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A Hybrid Measure of Efficiency in Performance Measurement: An Application to Concrete Industries

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ABSTRACT

In the present competitive age, the organization tries to gain competitive edge. In order to gain competitive edge, the managers need to consider and analyze performance of their organizations and to decide to improve them. There are various approaches to evaluate the performance in which the criterion of efficiency of under-study units is being placed in efficiency frontier: Data envelopment analysis is one of these approaches and designed in two radial and non-radial categories. Both of these models have advantages and shortcomings. So applications of these models in performance measurement are difficult. In this article, the Hybrid model is presented as the more accurate and more comprehensive measure of evaluation, and effort have been made to present more realistic results for decision making. Then the method is applied to measure the efficiency of 24 Concrete parts producer industries and shows their results.

Key Words: *Radial models, Non-Radial models, Data Envelopment Analysis Hybrid Model.*

1. INTRODUCTION

Data envelopment analysis (DEA) is a mathematical Programming method for evaluating the relative efficiency of decision-making units (DMUs) with multiple outputs and multiple inputs [1]. DEA calculates an efficiency score for each DMU under evaluation compared to a set of DMUs. Examples of DMUs that have been benchmarked with DEA are bank branches, fast food branches, and hospitals [2]. The DEA efficiency score measures the maximum radial (proportional) reduction in all inputs (increment in all outputs) that would increase the efficiency of a DMU to the level of the most efficient DMUs in a study set [3]. However, after this reduction (increment) is achieved, there may still exist slacks in some inputs and outputs, indicating that some additional in efficiencies remains in inefficient DMUs. To solve this problem, researchers [4] developed a set of non-radial DEA models where individual input reductions (output increments) are measured (see [5], [6] for a list of earlier non-radial DEA models). There are published papers that used non-radial approaches to measure efficiency and with application Japanese banking industry. The radial approach is represented by the CCR and BCC models. Its shortcoming is that it neglects the non-radial input/output slacks. The non-radial approach SBM deals with slacks directly, but it neglects the radial characteristics of inputs and/or outputs [7]. Differences exist in the characterization of input or output items if we divide inputs and outputs to radial and non-radial. To taking advantage of accurate measurement, it is necessary to compose of both radial and non-radial models. In this paper, we integrate these approaches in a unified framework and present a hybrid measure of efficiency (Hybrid).

2. THEORY

Data envelopment analysis known as DEA, developed by Charnes et al.(1978) [8] and Banker et al.(1989) [9]. DEA is a method used for the measurement of efficiency in cases where multiple input and output factors are observed and when it is not possible to turn these into one aggregate input or output factor. Since 1978, thousands of articles have been published using this analysis technique in various fields.

One of the most basic DEA models is CCR model [10] which was initially proposed by Charnes, Cooper and Rhodes. This section deals with CCR model as representative of radial models. The dual problem of (LP0) is expressed with a real variable θ and a non-

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negative vector $\lambda = (\lambda_1, ..., \lambda_n)^T$ of variables as follows:

$$
\begin{aligned}
(DLP_0) \quad & Min \quad \theta \\
& st: \\
& Y\lambda \geq y_0 \qquad r = 1,2,...,s \\
& \theta x_0 - X\lambda \geq 0 \qquad i = 1,2,...,m \qquad (1) \\
& \lambda \geq 0. \qquad j = 1,2,...,n
\end{aligned}
$$

Where:

θ : Efficiency score,

λ_s : Dual variables,

y_0 : Amount of output produced by DMU_0, and

x_0 : Amount of input utilized by

To θx_0 while remaining in P. Dual model is looking for an activity in P that guarantees at least the output level y_0 of DMU_0 in all components while reducing the input vector x_0 proportionally (radially) to a value as small as possible [8].

