G.U. Journal of Science
19(2): 119-125 (2006)

GUJS
www.gujs.org

Performance Evaluation of Goalkeepers of the World Cup

İhsan ALP[♣]

Gazi University, Faculty of Arts and Sciences, Department of Statistics, 06500, Ankara, ihsanalp@gazi.edu.tr
Received:15.02.2005 Accepted: 24.11.2005

ABSTRACT

Football is a very popular subdivision of sports not only in our country, but also all around the world. The money spent on football is on a large scale . In spite of this, scientific studies interested in the statistical, economical dimensions and performance evaluation for football are scarce in our country and even around the world. One of the reasons for this is because there is no recording of football statistics in our country. FIFA statistics can be held as a sample in this area. FIFA, holds about fifty different football statistics on the player basis or team level during the World Cup matches. It is obvious there is no debating that the performance of the players one by one and as an entire team is very important.

The aim of this study is to pioneer for how the performance analysis is applied and how we can benefit from the results obtained in football, if the statistics existed. So, displaying the numerical size of defectiveness mathematically helps the trainers and players. On the other hand, the practical adaptation of data envelopment analysis such as the output oriented Charness, Cooper, Rhodes (CCR) and super efficiency Andersen-Petersen (AP) models are tested for a football performance evaluation.

Key Words: *Football, Goalkeeper, Data Envelopment Analysis, Performance evaluation*

1. INTRODUCTION

During the last quarter of the 20th century, football became a more favourable sport then many other branches (such as volleyball, handball, basketball, athletics and boxing) and local games (baseball, golf, cricket, rugby), all over the world from Europe to South America and from Africa to Asia. In the struggle of many branches of sports, the advantages of football are its easiness, low cost, and no need for expensive tools.

Football, the unique branch of sport in the countries is becoming an important element of the culture and community. The referee's decision, technical director, team's tactics, and player's faults are the causes of argument. Gossip spreads everywhere, such as to TV, newspapers, on to streets, to business, schools and homes. So after the success of national teams, great celebrations are arranged on the city streets. The traffic stops and the streets become full of flags, peoples and cars. The amount of joy is as great as the victory. In this view, football has an important role of including the people in the national idea. Thus, by the association of people, football binds many people to build a community.

As a raising trend, football has became a commercial source. With the supporters of teams, sponsor income and publication, the truth income comes to a budget of hundreds of millions of dollars. For example, the income of Manchester United was 200 million euro from the English Premiere League in 2000 [1], from German Bundesliga Bayern München's income was 150 million euro [3]. All the teams in a country were taken into consideration, from here it is seen that this field is an independent commercial area.

In spite of the important economic and social functions, there is not much study scientific about the statistical, economical and performance evaluation viewpoint related with football. Unfortunately, in our country, there are not enough statistics to about teams and players make a conclusion opposing of FİFA's statistics. Consequently, it is impossible to arrange or make scientific studies about football in our country.

In foreign publications and in the other branches of sport, the appearance of studies related with performance evaluation is increasing each day. Examples especially are about professional baseball, basketball and American football exist [4,5,6,7].
Performance evaluation in football might be arisen in a few critics:
i-) The efficiency of the manager and technical director (coach): This kind of evaluation takes into consideration the input and output that depends on the experience of managers and directors. For example, the studies of [8,6,10].

ii-) The efficiency of the team in a match: This evaluation deal with inputs and outputs, inputs may be the players ownership of the ball, corner kicks, kicks, penalty kicks, and outputs may be the number of goals and the result of the match. For instance, the study of [5].

iii-) The efficiency of the team or teams in the season: In this point of view the season is appreciated as a process and cumulative efficiency is calculated with the data at the end of each week. Alternative approaches might be considered depending on the aim. The related studies of [2, 5, 7].

The 2002 FİFA World Cup goalkeepers were evaluated in this study, and this evaluation is the first part of the three types explained above. Teams and goalkeepers get the chance to join the FIFA World Cup at the end of a two year long period. It is seen that the FIFA World Cup period is a short season. Due to the corner of this short period the research data on goal keepers performances were only given for the 2002 FİFA World Cup matches. This study would be the first publication about goalkeeper-player evaluation in local and global literature.

