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Decomposition of Motor Unit Firing Pattern Using Kalman Filtering

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ABSTRACT

Interdischarge interval (IDI) is one of the basic parameters to study in motor unit firing analysis. Discharge intervals of a single motor unit vary over time and they are unpredictable. IDI sequences can be considered to comprise two components, namely a long term signal, an IDI trend, and a white noise process, instantaneous firing variability (IFV). In this paper a stochastic model of the IDI signal has been developed in order to estimate the elements of an IDI sequence. IDI sequences of several patients have been recorded at a clinic and a Kalman filter has been constructed based on the developed stochastic model. The Kalman filter is utilized to decompose the recorded IDI sequences into the IDI trend and IFV components. The obtained decomposed signals, especially the IDI trend component, may provide valuable information on motor unit firing performance and help diagnose neurological diseases.

Key Words: *Motor unit firing, IDI sequences, state-space modeling, Kalman filter*.

1. INTRODUCTION

Motor units are structural components of a muscle that are composed of a single motor neuron, its nerve fiber (i.e., axon) and all the muscle fibers to which the axon connects. Motor units are considered as the smallest functional units of the motor system. Muscular force is generated based on 2 mechanisms; *i)* order of recruitment and *ii)* modulation of motor unit firing frequency. Motor units are recruited according to the size principle. As shown in Figure 1, smaller motor units, i.e., fewer muscle fibers, (indicated by black color) have a small motor neuron and a low threshold for activation and these units are recruited first. As more force is demanded by an activity, progressively larger motor units (indicated by gray color) are recruited. Moreover, slow units operate at a lower frequency range than faster units. Within that range, the force generated by a motor unit increases with the increasing firing frequency. If an action potential reaches a muscle fiber before it has completely relaxed from a previous impulse, then force summation will occur. By this method, firing frequency affects muscular force generated by each motor unit. It is possible to extract

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information about motor unit discharge pattern by measuring the firing frequency of a single motor unit. Motor unit discharge depends on firing characteristic of lower motor neurons controlled by higher motor centers. Changes in motor unit discharge behavior have been documented for both upper [1, 2] and lower motor neuron lesions [3, 4]. Motor unit firing rate may be quantified by analyzing the time interval between consecutive discharges of the same potential, which is called as the *interdischarge interval* (IDI). Statistical properties of IDI, including mean, standard deviation, skewness and distribution of the density function have been extensively studied [5, 6, 7]. Joint interval histogram, another statistical analysis technique, has also been employed to study the temporal relationship between the consecutive discharges of a single motor neuron[8,9].