If an optimal solution $(\theta^*, \lambda^*, s^{-*}, s^{+*})$ of the LP above satisfies $\theta^* = 1$ and zero-slack $(s^{-*} = 0, s^{+*} = 0)$, then the DMU_0 is called CCR-efficient. In other words, the DMU_0 is called CCR-efficient, because:

$$(i) \quad \theta^* = 1$$

$$(ii) \quad (s^{-*} = 0, s^{+*} = 0)$$

Both of rules must be satisfied if full efficiency is to be attained. Input excesses and output shortfalls are defined following:

$$s^- = \theta x_0 - X\lambda \ , \ s^+ = Y\lambda - y_0$$

However, CCR model in efficiency improvement has structural weakness. The main shortcoming of the CCR model is the neglect of non-radial slacks in reporting the efficiency score θ^*. In many cases, we find a lot of remaining non-radial slacks. So, if these slacks have an important role in evaluating managerial efficiency, the radial approaches may mislead the decision making when this approach is utilized as the only index for evaluating performance of DMUs [6].

To solve this problem, the SBM[1] model have been designed by introducing a measure that makes its efficiency evaluation, as effected in the objective, invariant to the units of measure used for the different inputs and outputs. In order to estimate the efficiency of a DMU (x_0, y_0), fractional programming has formulated the following in λ, s^{-*} *and* s^{+*}.

$$(SBM) \quad \rho^* = \min \frac{1 - \frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{io}}{1 + \frac{1}{s}\sum_{r=1}^{s} s_r^+ / y_{ro}}$$

subject to :

$$
\begin{aligned}
x_0 &= X\lambda + s^- \qquad (2) \\
y_0 &= Y\lambda - s^+ \\
\lambda \geq 0, & s^- \geq 0, s^+ \geq 0.
\end{aligned}
$$

Where:

m : Number of inputs,

s : Number of outputs,

s_i^- : Amount of input slack

s_r^+ : Amount of output slack

y_{ro} : Amount of output i produced by DMU_0, and

x_{io} : Amount of input r utilized by DMU_0

To attain efficient performance it's necessary that in certain levels of outputs, is decreased amount of inputs.

In SBM model efficiency score, p = 1, reached in SBM model only if slacks are zero in all inputs and outputs. In other words, under study unit will be efficient if it doesn't have surplus recourses or it doesn't confront with shortage production. Previously it supposed that SBM model would be useful in performance measurement but its validation was finished when radial inputs and outputs become propounded. It is said that differences exist in the characterizations of inputs or outputs (radial and non radial inputs and outputs) items. Inputs and outputs divide to radial and non-radial. To gain maximum outputs, radial inputs should be reduced proportionally and non-radial inputs should be decreased non-proportionally. Therefore compose of two radial and non-radial approaches will present accurate measurement.

2.2. A Hybrid Measure

In this model, inputs and outputs classified two radial and non-radial categories and each one based on own specifications improve to efficient frontier. In other words, hybrid model is composed of radial (CCR) and non-radial (SBM) and exploits their strengths and compensates their shortcomings [7]. Figure 1 illustrate hybrid conceptual model with separation radial and non-radial data.

[1] Slack Based Measure

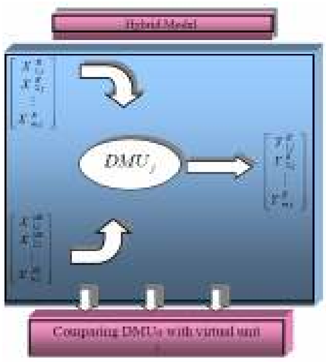

Figure1. Hybrid conceptual model.

Based on the classification term index ρ is defined as follows:

$$\rho = \frac{1-\frac{m_1}{m}(1-\theta)-\frac{1}{m}\sum_{i=1}^{m_2} s_i^{NR-} / x_{io}^{NR}}{1+\frac{s_1}{s}(\phi-1)+\frac{1}{s}\sum_{r=1}^{s_2} s_r^{NR+} / y_{ro}^{NR}} \qquad (3)$$

Where :

