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Fuzzy Wagner Whitin Algorithm and an Application of Class I Supplies

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ABSTRACT

The purpose of this study is to avoid unnecessary stocking of Class I supply materials via proper lot-sizing methods; to determine the level of stocks in food accountancy warehouses and to make appropriate stocking policies. The data that was used in this study was obtained from a unit of Turkish Armed Forces. All the data were determined and applied to the nine lot sizing methods and best order policies were obtained from Wagner Whitin algorithm. Because the future values of the parameters must be used in WW model, the data used at the beginning may change by the time. Related to the future changed data, the optimal order policy may differ and real total cost will be higher than the model. The fuzzy concept was utilized in Wagner Whitin algorithm to manage uncertain demands and costs. Fuzzy WW algorithm was proposed and shown for one of the items.

Keywords: *Order policy, inventory management, incapacitated lot sizing, Wagner Whitin algorithm, fuzzy sets.*

1. INTRODUCTION

Lot sizing means calculating the amount of end products or components that meet the demand and minimize the total cost which consists of order cost, inventory holding cost and item unit cost.

Class I supplies are items like food and provender that are consumed by personnel and animals everyday regularly independent of action and terrain circumstances [1].

The success of the missions in the Armed Forces depends on precise determination of the demands; appropriate planning and allocation of the supply items. Nevertheless, demands have to be met at the sufficient level with optimum cost; otherwise, it will not be possible to meet the demands in some of the periods while the inventory level will be higher than the optimum in some other periods. That is why the order policies of the items must be determined according to optimum inventory levels and order quantities.

It is stated by the Armed Forces Logistics Department that peacetime stock levels and proper lot sizing methods of Class I Supplies must be redefined while unnecessary

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stock levels are avoided. That is why this theme was studied. Class I Supply items included in this study have independent demands; that is the demand of the item to supply is not affected from the demand of any other item. However there are no resources constraints; if it's required even the annual need of any item can be supplied at once. Because of these specifications, the problem in this study is a SLUR (Single Level Lot Sizing without Resource Constraints) problem according to the classification made with respect to the problem characteristics in the literature [2]. There are a lot of preceding studies about this type of the problem and some of them were stated below;

Kaiman [3] compared the methods economic order quantity (EOQ) and Wagner Whitin algorithm (WW) using 25 test problems. In addition to these two methods, Berry [4] included in his study the periodic order quantity (POQ) method and the part period balancing (PPB) method and made his comparisons accordingly. Another study comparing the methods WW, PPB and Silver-Meal heuristic (SM) in the rolling schedule environment was implemented by Blackburn and Millen [5].

In deterministic models, demand can be considered constant. Chand [6] carried out several studies on such models with constant demand. Ganas and Papachristos [7] studied heuristics in situations where the demand was constant. Sanchéz *et al.* [8] compared total inventory costs of five lot sizing techniques; add-drop heuristic (ADH), lot for lot (L4L), fixed order period (FOP), least unit cost (LUC) and Silver Meal Heuristic (SM). In their study, Heuvel and Wagelmans [9] assessed the WW model with end effects after the short model horizon and expressed that the optimum result for the planning period in fact may not be optimum in the long run. In a lot-sizing problem the demand was not zero after the planning horizon. Thus, they developed an algorithm named extended Wagner Whitin (EWW) which uses predictive values of orders with respect to long term and carried out a comparative study using the ending inventory valuation (EIV) method of Fisher *et al.*[10]. Ho *et al.*[11] suggested two different heuristic methods; nLPC and nLPC(i), which they obtained improving the Silver Meal model (also known as least period cost). They carried out a performance evaluation comparing the results they obtained from the test problems with the results of the methods least total cost (LTC), POQ, LUC, PPB and least period cost (LPC). Şenyiğit and Yıldırım [12] applied the methods WW, PPB, LTC, LUC, L4L, POQ, EOQ and FOQ to 37 products of an enterprise, producing 209 different products and carried out a performance evaluation comparing the results they obtained from this study with the ones obtained using the double-coefficient algorithm they developed. Bahl [13] studied the method of Groff reorder procedure (GRP) and dynamic lot-size creation method (DLC) with the models L4L, PPB, LUC, which are used widely in ERP software. Koç [14] evaluated the models Wagner Whitin and Silver Meal to determine the economic lot sizes in an automotive maintenance and repair enterprise, and concluded that it is applicable put the Silver Meal model into practice.

