

# Adaptive Frequency Estimation of Distorted Power System Signals Using Modified Extended Kalman Filter

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## ABSTRACT

This research presents a method for frequency estimation of distorted power system signals using a modified Extended Kalman Filter (EKF). Distorted signals which have been obtained from measurements may involve noises which may affect the accuracy of frequency measurement in a power system. Thus, the proposed method is employed to eliminate and filter noise and hence to improve the efficiency of frequency estimation. Computer simulations have been carried out for the performance analysis of the proposed method and the comparison of the results of the proposed method with the former Extended Kalman Filter results are presented. The fast tracking performance of the new approach in power system frequency estimation was emphasized.

**Key Words:** *Modified Extended Kalman filter, Frequency estimation, Power system*

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## 1. INTRODUCTION

Fast and accurate estimation of frequency and its rate of change is vital for control and protection of power system since frequency is used to indicate the systems operation state, hence frequency estimation in a power system must be reliable. Most of the estimation problems originate from distortion including random noise and higher order harmonics. The accuracy, speed of convergence and noise immunity are among the most important parameters that frequency estimator of a power system should ensure. Several algorithms have been proposed for estimating the power system frequency. Zero-crossing [1], Discrete Fourier transform (DFT) [2], Kalman filtering [3] and adaptive filtering [4, 5] are among the most popular algorithms. In the presence of noise and harmonics, these well known methods cannot accurately

and quickly track the frequency due to the unnatural values of measurements. The Extended Kalman Filter (EKF) has been proposed to accelerate convergence and obtain better accuracy [3]. In this paper, in order to decrease convergence time and increase precision of estimator a modified EKF is proposed.

The modification of EKF is based on the noise variance technique [6]. The modified EKF adjusts the noise variances of state variables according to the estimation performance. When the estimator has a satisfactory result, the noise variance is set to zero. On the contrary, it is set to a larger value if the estimator result is within a threshold. This modification is capable of adjusting the tracking system according to estimation performance. The proposed modified EKF algorithm has better frequency and amplitude estimation performance than

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conventional EKF. Also modified EKF can easily be adapted to real time applications. The existing frequency detection algorithms such as conventional EKF, zero crossing and FFT were analyzed in previous studies. Moreover, the proposed method improves the accuracy and reliability of frequency estimation of distorted power system signals.

## 2. SIGNAL MODEL

The power system signal can be described with non-linear state space model. Several non-linear models have been proposed to estimate the frequency of a single sinusoid. Once the signal model is established the modified EKF can be adapted to this model. An observation signal  $z_k$  at time  $t_k$  can be denoted as a sum of  $x_k$  of a sinusoids with additive noise  $v_k$

$$z_k = x_k + v_k \quad k=1,2,3,\dots,N \quad (1)$$

where,

$$x_k = a \cos(\omega t_k + \Phi) \quad (2)$$

$$\omega = 2\pi f \quad (3)$$

in which  $a$ ,  $f$  and  $\Phi$  are amplitude, frequency and the phase of the sinusoid (power system signal)

respectively. The observation noise  $v_k$  is a Gaussian white noise with zero-mean and variance  $\sigma_v^2$ .

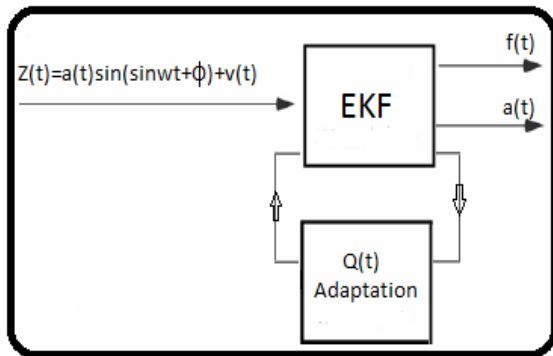


Figure 1. Block diagram of the proposed method.