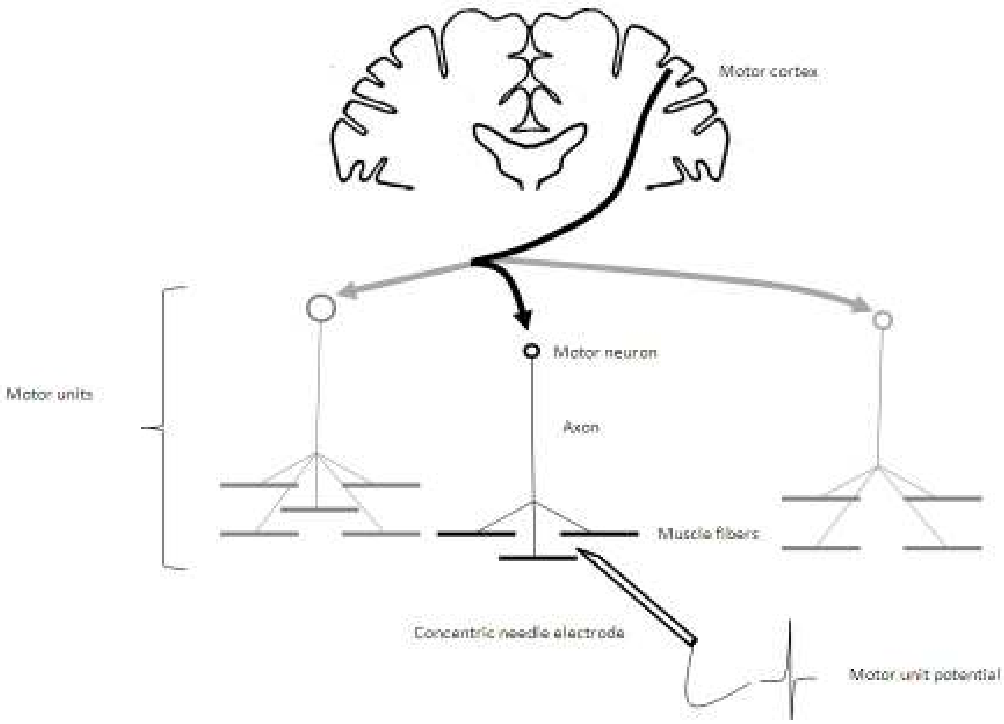

Figure 1. Illustration of motor unit; motor neuron, axon and muscle fibers.

Discrete-time linear state space models have been employed since 1960's, mostly in the control and signal processing areas, and Kalman filtering [10] has emerged as the most commonly used tool. The Kalman filter has been extensively employed in many areas of estimation the extensions and applications of discrete-time linear state space models can be found in almost all disciplines [10, 11, 12]. The Kalman filter has also been utilized in electrophysiological signal analysis and compared favorably with other approaches [13, 14].

This paper presents the use of Kalman filtering in the analysis of the IDI sequence of a motor unit. The approach takes a recorded IDI sequence and decomposes this sequence into a long term signal, an IDI trend, and a white noise process, instantaneous firing variability. This decomposition is expected to provide insight into motor unit firing analysis and help diagnose neurological diseases. Although there have been other approaches to analyze IDI sequences such as random walk and Poisson process models [15] and the 1/f model [16], the Kalman filter has never been considered for such an application. Kalman filter solves the problem of estimating the instantaneous states of a linear dynamic system perturbed by Gaussian white noise, using measurements that are linear functions of the system state but corrupted by additive white noise. As described in details in the following section, discharge intervals of a single motor unit constitutes a stochastic process which is best solved with the Kalman filter. Thus, in comparison to other approaches utilized for this specific application, the Kalman filter will provide a simple but more efficient solution to the problem in hand.

The paper is organized as follows; the following section outlines the approach and the use of Kalman filtering in IDI sequence analysis where section 3 describes the experimental set up and how IDI sequences were obtained. Results are given in the following section and the paper concludes with some closing remarks.

2. THE APPROACH

When discharge intervals of a single motor unit are examined it will be revealed that these intervals vary over time and they are unpredictable (i.e., random), which could be classified as a stochastic process. Several stochastic process models such as the random walk model and the Poisson process model have been proposed to characterize discharge patterns of single motor units [15]. Statistical models have also been employed for motor unit firing analysis, for instance, in [16] the 1/f process model was developed to study temporal discharge patterns of single motor units where fractional parameter of the model was shown to be an indicator for distinguishing between discharge behaviors of single motor units of normal subjects and patients with upper motor neuron lesions. The study also considered the IDI sequence as a summation of a long-term signal, the trend, and a white noise sequence, the firing variability. The trend was assumed to fluctuate slowly around a mean that belongs to the IDI sequence where the instantaneous firing variability (IFV) reflected the correlation between adjacent IDIs. In [17] this fact was mathematically represented and IDI sequence was defined as

$$I_n = T_n + V_n \tag{1}$$

where I_n is the nth IDI data set, T_n is the nth data set of the IDI trend, and V_n is the nth data set of the IFV belonging to the nth IDI data set respectively. The use of Eq. (1) makes it possible to simulate the IDI trend by passing a white noise sequence through a moving average (MA) process and V_n as a stationary first-order autoregressive (AR) process. This fact was exploited in [17], where singular value decomposition method was employed to quantify joint interval histogram plots of simulated IDI sequences. Although this study provided a simulated environment for investigating possible influencing factors on IDI sequences, however, it fell short for providing any information about the real IDI data sequence decomposition.

