 0.15
Sixth sample	- 0.15 + 0.12

4.1. First Group of Experiments:

In this group average equivalent diameter of particles for the chosen six samples was determined. These experiments were performed at:

1. Diameter each of sample and mercury tubes = 4.12 mm
2. Mercury weight = 2.00 gm
3. Average L_1 = 33.30 cm
4. Average L_2 = 11.80 cm

$\overline{V}$ was determined by the rate of displacement of mercury with empty U – tube. The results are given in Table 2.

The results of the experiment on the first sample are given in Table 3 and 4.

Average V1 = 11.88 cm/Sec.
Average V2 = 28.10 cm/Sec.
VR = 0.12
$\theta_1 = 90^o (0.12) = 10.80^o$
$\theta_2 = 45^o - 10.80^o = 34.20^o$
$\theta_3 = (1 - 0.12) * \theta_2 = 30.90^o$
$\theta_4 = \text{Tan}^{-1} (1 - 0.12) = 41.35^o$
$\theta_5 = 41.35^o - 30.90^o = 11.25^o$
Average equivalent diameter of particles = Tan (11.25)* 4.12 = 0.82 mm.
Table 5 Summarizes results for all the experiments pertaining to group 1.

4.2. Second Group of Experiments

These experiments were made to check the result of the first group of experiments in which weight of mercury was reduced from 2.0 gm to 1.65 gm. The length of the samples was also reduced.

1. Diameter each of the two tubes of sample and mercury = 4.12 mm
2. Mercury weight = 1.65 gm
3. Average L_1 = 19.20 cm.
4. Average L_2 = 5.90 cm.

Table 6 gives the results of free air current results to determine $\overline{V}$.
Table 7 summarizes results for all the experiments pertaining to group 2.
Table 8: summarizes results for First and second group of experiments.

Table 2. Free Air Current Results of First Group

Distance (cm)	Time (Sec.)	$\overline{V}$ (cm/Sec.)
71.5	0.75	95.33
73.0	0.73	100.00
73.0	0.72	101.39
71.0	0.72	98.61
71.0	0.72	98.61
73.0	0.69	105.80
73.0	0.75	97.33
73.0	0.75	97.33
73.0	0.75	97.33
73.0	0.78	93.59
72.5	0.72	100.69
72.0	0.75	96.00
72.0	0.65	110.77
70.5	0.75	94.00
71.5	0.72	99.30
73.0	0.75	97.33
73.0	0.75	97.33
73.0	0.74	98.65
72.5	0.71	102.11
71.0	0.75	94.67

Average $\overline{V}$ = 98.81 cm/Sec.

Table 3. Air Velocity Results at L_1

L1 = 33.9 cm		
Distance (cm)	**Time (Sec.)**	**V_2 (cm/Sec.)**
30	2.54	11.81
30	2.50	12.00
30	2.66	11.28
30	2.60	11.54
30	2.47	12.14
30	2.62	11.45
30	2.50	12.00
30	2.47	12.14
30	2.44	12.29
30	2.47	12.14

Table 4. Air Velocity Results at L_2

L2 = 12.1 cm		
Distance (cm)	**Time (Sec)**	**V_2 (cm/Sec.)**
30	1.00	30.00
30	1.13	26.55
30	1.09	27.52
30	1.03	29.13
30	1.00	30.00
30	1.10	27.27
30	1.13	26.55
30	1.13	26.55
30	1.06	28.30
30	1.03	29.13

Table 5. Results of First Group of Experiments

Sample	L_1 (mm)	L_2 (mm)	Av V_1 (cm/s)	Av V_2 (cm/s)	Void Ratio VR	θ_1	θ_2	θ_3	θ_4	θ_5	Av Eq. Diameter (mm)	Av. Size (mm)
1	33.9	12.1	11.88	28.1	0.12	10.80	34.20	30.90	41.35	11.25	0.82	0.855
2	29.1	10.9	6.65	17.70	0.09	8.10	36.90	33.57	42.30	8.73	0.63	0.605
3	33.5	11.9	2.56	6.33	0.05	4.92	40.08	37.89	43.39	5.50	0.40	0.425
4	36.9	13.5	1.29	3.04	0.04	3.48	41.52	39.91	43.87	3.96	0.28	0.300
5	34.2	11.2	0.59	1.54	0.03	2.36	42.64	41.52	44.24	2.71	0.20	0.200
6	32.4	11.35	0.30	0.66	0.02	1.64	43.36	42.57	44.47	1.90	0.14	0.135

Table 6. Free air current results of second group

Distance (cm)	Time (Sec.)	$\overline{V}$ (cm/Sec.)
72.0	0.88	81.82
70.0	0.84	83.33
69.5	0.72	96.53
68.5	0.78	87.82
64.5	0.71	90.84

Average $\overline{V}$ = 88.07 cm/sec

Table 7. Results of Second Group of Experiments

Sample	L_1 (mm)	L_2 (mm)	Av V_1 (cm/s)	Av V_2 (cm/s)	Void Ratio VR	θ_1	θ_2	θ_3	θ_4	θ_5	Av Eq. Diameter (mm)	Av. Size (mm)
1	21.8	6.20	12.47	31.85	0.13	11.41	33.59	29.33	41.13	11.79	0.86	0.855
2	17.40	6.20	7.52	16.51	0.10	8.56	36.44	32.98	42.14	9.17	0.66	0.605
3	18.30	6.30	3.06	7.70	0.06	5.49	39.51	37.10	43.20	6.10	0.44	0.425
4	18.40	5.70	1.74	4.86	0.05	4.15	40.85	38.97	43.65	4.68	0.34	0.300
5	18.80	5.60	0.83	2.73	0.03	2.92	42.08	40.71	44.05	3.34	0.24	0.200
6	20.50	5.50	0.28	0.70	0.02	1.63	43.37	42.59	44.48	1.89	0.14	0.135

Table 8. Summary of Results for First and Second Group of Experiments

The samples	Sieve Size (mm)	AV. Fraction size (mm)	Average Equivalent Dia. of particles (mm)	
			First Group	Second Group
The first	- 1 + 0.71	0.855	0.82	0.86
The second	- 0.71 + 0.5	0.606	0.63	0.66
The third	- 0.5 + 0.35	0.425	0.40	o.44
The fourth	- 0.35 + 0.25	0.300	0.28	0.34
The fifth	- 0.25 + 15	0.200	0.20	0.24
The sixth	- 0.15 + 0.12	0.135	0.14	0.14

4.3. Conclusion

1. There is a very close comparison of the values of experimentally determined average equivalent diameter and the average size of the respective fractions of each sample. The results are very encouraging in that the method can be satisfactorily employed for determining average equivalent diameter of a granular medium.

2. The results of the second group of experiments are also quite good but slightly less accurate than those of the first group of experiments. This may be because of the quantity of mercury being less. Therefore, using 2 grams of mercury is better than using 1.65 gm. The length of the sample might also have some bearing on accuracy of the results.

3. Further research work may be recommended for:
   a. Applications of the method on finer particles.
   b. Effect of the amount of mercury used for the experiments.
   c. Effect of the length of samples.
   d. Application of the method to samples comprised of wide range of particle size.

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