The block diagram of the proposed method is given in Figure 1. In power systems, fundamental frequency is expected to be 50 Hz. However, the fundamental frequency may deviate from the nominal value due to harmonics. In the presence of harmonics the equation will be modified as

$$S_k = \sum_{m=1}^M a_m \cos(m\omega t_k + \Phi_m) + v_k \quad (4)$$

where,  $m$  is the harmonic order and  $M$  is the highest harmonic component of the signal [7].

## 3. EXTENDED KALMAN FILTER (EKF)

Considering the non-linear systems, EKF is a non-linear extension of the conventional Kalman filter. In the EKF, the state transition and observation models may be differentiable functions. One disadvantage of this version is that it needs more time consuming calculations than ordinary Kalman filter. EKF is generally used for estimating the unknown parameters of a system with corrupted measurements and known stochastic model. Moreover, it can also define the state vector and stochastic model of future measurements of the system [5]. In the dynamic system model of EKF the Eq. (5) represents state model and the Eq. (6) represents observation (measurement) model.

$$x_k = Ax_{k-1} + Bu_k + w_{k-1} \quad (5)$$

$$z_k = Hx_k + v_k \quad (6)$$

In the model  $x_k$  is the state vector at  $k$ th update which represents the system signal to be estimated in EKF. We assume that the control signal  $u_k$  is zero in our system.

The  $w_{k-1}$  and  $v_k$  are the process and measurement noises respectively. Noise vectors are Gaussian and statistically independent. By substituting process and measurement noises in Eq. (7) and Eq. (8) the process noise covariance  $Q$  and measurement noise covariance  $R$  can be attained. Also  $A$  is the state transition matrix,  $H$  is the observation matrix and  $B$  is the control matrix in our model.

$$Q_k = E\{w_k w_k^T\} \quad (7)$$

$$R_k = E\{v_k v_k^T\} \quad (8)$$

In order to implement the EKF, prediction and correction equations are summarized as follows:

$$\hat{x}_k^- = K_k Z_k + (1 - K_k) \hat{x}_{k-1} \quad (9)$$

The relationship between estimated signal at the  $k$ th step of update and the estimated signal at the previous step is given in Eq. (9). Here  $K_k$  is the Kalman gain and the  $Z_k$  is the measurement data. The main purpose of the Eq. (9) is to calculate the estimation value of each time step by using time updated Kalman gain. For this purpose EKF uses two different groups of equations. First group is called prediction (time update) equations and given in Eqs. (10-11):

$$\hat{x}_k^- = A \hat{x}_{k-1} \quad (10)$$

$$P_k^- = A P_{k-1} A^T + Q \quad (11)$$

The second group is called correction (observation update) equations and given in equations (12) through (14).

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \quad (12)$$

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H\hat{x}_k^-) \quad (13)$$

$$P_k = (1 - K_k H) P_k^- \quad (14)$$

Here,  $P_k$  is the error covariance matrix. The estimation values, which are calculated by using prediction equations are the priori estimations ( $\hat{x}_k^-$  and  $P_k^-$ ) before correction. The updated posterior estimations ( $\hat{x}_k$  and  $P_k$ ) are obtained by substituting priori estimations in correction equations. The next step is to recalculate priori estimations by using updated posterior estimations that are yielded in the previous step. This update process continues iteratively.

### 3.1. Modification Algorithm of EKF

The process noise covariance  $Q$  can be represented in the following form,

$$Q_k = \begin{bmatrix} \sigma_{w1}^2 & 0 \\ 0 & \sigma_{w2}^2 \end{bmatrix} \quad (15)$$

where  $\sigma_{w1}^2$  and  $\sigma_{w2}^2$  are both zero mean Gaussian random variables. We chose the state variable as;

$$\hat{x}_k = [a_k, f_k]^T \quad (16)$$

where  $a$  and  $f$  are magnitude and frequency of the system signal. The lock detection algorithm [6] adjusts the covariance matrix  $Q_k$  given by Eq. (14) adaptively according to the tracking performance of the EKF. If the estimation performance is satisfactory,  $\sigma_{w1}^2$  and  $\sigma_{w2}^2$  are set to zero. However, if the performance is not appropriate for the desired estimation, their values are defined by the modification algorithm. The tracking performance of the EKF can be interpreted by the error function at each step  $k$ . The error function is given in Eq. (17).