θ : decrease ratio in inputs

ϕ : Increase ratio in outputs

s_i^{R-} : Amount of radial input slack

s_i^{NR-} : Amount of non-radial input slack

s_r^{R+} : Amount of radial output slack

s_r^{NR+} : Amount of non-radial output slack

And θhis index p is designed so that it is decreasing with respect to decreases in θ and increases in ϕ, $s_i^{NR-} \forall i$ and $s_r^{NR+} \forall r$, but is not affected by s^{R-} and s^{R+} directly, reflecting free disposability of these radial slacks. So the Hybrid model presented as follows:

$$(Hybrid) \quad \rho^* = \min \frac{1-\frac{m_1}{m}(1-\theta)-\frac{1}{m}\sum_{i=1}^{m_2} s_i^{NR-} / x_{io}^{NR}}{1+\frac{s_1}{s}(\phi-1)+\frac{1}{s}\sum_{r=1}^{s_2} s_r^{NR+} / y_{ro}^{NR}}$$

$$\begin{aligned} \textit{subject to}: \quad & \theta x_0^R \geq X^R \lambda \\ & x_0^{NR} = X^{NR} \lambda + s^{NR-} \\ & \phi y_0^R \leq Y^R \lambda \\ & y_0^{NR} = Y^{NR} \lambda - s^{NR+} \\ & \theta \leq 1, \phi \geq 1, \lambda \geq 0, s^{NR-} \geq 0, s^{NR+} \geq 0. \end{aligned} \qquad (4)$$

The ρ is decreasing with respect to decreases in θ and increases in ϕ, $s_i^{NR-} \forall i$ and $s_r^{NR+} \forall r$, but is not affected by s^{R-} and s^{R+} directly, reflecting free disposability of these radial slacks. This index is also units invariant, i.e., invariant with respect to the measurement units of the data [7].

2.2. Hybrid Efficient status

The $DMU(x_0, y_0)$ is hybrid efficient if and only if $\rho = 1$ for every feasible expression of (3) and:

$$\theta = 1, \phi = 1, s^{NR-} = 0, s^{NR+} = 0$$

2.3. Decomposition of Inefficiency

Using the optimal solution $(\theta^*, \varphi^*, s^{NR-*}, s^{NR+*})$, we can decompose the hybrid efficiency indicator ρ^* into four factors as follows:

Radial input inefficiency:

$$\alpha_1 = \frac{m_1}{m}(1-\theta^*)$$

Non- radial input inefficiency:

$$\alpha_2 = \frac{1}{m}\sum_{i=1}^{m_2} s_i^{NR-} / x_{io}^{NR}$$

Radial output inefficiency:

$$\beta_1^* = \frac{s_1}{s}(\phi-1)$$

Non-radial output efficiency:

$$\beta_2^* = \frac{1}{s}\sum_{r=1}^{s_2} s_r^{NR+} / y_{ro}^{NR}$$

Inefficiencies also are defined input and output as:

Input inefficiency:

$$\alpha = \alpha_1 + \alpha_2$$

Output inefficiency:

$$\beta = \beta_1 + \beta_2$$

This expression is useful for finding the sources of inefficiency and the magnitude of their influence on the efficiency score ρ^* [7].

2.4. Inefficiency Improvement

Current research follows to find method with mathematical assurance and to show efficiency and inefficiency of production units. Obviously, performance improvement will be done when inefficiencies is recognized. According to explanations, using hybrid model determines efficient units and for inefficient units present, improvement benchmark by decrease in input wasting. In researcher's view applying this method will have better and more precise results in performance measurement. Then this method recommend for government, private, production and service organizations. Figure 2 illustrate above mentioned method