In this study, IDI sequences recorded in a controlled laboratory have been examined and the long term trend and the instantaneous firing variability components are estimated from the observed IDI sequence. It has to be noted that through experiments only IDI sequences could be observed, however, all the useful information is buried in the trend and IFV components and they have to be extracted for further processing and analysis. Thus, it is important to obtain these components for investigation. In this study, Kalman filtering[1] has been used to estimate the trend and IFV components from the observed IDI sequences. Kalman filter is a recursive estimator for what is called the linear quadratic Gaussian problem, which is the problem of estimating the instantaneous state of a linear dynamic system perturbed by Gaussian white noise, by using measurements related to the state but corrupted by Gaussian white noise. Thus, as long as a correct model that belongs to the system of interest is constructed, Kalman filtering will be the answer to the linear estimation problem in hand. The assumption that an IDI sequence can be considered to comprise two parts, namely the trend and IFV, which provides the means for the system to be modeled appropriately using the Kalman filter.

In [18] a simple linear model was proposed to describe the time course of stochastic time-series. This so-called "dynamic linear model" (DLM) is defined in terms of state space representation through

$$y(k) = \mu(k) + e_1(k) \tag{2}$$

$$\mu(k) = \mu(k-1) + \beta(k) + e_2(k) \tag{3}$$

$$\beta(k) = \beta(k-1) + e_3(k) \tag{4}$$

where, Eq. (2) is the output where it describes the measurement at time k, the added term $e_1(k)$ is the Gaussian measurement noise that corrupts the observed value of the state $\mu(k)$. The state $\mu(k)$ is assumed to evolve in time as described by Eq. (3) and its value at time k depends on its value observed at $(k\text{-}1)$ plus the value of the second state at time k. The noise term $e_2(k)$ is the process noise component corresponding to that state, used to model unexpected changes in the state. Eq. (4) describes the evolution of the second state $\beta(k)$ which depends on its previous value, $e_3(k)$ is the process noise component corresponding to the second state that is used to model the unpredicted changes in this state. Relating the IDI model to Eqs. (2) - (4), $\mu(k)$ represents the trend that fluctuates around a mean and is corrupted by noise where $\beta(k)$ is the systematic variation that is added to the trend, i.e., the IFV. Thus, through this representation both the IDI trend and IFV can be estimated from an observed (recorded) real IDI sequence with an appropriate tool, Kalman filter. But first, it would be useful to put these equations in vector-matrix form in order to obtain the state-space model for an IDI sequence. Re-arranging Eqs. (2) - (4) would yield

$$\begin{bmatrix} \mu(k) \\ \beta(k) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mu(k-1) \\ \beta(k-1) \end{bmatrix} + \begin{bmatrix} e_2(k) + e_3(k) \\ e_3(k) \end{bmatrix} \tag{5}$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \mu(k) \\ \beta(k) \end{bmatrix} + e_1(k) \tag{6}$$

where Eq. (5) is the state equation that defines the evolution of system states and Eq. (6) is the output equation that relates system states to the observations. In above equations, process noise (e_2 and e_3) and measurement noise (e_1) sequences are assumed to be Gaussian and independent of each other. If we introduce general Kalman filter representation[2] at this point, it will be easier to see the suitability of the Kalman filter approach to this specific problem. Lets consider a general discrete-time stochastic system represented by the state and measurement models given by

$$x_{k+1} = \Phi_k x_k + B_k u_k + w_k \tag{7}$$

$$y_k = H_k x_k + v_k \tag{8}$$

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[1] Kalman filter is in fact an estimator rather than a conventional filter, however it is employed to estimate parameters from a noisy data sequence, hence the name filter.

[2] Please see the Appendix for the derivation of Kalman filter equations.

where, x_k is an $n\times1$ system vector, y_k is an $m\times1$ observation vector, Φ_k is an $n\times n$ system matrix, u_k is a $p\times1$ vector of the input forcing function, B_k is an $n\times p$ matrix, H_k is an $m\times n$ matrix, w_k an $n\times1$ vector of zero mean white noise sequence and v_k is an $m\times1$ measurement error vector assumed to be a zero mean white sequence uncorrelated with the w_k sequence. The matrices $\Phi_k, B_k, H_k, Q_k, R_k$ are assumed known at time k. The covariance matrices w_k and v_k are defined by

$$E(w_k w_k') = Q_k \delta_{kl}$$

$$E(v_k v_k) = R_k \delta_{kl}$$

$$E(w_k v_k) = 0$$

where δ_{kl} is the Kronecker delta function. Please note the similarity between the Eqs. 5-7 and 8, thus, applying the state-space representation given by the Eqs. 5 and 6 to the Kalman filtering would result in a decomposed IDI sequence and estimated values of IDI trend and IFV components. This is a rather simple but very effective approach and to the authors' best knowledge it has not been employed for this purpose before.