$$e_k = (z_k - H\hat{x}_k^-) \quad (17)$$

The tracking performance strictly depends on the error function. In our assumption average of recent errors is used for comparison. If this average is smaller than  $\gamma$  times the square root of measurement noise covariance  $R_k$  (standard deviation), the performance is said to be satisfactory. ‘ $\gamma$ ’ is a constant with a pre-defined threshold value range ( $1.0 \leq \gamma \leq 3.0$ ). Otherwise, modification system adjusts the covariance matrix.

$$\sigma_{w1}^2 = \sigma_{w2}^2 = \frac{f_k^2}{12} \quad (18)$$

Kalman gain increases with the increasing  $\sigma_{w1}^2$  and  $\sigma_{w2}^2$  values. This lock detection algorithm modifies the EKF by locking and unlocking Kalman gain. Lock detection algorithm is summarized mathematically given as below:

$$\begin{aligned} \sigma_{w1}^2 = \sigma_{w2}^2 = 0 & \quad \text{as } \frac{1}{M} \sum_{m=1}^M e_m \geq \gamma \cdot \sqrt{R_k} \\ \sigma_{w1}^2 = \sigma_{w2}^2 = f_k^2 / 12 & \quad \text{as } \frac{1}{M} \sum_{m=1}^M e_m < \gamma \cdot \sqrt{R_k} \end{aligned} \quad (19)$$

The lock detection algorithm improves performance of EKF by reducing the convergence time and reducing estimation error.

### 4. COMPUTER SIMULATIONS

Computer simulations were carried out for power system signal with both noise and harmonic distortion. Matlab programming language and ordinary personal computer with 2.4 GHz double core processor are used for computer simulations. Computer simulations were carried out also for 50.1 Hz or 49.9 Hz. But the results of frequency and amplitude estimations of distorted signals were almost same with the 50 Hz results. In real world applications it is very rare to face much frequency-deviated signals. So simulations were carried out for 50 Hz in our model. The modified EKF is applied on the distorted voltage waveform as shown in Figure 1. For a 50 Hz power system, the recommended window length is 10 cycles which corresponds to 200 ms time [9, 10]. In the proposed method sampling rate  $f_s$  is 1 kHz and the window length is 200 ms. In other words, each window contains signal of 200 samples length. There are 0.9 ms (0.045 cycle) time delays between data windows.

#### 4.1. Noise Distortion

In the first part of the study we have used a pure sinusoidal signal with unity amplitude and 50 Hz is corrupted with a zero-mean white Gaussian noise with variance  $\sigma^2$ .

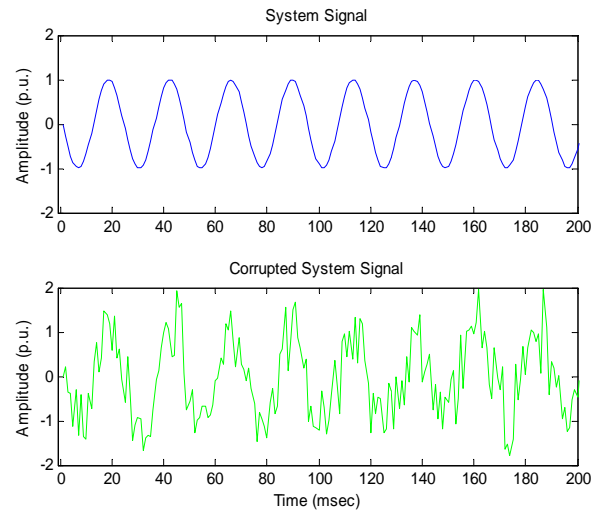


Figure 2. Power system signal and the corrupted system signal.

Focusing on the worst case scenario signal to noise ratio (SNR) is set to 10 dB. Comparison graphics of the results of the proposed method with conventional EKF results has been given. The power system signal and the distorted system signal are given in Figure 2.