When the goalkeeper, player, manager or team performance evaluation is the object, multiple input and multiple output should be included in the calculation simultaneously. This could be made by the distribution function related with the process determined or the weights given to the inputs and outputs. Determination of the distribution function takes a lot of time and the probability of making an error is high in this approach. The weights given to input and output are found before hand in a subjective way or objectively by a model. The weights determined subjectively are open to dispute and a target for argument every time. Weights in performance evaluation may be found objectively by efficiency analysis, Data Envelopment Analysis, DEA, from operation research techniques with the help of the model [12]. This method chooses the weights by maximizing the performance of the evaluated unit or the decision maker unit DMU, in where each one of the goalkeepers is one DMU. DMUs are divided in to two groups and named as efficient ones and non efficient ones in proper quality with the use of Efficiency Analysis. The nonefficient ones are evaluated according to the efficient ones. In this study, Anderson and Peterson's (AP, 1993) super efficiency separation technique was applied, then the best ones were arranged in order additionally [17].

In the second section of the study, the method applied for the analysis, in the third section FIFA World Cup and data, in the fourth section the application is explained. In the fifth section the conclusion and interpretations are given.

2. EFFICIENCY ANALYSIS

DEA is a new methodology that calculates the performance scores of various decision maker units with the operation research technique. Performance evaluation has a multi-variable and complex structure. Other methods used for this aim include weights determined prior or different harm sides. DEA is a technique that is an entirely objective way of performance evaluation.

In the beginning, DEA was thought/designed for the calculation of the performance of decision making units with a nonprofit aim. But whether it has an object of profit or not, it has an application in various fields like in the efficiency of military operations, commercial banks, universities, hospitals, agricultural cooperatives, common foundations and its applicability exists all the

$$Enbh_o = \phi_o + \varepsilon(\sum_{i=1}^{m} s_j^- + \sum_{r=1}^{s} s_r^+)$$

Kisitlar

$$\phi_o y_o - \sum_{j=1}^{n} y_{rj}\lambda_j + s_r^+ = 0 \qquad (1)$$

$$\sum_{j}^{n} x_{ij}\lambda_j + s_i^- = x_{io}$$

$$\lambda, s_r^+, s_i^- \geq 0$$

$$i = 1,...,m \quad r = 1,...,s \quad j = 1,...,n$$

time [13, 14, 15].

Out of a certain function defined by inputs and outputs, the efficient production frontier is determined in DEA. DEA constructs reference points with the aid of the data. When DMUs constructs reference points, they take the exact (1 or 100) performance score, a little less efficient DMUs take performance score values less then 1 or 100.

There are various models in DEA [12]. The CCR model that would be used in this study, is one of those models. Let the ones that turn the inputs into outputs or that manage the converted system are DMUs number is n. The number of inputs and outputs number is changeable for each DMU but the number of inputs and output used are the same. In mathematical symbols j.th DMU s-dimensional output vector produces y_{rj} (r = 1, 2,…,s) and m-dimensional input vector x_{ij} (i = 1, 2,…,m) is used. The DMU that would be evaluated is denoted by that index (subscript) o, but the others are denoted by j index. Output directed CCR model is defined as below:

Where h_0 is the objective function that will be maximized, efficiency score ϕ_0 , a small number ε, weights λj , s^+ and s^- are slack variables. If performance score h_0 found in the result $h_0 = 1$ (or 100) and all the s^+ and s^- slack variables are zero, then DMU_0 is efficient. But if performance score $h_0 > 1$ (or 100), then DMU_0 is called a nonefficient DMU.