3. EXPERIMANTAL SETUP

This section outlines the experimental setup and the recording of IDI sequences used in this study. Recordings were taken from 5 normal (healthy) subjects. Motor unit discharge patterns were recorded from the biceps brachii muscle. Concentric needle electrode was utilized to record motor unit potentials where EMG recording was performed by commercially available EMG equipment. Motor unit potentials were recorded at two different contraction levels, namely, 10% and 20% of the maximum voluntary contraction. A strain gauge was employed to monitor the force contraction. The same procedure has been utilized to take motor unit discharge recordings from 5 normal patients. Single motor units in these recordings were identified in the usual way, that is, by observing the occurrence of a motor unit potential which has a uniform pattern. Figure. 2a displays a typical motor unit firing trace that was recorded for a constant level of contraction. The IDI sequence related to this firing trace was extracted by measuring the time intervals between the consecutive discharges of the same motor unit and is shown in Figure (2b). Measurement of time intervals between successive discharges of the same motor unit was performed by using MATLAB. Recording segments that were contaminated with discharges of other motor units were rejected.

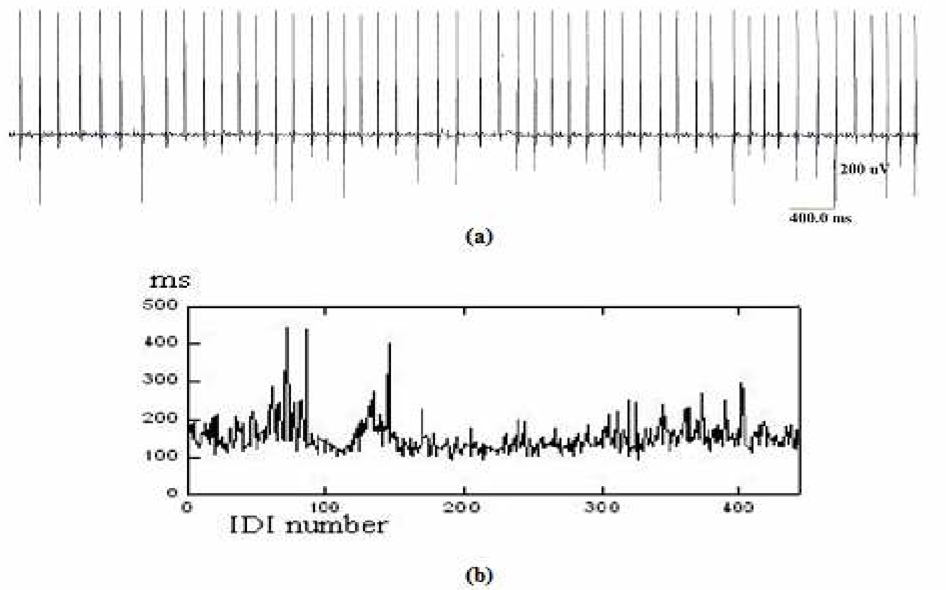

Figure 2. (a) Discharges of motor unit action potentials created by single motor unit, (b) IDI sequence related to this firing trace.

4. RESULTS

Data processing of the IDI sequence, extracted from the recorded firing trace, was performed off-line. The IDI sequence has been put through the Kalman filter that has been designed to model the system described by Eqs. (5) and (6). Initial values of the IDI and IFV have been arbitrarily taken as $x(0) = [1 \;\; 1]^T$. Selection of the initial values is not critical as the properly constructed model will yield these initial values to converge to the measurements. Standard deviation of the process noise has been chosen as 1 while the measurement noise standard deviation has been set at 35. Here the measurement noise standard deviation is determined from the experimentally recorded data and it is fixed. On the other hand the process noise variance is a modeling parameter and it can be thought as a parameter that determines the level of variation of the states, in this case IDI trend and the IFV. However, although the band that the states fluctuate change by changing the process noise standard deviation, their mean and standard deviation remain almost the same. Thus, since the main goal in this study is decomposing the IDI sequence into its components, namely IDI trend and the IFV the selection of process noise standard deviation is not critical.