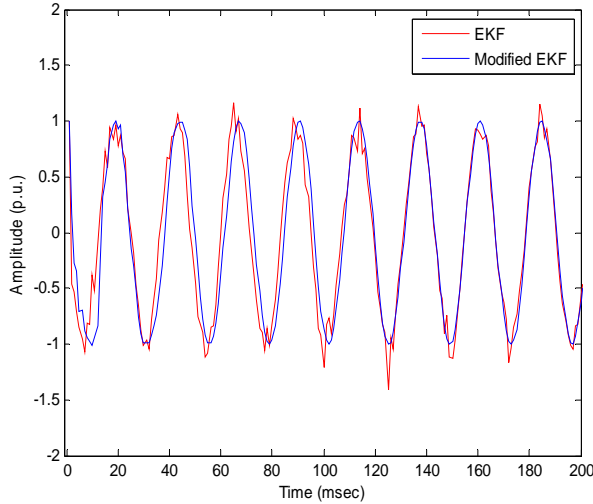


Figure 3. Power system signal estimation with EKF and Modified EKF.

The system signal waveform tracking with EKF and modified EKF algorithm is given in Figure 3. As shown in Figure 3 Modified EKF algorithm estimates the system signal better than conventional EKF. The EKF algorithm estimates the power system signal frequency by using corresponding signal estimation. The frequency tracking performance of EKF algorithm and modified EKF algorithm is given in Figure 4. The EKF estimates the original frequency approximately in 35ms (The correct estimation is in 90ms) and the maximum deviation of the frequency from 50 Hz is about 0.4 Hz.

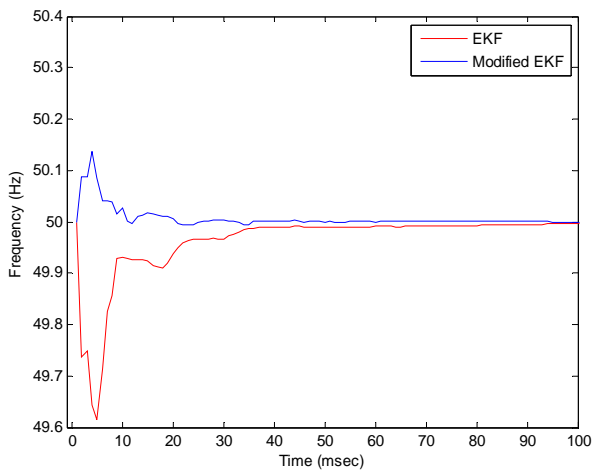


Figure 4. The frequency estimation of system signal with EKF and Modified EKF.

The modified EKF estimates the original frequency approximately in 35ms and the maximum deviation of the frequency from 50 Hz is less than 0.2 Hz.

**4.2. Harmonic Distortion**

In the second part of the study a 50 Hz pure sinusoidal signal with unity amplitude is corrupted with harmonic distortion and zero-mean white Gaussian noise with variance  $\sigma^2$  [8]. The base band signal was subjected to 3rd, 5th, 7th and 9th harmonics. The 11th harmonic and the rest of the harmonics were ignored due to the insignificance distortion effects on the signal. The power system signal and distorted system signal is given in Figure 5.

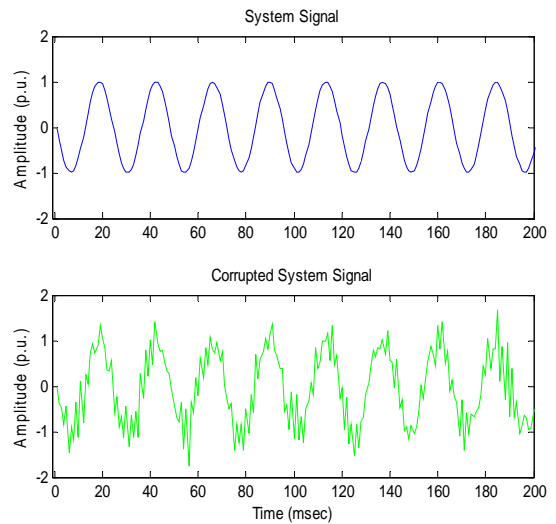


Figure 5. Power system signal and the corrupted system signal.