DMUs are separated in two groups with the classical way of DEA. DMUs could be ordered in a sequence according to the performance score of nonefficient DMUs that have scores of 1 or less than 100 (these scores are greater than 1/100 when the model is output directed) but all the efficient ones have the same performance scores so they couldn't be ordered in a sequence as well. Andersen and Petersen (1993) have developed a new model that can set up in order the efficient DMUs [4]. In this model scores are calculated from the coefficient matrix which is subtracted by the subject related to the DMU with the half of all other DMUs.

In brief, the super-efficient model named the AP model may be given in dual form as:

$$Enk\, h_k = \phi_k$$

Kisitlar

$$\sum_{j \in J} \lambda_{kj} x_{ij} \le \phi_k x_{ik} \qquad (2)$$

$$\sum_{j \in J} \lambda_{kj} y_{rj} \ge y_{rk}$$

$$\lambda_{kj} \ge 0$$

$$i = 1,..m \;\; r = 1,...s \;\; j = 1,..n \;\; j \ne k.$$

3. FIFA WORLD CUP AND DATA

The international football association written in capital letters in French by FIFA was organized by seven European countries with the leader of France in 1904. In the following years participation has occurred from African and American continents. The first world cup was made with an extensive participation. This organization for football was the beginning of a new and brilliant age. Today FIFA is the upper foundation of more than 200 sport associations. It meets its expenditures from income of the publication of its organizations excessively.

After teams have successes in various regions, they can join the FIFA world cup. This consists of the regions of Europe (UEFA), Asia (AFC), Africa (CAF), North-Middle America and Caraibean İslands (CONCCAF), Oceania (OFC) and South America (CONMEBOL). For each region, different numbers of teams join the FİFA world cup in a different manner. In other words 15 countries from UEFA, 4 countries from AFC, 5 from CAF, 3 from CONCACAF, none from OFC and 5 from CONMEBOL so a total of 32 countries could join the FİFA world cup successively in 2002. The World cup begun on the 31st of May, 2002 with the match of France-Senagal in Seul, the capital of Korea, and ended on 30th of June, 2002 with the match of Germany-Brazil in Yokohomo, Japan. The statistics of the players and teams during the competition arranged are found on FİFA's official site (http://fifaworldcup.yahoo.com/02/en/t/st/gk/mp/)
The statistics of raw data of 2002 FİFA World Cup Goalkeepers is seen in Table 1.

Table 1. The statistics of 2002 FİFA World Cup Goalkeepers

	Matches Played	Goal Againts	Goals Against per Game Ratio	Penalty Kicks saved	Free Kicks saved	Corner Kicks saved	Fast Breaks saved	Individual saves
Average	3,88	4,84	1,398	0,16	1,25	0,53	0,88	13,09
Median	3,00	5,00	1,250	0	1,00	0	0,50	11,50
Mode	3	3	1,000	0	1	0	0	8
Variance	2,11	4,01	0,586	0,20	0,90	0,39	1,60	44,67
Range	5	10	3,571	2	3	2	6	33
Minimum	2	2	0,429	0	0	0	0	1
Maximum	7	12	4,000	2	3	2	6	34

In the raw data of FİFA' s web site the statistics of 36 goalkeepers exist. The second goalkeepers of the teams, named Dabanovic M., Enyeama V., Tavarelli R., Catkic O., and Majdan R. have low data because they played very little in the matches. These goalkeepers were kept out of the analysis because of their insufficient statistics.
In the raw data set, the output variables could be the ones from the matches played of to the number of Individual saves, which are 8 variables (Table 1). For, the method had to wrk properly, some of the variables are transformed to a form in a sense, some of the other variables represented by a new variable will not be presented in the method. The input value of all goalkeepers will be 1 "one". The output variables of the analysis are:

-The ratio/number of Goals Againts per match: the Goals Against divided by number of Matches Played gives Goals Against per Game Ratio , this value affects the performance negatively. The maximum value of that ratio/number (Al Deayea M. 12/3) is 4. A new variable which is A1 Deayea M's minimum value is defined as: Goals Against per Game Ratio (GOALSAGAINTSr.)