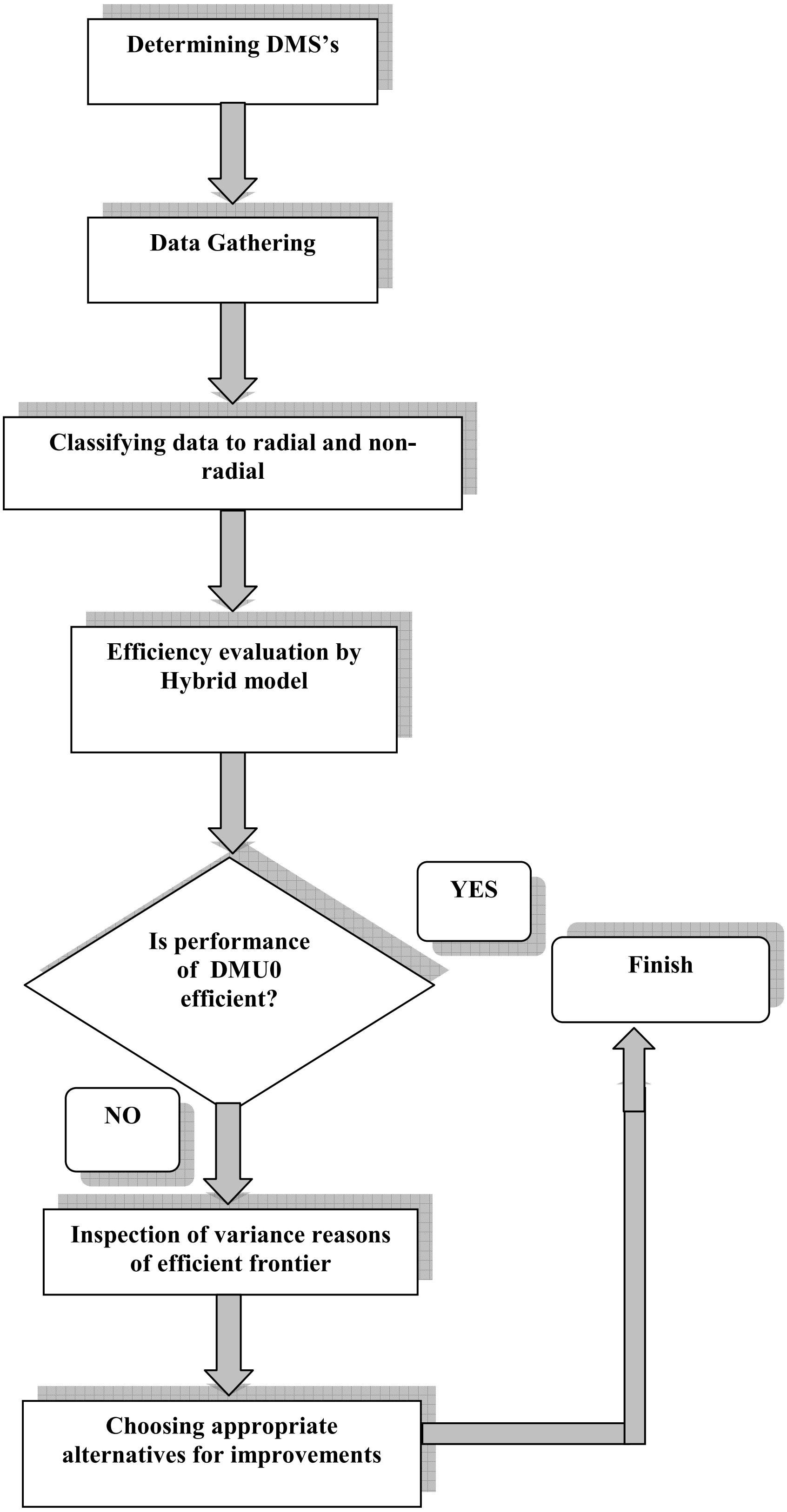

Figure 2. Efficiency improvement procedure.

3. AN APPLICATION

To lighten the weight and non conducting heat of building concrete parts industries have quickly grow and a lot of companies produce concrete products. In order to these companies use similar technologies therefore it's very critical to access better performance and attain more market shares.

Companies of concrete parts in this research are located in Tabriz industrial towns (Rajaei-pak-mayan) and mainly are producing wall and ceiling parts. This society consists of 24 factories and they are established research's DMUs. Performance measurement and efficiency evaluation of them is research scope and in this way we explicit efficient and inefficient DMUs. Case Data have been used as the followings as:

Inputs: x_{ij}

Human recourse:

The total number of employees (Based on indicator worker)

Material:

Amount of consumed row material (ton)

Machinery:

The number of used machines (Quantity)

Other costs:

Cost of consumed water and electricity (in 1000 Rials)

Outputs: y_{ij}

Product1: wall parts (Quantity)

Product 2: ceiling parts (Quantity)

Current research had used Hybrid method, and each production line is considered as a DMU. So, in this study we will have 24 DMUs with 4 Inputs and 2 Outputs. Data on the above-mentioned factors for the DMUs are reported in Table 1, where the last columns give the efficiency scores obtained from the DEA models in. These scores are calculated with DEA Solver software. After data gathering efficiency scores of DMUs will calculated separately with used of three CCR, SBM and Hybrid models and then difference between reported inefficiency will be considered.