The power system signal estimation with harmonic distortion of proposed modified EKF is given in Figure 6. Modified EKF algorithm estimates the system signal better at the beginning of the data window but after the beginning the tracking performances of conventional EKF and the modified EKF are approximately same.

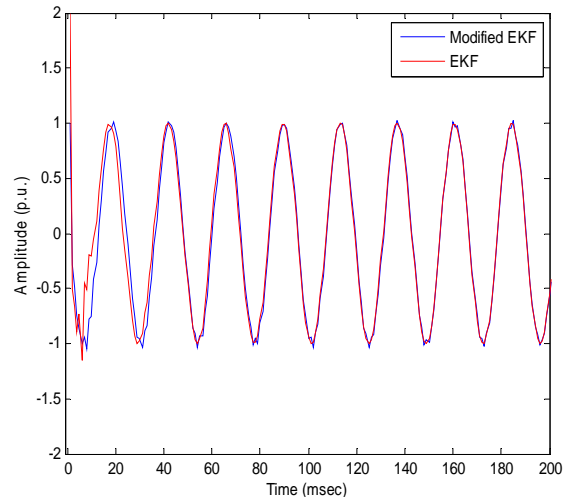


Figure 6. Power system signal estimation with EKF and Modified EKF.

The frequency tracking performance of EKF algorithm and modified EKF algorithm for harmonic distortion is given in Figure 7.

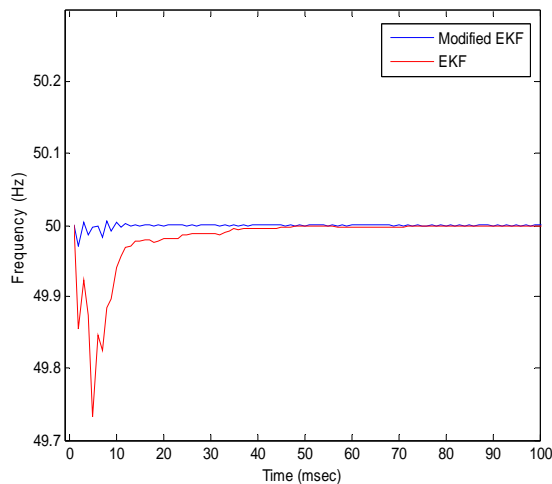


Figure 7. The frequency estimation of system signal with EKF and Modified EKF.

The EKF algorithm estimates the system frequency approximately in 40ms and the maximum deviation of the frequency from 50 Hz is about 0.3 Hz. On the other hand the modified EKF algorithm estimates the system frequency approximately in 15ms and the maximum deviation of the frequency from 50 Hz is about 0.02 Hz.

## 5. DISCUSSION AND CONCLUSION

In this paper, a modified extended Kalman filter (EKF) approach is proposed for frequency and amplitude estimations of distorted signals in a power system. The EKF structure not only estimates the frequency as a Kalman state variable, but also estimates the power system signal amplitude. The proposed modification is based on the lock detection algorithm which adjusts the covariance matrix adaptively according to the tracking performance of the EKF. In this study in order to simulate distortion of power system signal, the noise and harmonic contamination is used. The results of amplitude estimations of distorted signals using the modified EKF are better than those of using the conventional EKF algorithm. The frequency detection time for noise and harmonic distortion of modified EKF is shorter than conventional EKF. Especially for harmonic distortion frequency detection of modified EKF is faster (25ms) than conventional EKF. Also frequency estimation of modified EKF much less deviated from system frequency. Simulation results monitored that the performance of proposed algorithm under harmonic distortion is better than the one with the noise distortion. With the acceptable time delay between data windows, proposed algorithm is suitable for real time applications where the noise and the harmonic disturbances are high. Moreover, the proposed method can facilitate the implementation of field measurements.

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