-Number of Penalty Kicks Saved affects the performance positively. Because of the neutrality of the ratio of Penalty Kicks saved per matchs, this is divided by the number of Matches Played. (PENALTYKICKS S.r.)

-Number of other saves effects the performance positively. With the same logic formed Penalty Kicks saved ratio other variables are divided by the number of Matches Played too.

- Free Kicks saved ratio per match (FREEKICKS S.r.),
- Corner Kicks saved ratio per match (CORNERKICKS S.r.),
- Fast Breaks saved ratio per match (FASTBREAKS S.r.),
- Individual saves ratio per match (INDIVIDUAL S.r.),

the variables were defined and calculations were done with these variables.

4. APPLICATION

Output directed CCR application model solutions are obtained by the EMS (Efficiency measurement systems, version 1.3, Scheel, 2000) decision supporter system. It is found that 12 from 32 national goalkeepers are efficient and performance scores are exact (1 or in other words 100%). In Table 2, efficient goalkeepers and their exact performance scores which variables they had come from and what their weights were and also their number of references can be seen.

Kahn O. took the Goals Against per Game Ratio (GOALSAGAINTS.r) from performance exact score weighted by 99% and Fast Breaks saves ratio (FASTBREAKS S.r..) weighted by 1%. However; Reçber R. took the Goals Against per Game Ratio (GOALSAGAINTSr.) variable with the weight of 63% from Individual saves ratio (INDIVIDUAL S.r.) weighted by 37%. Barthez F. is an efficient goalkeeper and he has a Fast Breaks saves ratio (FASTBREAKS S.r.) variable weighted by 100%, and by Chilavert J.L.'s exact performance score, his personal Individual saves ratio (INDIVIDUAL S.r.) value was important with the weight of 95%. However; with Shorunma I's performance score, Free Kicks saves ratio per match (FREEKICKS S.r.) value has a role with the weight of 90%.

Table 2. Efficient Goalkeepers

Goalkeepers (DMU)	Goals Against per Game Ratio weight	Penalty Kicks saved weight	Free Kicks saved weight	Corner Kicks saved weight	Fast Breaks saved weight	Individual saves weight	Number of References
KAHN Oliver(GER)	0,99	0	0	0	0,01	0	14
RECBER Rustu(TUR)	0,63	0	0	0	0	0,37	7
FRIEDEL Brad(USA)	0	0,74	0,11	0	0	0,14	1
IKER CASILLAS(ESP)	0,82	0,13	0	0,05	0	0	0
BUFFON Gianluigi(ITA)	0,28	0,22	0	0,13	0	0,36	0
PLETIKOSA S(CRO)	0,58	0	0	0,35	0	0,08	7
CHILAVERT J L(PAR)	0	0	0	0,05	0	0,95	3
BARTHEZ Fabien(FRA)	0	0	0	0	1	0	7
BOUMNIJEL Ali(TUN)	0,45	0	0,34	0,21	0	0	2
ALIOUM Boukar(CMR)	0,79	0	0,21	0	0	0	7
SHORUNMU Ike(NGA)	0	0	0,9	0	0,1	0	8
DUDEK Jerzy(POL)	0	0	0,14	0,76	0,1	0	2

Instead of non efficient goalkeepers, the defective sides of efficient goalkeepers in DEA are directions-variables with the references (sample, peer, benchmark). The amount of the number of references that appeared are favoured for a goalkeepers' career. Kahn O. has 14 samples. This frequency is twice the amount of references for the other efficient goalkeepers. Shorunmu I. has 8 and Rençber R., Pletikosa S., Barthez F., Alioum B. with the mode value of 7 are good samples for inefficient goalkeepers. Iker C., Buffon G. were not sample units according to the measured variables for none of the inefficient goalkeepers.

Goalkeepers that don't have exact performance scores (1, 100%), in other words inefficient goalkeepers and their performance scores can be found in Table 3. The efficient goalkeepers referred to inefficient goalkeepers, these efficient goalkeepers' reference numbers and their lack of output variables which is the reference of the research can be found in Table 3. Table 3 was sorted according to the degree of efficiency of the goalkeepers.