Table 1. Data and efficiency scores.

DMUs	Radial Input		Non-radial Input		Radial Output		Efficiency score		
	x_1	x_2	x_3	x_4	y_1	y_2	Hybrid	CCR	SBM
1	13	2	3.7	110	700	260	0.744	0.918	0.650
2	8	2	2.6	80	440	200	0.595	0.640	0.595
3	15	3	4.8	120	1000	500	1	1	1
4	8	2	2.9	105	520	200	0.579	0.689	0.569
5	12	2	3.5	140	600	200	0.743	0.850	0.680
6	13	2	3.5	140	600	200	0.605	0.763	0.535
7	7	2	5	75	340	100	0.415	0.514	0.409
8	13	3	6.5	115	800	400	0.768	0.847	0.768
9	15	3	7.8	110	1000	300	0.809	0.981	0.792
10	17	4	3.8	190	1600	500	1	1	1
11	25	5	2.8	210	1700	800	1	1	1
12	17	4	8	85	800	400	1	1	1
13	12	2	2.6	89	500	200	0.591	0.692	0.517
14	17	5	3	210	1800	800	1	1	1
15	11	2	2.2	75	400	200	0.659	0.659	0.594
16	16	3	1.8	70	800	200	1	1	1
17	10	2	1.8	64	300	260	0.977	0.977	0.795
18	6	1	2.1	75	100	80	0.332	0.4	0.299
19	13	2	2	62	360	200	0.782	0.782	0.628
20	9	2	1.8	120	380	100	0.455	0.481	0.455
21	17	3	4.5	130	800	500	0.915	0.915	0.858
22	17	3	4.3	140	900	500	0.851	0.934	0.769
23	16	4	5.8	190	1100	800	1	1	1
24	21	4	6.4	210	1600	500	0.874	1	0.827

4. A COMPARISON OF THE MODELS

In this section, the performance of the CCR , SBM and the Hybrid DEA models are compared using generated information, where the performance is measured in terms of the radial and non-Radial data. The results demonstrate the important differences in score of three used methods based on non-zero slacks. For example, we find that $\theta_1^{*CCR} = 0.918$ as radial score for DMU_1 but efficiency scores of two DEA models are

$\rho_1^{*SBM} = 0.65$ and $\rho_1^{*Hybrid} = 0.744$ .This indicates that non-zero slacks have been ignored in radial efficiency measuring. Also radial specification has been neglected in non-radial models. In hybrid model radial variables improve as proportionally and non-radial variables improve variant to reach maximum efficiency. In some units are seen that hybrid efficiency score is equal with one of SBM or CCR models. To instance in second unit SBM score and hybrid score are similar. Reason of this equality is existence of non-radial inefficiency and lack of radial inefficiency. The other considerable point is efficiency scores in 24th unit. This DMU have perfect score (1) but another models calculate inefficient amounts in input slacks and recognize said unit as inefficient DMU. Then, significance of usage with model that could calculate slack inputs with type of them becomes clearer. This property makes evaluation by hybrid model more exact and between radial and non-radial scores. Reminder units with efficiency score (1), are perfect efficient and don't have input slacks. In other word, in certain level of output they use minimum input. This conclusion is further supported by the comparative bar graph displayed in Figure 3, which shows that the 3 the three models yield similar efficiency values for all 24 Concrete part producer industries, as can be deduced from Figure 3.

Goal of this study is description of more accurate of inefficiencies(radial and non-radial) and for complete ranking should used to another methods with mentioned method (Hybrid).