All 5 recordings of IDI sequences have been decomposed and analyzed with the above given parameters and utilizing the proposed method in the same way. However, in this paper, results of only one recording have been presented as the analysis of each recording has produced similar outcomes. The investigated IDI sequence in which the contraction level has been altered while recording is shown in Figure 3 at the top. In the figure the arrow indicates the point where the force contraction was changed. The vertical axis in Figure 3 is the time lapse between consecutive motor unit firings, inverse of which is the motor unit firing frequency. As it can be seen, the increase in the contraction level, from 10% to 20%, as expected, has caused the time between firings to decrease, in other words an increase in the motor unit firing frequency. The reason for taking motor unit potential recordings at two different levels of contraction is to investigate whether the proposed approach can successfully decompose the IDI sequence and estimate the IDI trend and IFV and detect the change in the muscular force. Note that, since the IFV is characteristically white noise, no change in the IFV component is expected with the changing contraction level.

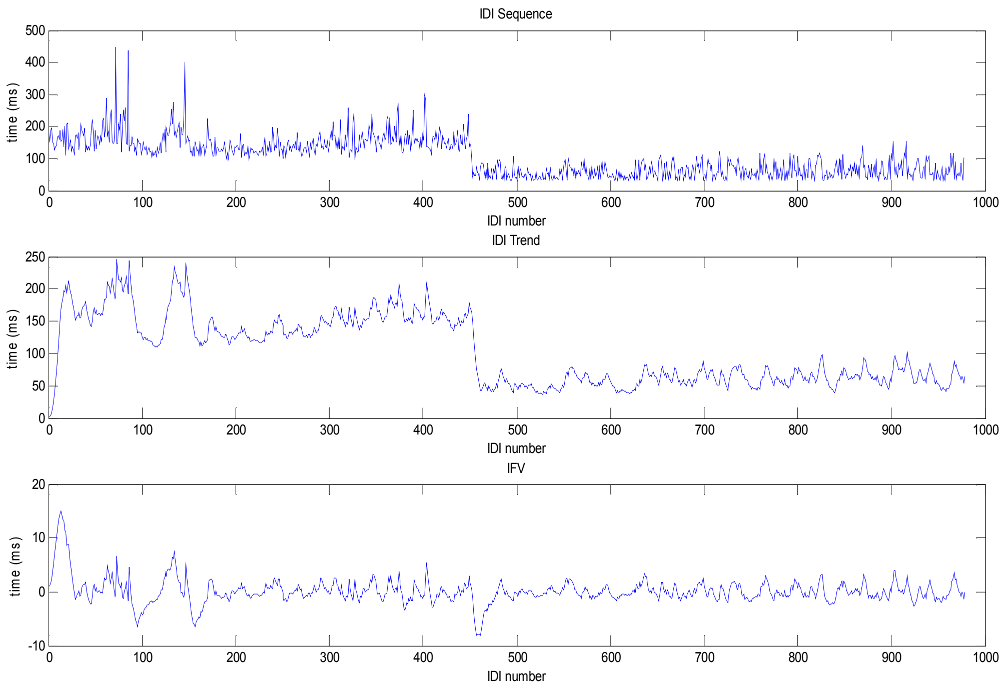

Figure 3. Decomposition of the IDI sequence.

On the other hand the IDI trend is directly related to the force contraction and any change in the contraction level should result in a change of firing frequency which is expected to be reflected as a change in the mean of the IDI trend. The middle and bottom graphics in Figure 3 display the estimated IDI trend and IFV obtained at the output of the Kalman filter. As it can be clearly seen from the middle graphic the estimated trend, as expected, follows the change in the contraction level. Moreover, still conforming to expectations, the estimated IFV is oblivious to the force contraction change and exhibits a relatively stable level. These results have revealed that with the given system model and the assumptions, the Kalman filter could successfully be used to decompose a real IDI sequence into the IDI trend and IFV components and estimate these components from the observed firing patterns.