An output directed model was used in the analysis of the performance of goalkeepers. The exact performance score (1 or 100) is assigned for efficient decision makers in the output directed models and of course the value of 1 or more than 100 for inefficient decision maker units. Performance scores higher than 100 shows that the decision maker is inefficient, and if the output value rose in that rate, it will be in an efficient position. For example, if Lee Woon J. is able to increase his outputs with the rate of 104.18%, he will be efficient. Otherwise the goalkeepers that have shortage of outputs or shortage of aggregate un efficiency should remove their efficiencies. For example, Narazaki S.'s performance score is 103.09% ; thus if 103.09% for outputs, 0,09 point for Free Kicks saved ratio (FREEKICKS S.r.), 0,022 point for Corner Kicks saved ratio per match (CORNERKICKS S.r.), 1,28 point for Individual saves ratio (INDIVIDUAL S.r.) could be increased, he would be an efficient goalkeeper.

Table 3. Inefficient Goalkeepers

Goalkeepers(DMU)	Score (%)	Arranged Degree of Efficiency (%)	References	Shortage of Goals Against per game Ratio	Shortage of Penalty Kicks saved	Shortage of Free Kicks saved	Shortage of Corner Kicks saved	Shortage of Fast Breaks saved weight	Shortage of Individual saves
MARCOS(BRA)	100,01	99.99	2(0,75) 18(0,21) 29(0,04)	0	0	0	0	0,18	0,27
SEAMAN David(ENG)	100,90	99.10	2(0,69) 4(0,26) 26(0,05)	0	0	0,01	0,04	0	0
CAVALLERO Pablo(ARG)	102,38	97.67	2(0,72) 26(0,12) 29(0,16)	0	0	0	0	0	0,6
NARAZAKI Seigo(JPN)	103,09	97.00	2(0,61) 18(0,39)	0	0	0,09	0	0,22	1,28
Lee Woon JEE(KOR)	104,18	95.98	2(0,45) 4(0,21) 18(0,18) 29(0,13) 30(0,03)	0,004	0	0	0	0	0.003
GIVEN Shay(IRL)	105,63	94.67	2(0,68) 4(0,32)	0	0	0,19	0,05	0,1	0
SORENSEN T.(DEN)	106,28	94.09	4(0,61) 18(0,14) 22(0,25)	0	0	0,07	0	0,03	0
HEDMAN Magnus(SWE)	106,38	94.00	2(0,34) 7(0,66)	0	0	0,05	0	0,05	1,59
VITOR BAIA(POR)	111,80	89.44	18(0,56) 26(0,26) 29(0,14) 30(0,04)	0	0	0	0	0	0,08
ARENDSE Andre(RSA)	112,50	88.89	18(0,37) 27(0,38) 30 (0,25)	0	0	0	0	0,37	1,25
SYLVA Tony(SEN)	112,70	88.73	2(0,58) 26(0,01) 29(0,07) 30(0,35)	0.006	0	0	0.004	0	0
PEREZ Oscar(MEX)	112,80	88.65	2(0,67) 4(0,21) 26(0,07) 30(0,05)	0	0	0	0,03	0	0
CEVALLOS Jose(ECU)	116,08	86.15	2(0,46) 29(0,21) 30(0,33)	0	0	0	0	0,4	0
AL DEAYEA M.(KSA)	128,57	77.78	22(0,86) 32(0,14)	2,14	0	0,21	0	1	0
SIMEUNOVIC M.(SVN)	129,96	76.95	4(0,33) 22(0,67)	0	0	0,54	0,27	0,67	0
NIGMATULLIN R.(RUS)	133,93	74.66	2(1,00)	0	0	0,14	0	0,14	0,91
CARINI Fabian(URU)	137,48	72.73	2(0,44) 4(0,36) 30(0,19)	0	0	0	0,05	0,26	0
DE VLIEGER Geert(BEL)	150,40	66.49	2(0,67) 26(0,33)	0	0	0,31	0	0	0,04
LONNIS Erich(CRC)	162,61	61.50	2(0,44) 26(0,24) 29(0,32)	0	0	0	0	0	0,81

Output increase rates were arranged properly in the efficiency degree column, and so they could be explained clearly. In this manner, the arranged performance score of Narazaki S is 97.00% (Table 3).