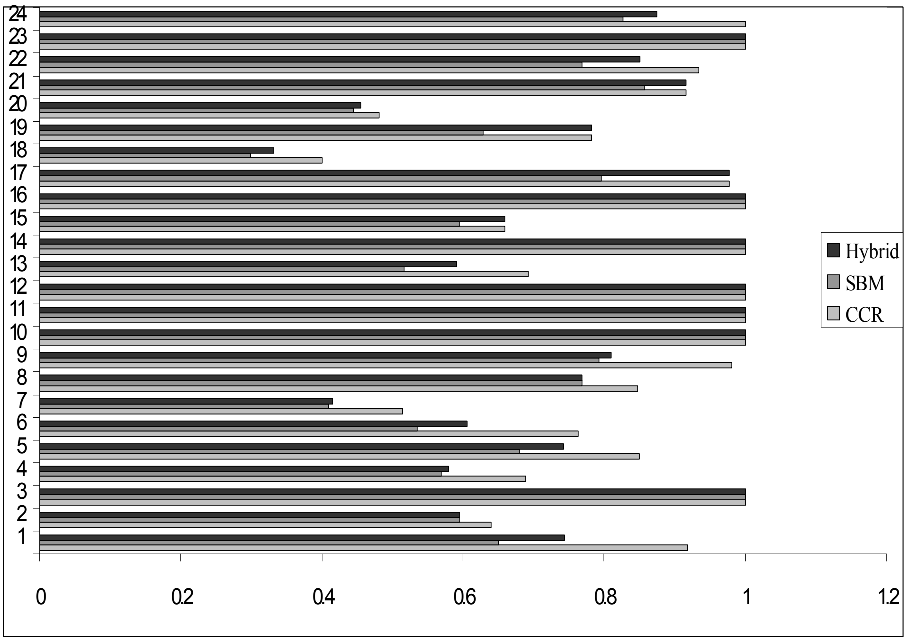

Figure 3. The Comparison of the relative efficiency resulting from the CCR, SBM and Hybrid DEA models for the 24Concrete part producer industries data set.

5. VALIDITY

To evaluation method's validity has been compared results of three models with presentation ranking by experts. This ranking is called real ranking and has been attention to view of experts with themselves attributes. Real ranking and ranking of three models have been shown in table2:

To compare of results of hybrid model and real ranking have used Wilcoxon test (SPSS v.14) and is comment as the follows:

Table 2. Ranking of DMUs by Real, Hybrid, SBM and CCR ranking.

CCR Ranking	SBM Ranking	Hybrid Ranking	Real Ranking
12	15	15	15
21	17	19	19
1	1	1	3
19	19	21	22
14	14	16	13
17	20	18	20
22	23	23	18
15	13	14	14
9	11	12	12
1	1	1	5
1	1	1	2
1	1	1	6
18	21	20	21
1	1	1	1
20	18	17	17
1	1	1	4
10	10	8	10
24	24	24	24
16	16	13	16
23	22	22	23
13	8	9	9
11	12	11	11
1	1	1	7
1	9	10	8

Table 3. Wilcoxon test Statistics.

Wilcoxon Signed Ranks Test	
Real Rank - Hybrid	
45.50 45.50	Sum of Ranks
-1.713a)	Z
0.087	Asymp. Sig. (2-tailed)

a The sum of negative ranks equals the sum of positive ranks.

P-value and statistic of wilcoxon test is shown in table 4 and its amount equal 0.087 and this prove that exist smaller variance between hybrid's rank and real's rank. Also, Table 5 shows comparing of each ranking with real's rank and presents more realistic results for realistic decision making by hybrid results.

Table 4. P_value for 4 ranking.

Comparing methods with real ranking	Asymp. Sig. (2-tailed)
CCR	0.079
SBM	0.048
HYBRID	0.087
REAL RANK	0.087

6. CONCLUSION

In this research, the Hybrid model was presented to performance measurement and its results was compared with radial and non-radial model. Radial models neglect the non-radial input/output slacks and may mislead the decision making. The non-radial approach deals with slacks directly, but it neglects the radial characteristics of inputs and/or outputs. Above mentioned shortcomings had been shown in concrete industries and presented variance in efficiency score based on radial and non-radial slacks. Also, in this article, the Hybrid model was presented as the more accurate and more comprehensive measure of evaluation. Also, wilcoxon test is done between ranking of CCR, SBM, Hybrid and real's rank. Hybrid results are presented as more realistic results for realistic decision making. Then, it is recommended to classify data to radial and non-radial then measure each section in its own method. Finally, there must be attention to this fact that current study is based on input-oriented DEA models. However, hybrid model can be used in output-oriented DEA approach. For complete ranking, it is recommended that hybrid model be composed with another model (for example AHP)

Further research opportunities include studying the relationship between our implicit value judgments in the primal DEA model and the explicit weight restrictions in the multiplier DEA model.

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