Also, statistical analysis of the estimated components might produce useful information regarding possible diseases. The mean, taken for every 20 segments, of the estimated IDI trend and IFV signals are shown in Figures 4-5.

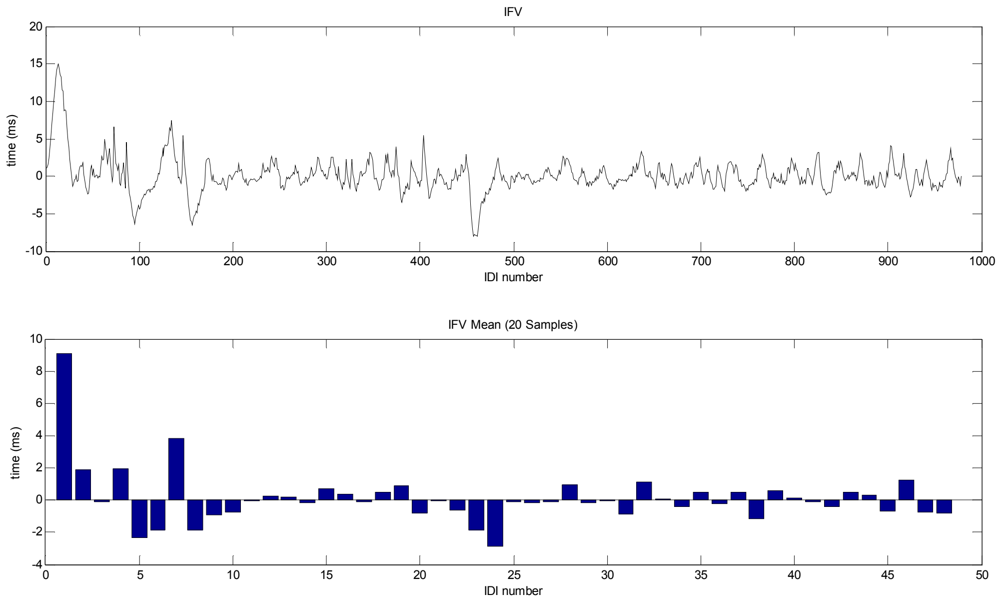

Figure 4. Estimated IFV and its mean.

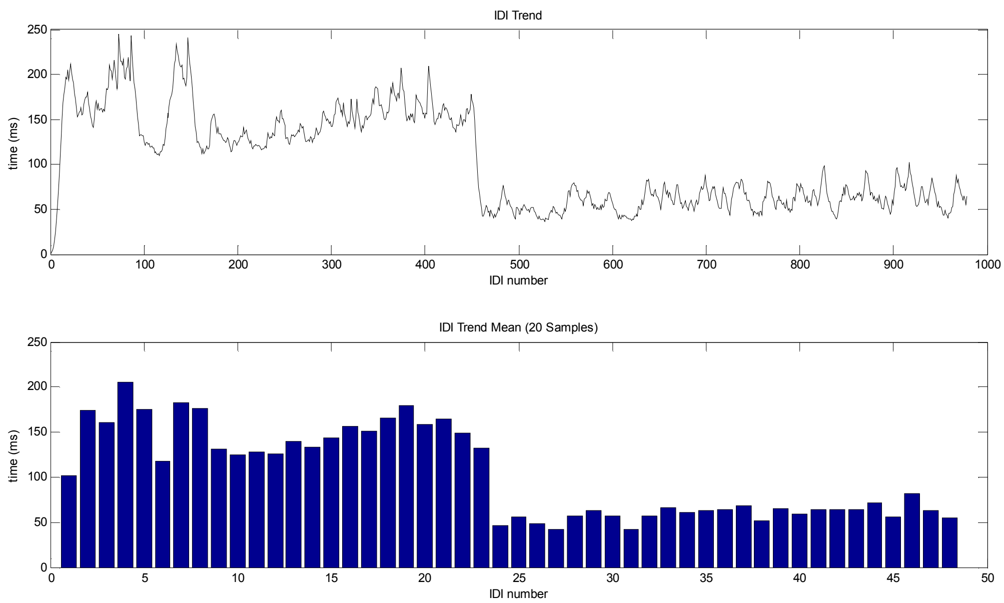

Figure 5. Estimated IDI trend and its mean.