Performance scores of Marcos and Seaman D. are nearly 100%, Marcos could be especially regarded as an efficient goalkeeper (deficiency 0.01%). The performance score of Jiang J. is the lowest and 51.85% of the exact efficient goalkeepers' quantity.

The projection of inefficient decision maker units to the efficiency frontier, in other words to be efficient could, be constructed by taking the proper rate of inputs and outputs of the efficient decision makers as reference. These calculations could also be done in the DEA. For example, if the outputs F Given S. were similar to Kahn O. with a rate of 68% and similar to Reçber R. with a rate of 32%, he could be an efficient goalkeeper (Table 3). There is only one goalkeeper that would be a reference for Nigmatullin R, he is Kahn O.

DMUs (decision maker unit) are divided by DEA in two groups as the efficient (performance scores 100) and the inefficient, their performance scores are less than 100 or 1. Efficient DMU could be arranged in order with the help of a new concept of super efficiency. The results of explanation could be seen in Table 4.

Table 4. Order of Efficient Goalkeepers

Goalkeepers(DMU)	Super Efficiency Score	Arranged Super Efficiency Score
BOUMNIJEL Ali (TUN)	99.36	100.00
BUFFON Gianluigi (ITA)	98.86	99.46
IKER Casıllas (ESP)	97.66	98.28
ALIOUM Boukar (CMR)	97.22	97.48
KAHN Oliver (GER)	95.45	96.06
RECBER Rustu (TUR)	95.34	95.95
PLETIKOSA Stipe (CRO)	79.62	80.13
CHILAVERT Jose L. (PAR)	75.00	75.48
FRIEDEL Brad (USA)	60.71	61.10
SHORUNMU Ike (NGA)	60.00	60.38
DUDEK Jerzy (POL)	53.97	54.31
BARTHEZ Fabien (FRA)	50.00	50.32

At the end of the evaluation of efficient goalkeepers, Boumnijel A. is determined as a super goalkeeper with the study of the super efficiency approach. In the arrangement of Table 4, the performance values of the first 6 goalkeepers are more than 95% with little differences between each of them. According to each of them, their values could be regarded as they are at the same level. In the evaluation of Table 4, the last 6 goalkeepers have different performance values. Barthez has a performance value with the rate of 50.32%, half that of the super efficient goalkeeper's performance.

5. CONCLUSION AND SUGGESTIONS

2002 FİFA world cup was a very important organization for our country, Turkey. The Turkish National Team became third in the world. Rüştü Reçber, the goalkeeper in the national team was successful. This success has been seen clearly in the mathematical analysis.

During the competition of the FİFA World Cup, more than 50 different statistics about people and teams have been registered and published. Unfortunately, in our country, statistics about football are not sufficiently registered for scientific researches as far as the interest of the technical crew and media for sport commentators. The football federation/association and teams should construct the background for statistical researches by registering the statistics about players and teams as soon as possible.

Most certainly, it is not enough to register the statistics about football alone. It will be significant that the background studies should be built for the explanation of statistics taken and development of forecasts about football, as well.

In this study, the output directed CCR which is the one of the efficiency analysis models, is used to calculate the performance scores of the players and also to determine the lack of aspects and the amount of lack of the inefficient players. This way, inefficient players make up for their shortages regarding the deficient variables, and therefore become efficient players. Along with their general observations and experience, technical managers can take into account the developing efficiency analysis, when giving advice to inefficient players. As the players work on their insufficiencies, the success level of the team will be positively affected at the end of the process.

Observing the gaming styles of teams from video record and determining the excessiveness and defectiveness of the teams and players, scientifically found in numerical values, will support the opinions.

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