Detailed analysis of IDI trend and IFV statistics may lead to useful information regarding physiological characteristics of motor unit firing. Moreover, the effects of pathological conditions may be analyzed quantitatively in this manner which may provide additional information regarding human motor unit performance both in healthy persons and people with diseases. For example, pathological processes such as myopathy or neurogenic conditions that effect the force generation can be evaluated. However, application of the clinical data is needed to evaluate the performance of this technique for detecting motor unit firing abnormality. For such a study a data base of IDI sequences collected from patients with different diseases and normal subjects is being established. All the IDI sequences from this data base will be decomposed with the proposed approach and classical statistical evaluation methods as well as singular value decomposition will be applied to the decomposed components for observing possible outcomes.

5. CONCLUSIONS

In this paper, a novel method to decompose a real IDI signal has been proposed for motor unit analysis. The evolution of the IDI signal has been modeled in state space and the Kalman filter has been employed to decompose the recorded IDI signal into the IDI trend and IFV parts. Previous studies have either attempted simulate these parts to work on [17] or studied the motor unit firing as a whole without decomposing [16]. Decomposition of motor unit firing using the proposed technique may provide additional information about human motor unit performance both in normal persons as well as patients.

APPENDIX:

A general state space model takes the following form:

$$x_{k+1} = \Phi_k x_k + G_k w_k$$

$$y_k = H_k x_k + v_k$$

where, x_k is an nx1 state vector, y_k is an mx1 observation vector, Φ_k is a nxn state transition matrix, H_k is the mxn observation matrix. w_k is an nx1 vector of zero mean white noise sequence and v_k is an mx1 measurement error vector assumed to be a zero mean white sequence uncorrelated with the w_k sequence., The matrices Φ_k, G_k, H_k, Q_k, R_k are assumed known at time k. The covariance matrices for the w_k and v_k vectors are defined by

$$E\left[v_k v_j'\right] = R_k \delta_{kj},$$

$$.E\left[w_k w_j'\right] = Q_k \delta_{kj},$$

$$E\left[v_k w_j'\right] = 0$$

In the above equations δ_{kj} is the Kronecker delta function.

As introduced in [10], the filtering problem in question is to estimate the state vector x_k, given the observation vector $Y_k = \{y_0, y_1, ..., y_k\}$, which can be denoted as:

$$\hat{x}_{k|k} = E\left[x_k \middle| y_0, y_1, ..., y_k\right] = E\left[x_k \middle| Y_k\right]$$

with the covariance matrix:

$$P_{k|k} = E\left[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})' \middle| Y_k\right]$$

Let the observation matrix take the form: $Y_{k-1} = \{y_0, y_1, ..., y_{k-1}\}$, then estimating the state vector x_k will be as

$$\hat{x}_{k|k-1} = E\left[x_k \middle| y_0, y_1, ..., y_{k-1}\right] = E\left[x_k \middle| Y_{k-1}\right]$$

with the covariance matrix

$$P_{k|k-1} = E\left[(x_k - \hat{x}_{k|k-1})(x_k - \hat{x}_{k|k-1})' \middle| Y_{k-1}\right]$$

In this case, the Kalman Filter, depending on the initial values, $P_{0|-1} = P_0$, $\hat{x}_{0|-1} = \bar{x}_0$ is characterized by the following expressions:

$$\hat{x}_{k|k-1} = \Phi_{k-1}\hat{x}_{k-1|k-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left[y_k - H_k\hat{x}_{k|k-1}\right]$$

$$K_k = P_{k|k-1}H_k'\left[H_k P_{k|k-1}H_k' + R_k\right]^{-1}$$

$$P_{k|k} = \left[I - K_k H_k\right]P_{k|k-1}$$

$$P_{k|k-1} = \Phi_{k-1}P_{k-1|k-1}\Phi_{k-1}' + G_{k-1}Q_{k-1}G_{k-1}'$$

As described in [19], K_k is also known as the "Kalman Gain".

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