2. LOT SIZING

The data of daily existing persons obtained from a division level unit of Armed Forces, which is the universe of this study. Six Class I items are chosen as sample out of 38 main ones of which consumption levels are determined. Because the staff of the division is determined, the lot sizing problem of Class I items is suitable for the deterministic models. Nevertheless, order costs, inventory holding costs and item unit costs are changeable in the periods because of the economic situation. In addition, demands are changeable in the periods because of the personnel in and out movements.

These features provide a dynamic structure for the problem. Existing models in literature were examined; the ones which are suitable for the problem and which have previously been used in similar studies, namely L4L, FOQ, EOQ, FOP, POQ, PPB, LUC, SM and WW methods, were put into practice.

2.1. Data

It is necessary to know the demand, item unit cost, order cost and inventory holding cost of each item in order to apply the lot sizing models. In this study, the demand for each item was calculated based on the consumption factors for daily existing persons and in order to make ordering easier, these values were converted into units (bags, parcels, canisters etc.) according to the type and weight of packaging before applying to the models. The monthly demand for each item used in this study is presented in the Table 1.

Table 1. Monthly demand for each item (unit).

Months	Sunflower oil	Red Lentil	Haricot Beans	Tomato Paste	Green Lentil	Black Olives
January	1016	16	174	1030	32	396
February	662	10	114	671	21	258
March	903	14	155	916	28	352
April	918	14	157	931	29	358
May	827	13	142	839	26	323
June	931	15	80	182	29	363
July	900	14	77	175	28	351
August	832	13	71	162	26	324
September	961	15	82	187	30	375
October	929	14	80	181	29	362
November	692	11	59	135	22	270
December	843	13	145	855	26	329

Unit cost of each item is determined by adjudications and may vary depending on the supplier. In this study, the mean price obtained from the division in question was determined based on the global increase rates for wholesale commodity prices index. Table 2 presents monthly item unit costs.

Table 2. Monthly item unit costs (Euros).

Months	Sunflower Oil	Red Lentil	Haricot beans	Tomato Paste	Green Lentil	Black Olives
January	28.68	30.71	43.88	5.46	44.30	11.33
February	28.82	30.86	44.10	5.49	44.52	10.98
March	29.40	31.48	44.98	5.60	45.41	11.20
April	30.10	32.23	46.06	5.73	46.50	11.47
May	30.91	33.10	47.30	5.89	47.75	11.78
June	32.24	34.53	49.33	6.14	49.81	12.28
July	33.63	36.01	51.46	6.41	51.95	12.81
August	34.54	36.98	52.84	6.58	53.35	13.16
September	35.68	38.20	54.59	6.80	55.11	13.59
October	36.28	38.85	55.52	6.91	56.05	13.82
November	36.39	38.97	55.68	6.93	56.22	13.86
December	36.97	39.59	56.57	7.04	57.12	14.09

Order cost, which is the total value accounting for the costs from the moment demands are determined and the ordering decision is made, to the moment the items are received, was calculated according to the sum of the costs of adjudication declaration, reports, notary public, personnel, vehicles, stationery, communication and electricity, taking into consideration the variations, wholesale commodity prices index and consumer prices index increments. Order cost does not differ according to the item. Table 3 presents monthly order cost.

Table 3. Order costs.

Months	Ordering costs (Euros)
January	1341.15
February	1345.70
March	1367.06
April	1395.59
May	1425.74
June	1476.64
July	1568.00
August	1599.40
September	1641.36
October	1662.85
November	1667.63
December	1688.35
Mean	**1514.95**

Inventory holding costs which are calculated according to the sum of the costs electricity, maintenance, personnel, security and opportunity, are the costs to bear while keeping the previously bought items for future use. It was calculated taking into consideration the variations, increase rates of wholesale commodity prices index and monthly interest rates. Monthly inventory holding costs for the items used in this study are presented in Table 4.

Table 4. Monthly inventory holding costs (Euros).

Months	Sunflower Oil	Red Lentil	Haricot Beans	Tomato Paste	Green Lentil	Black Olives
January	1.43	1.71	2.35	0.27	2.37	0.57
February	1.42	1.70	2.34	0.27	2.36	0.54
March	1.34	1.62	2.22	0.26	2.24	0.52
April	1.27	1.55	2.12	0.24	2.14	0.49
May	1.27	1.55	2.12	0.24	2.14	0.49
June	1.32	1.62	2.21	0.26	2.23	0.51
July	1.39	1.70	2.33	0.27	2.35	0.54
August	1.43	1.75	2.39	0.28	2.41	0.55
September	1.48	1.80	2.47	0.28	2.49	0.57
October	1.50	1.83	2.51	0.29	2.53	0.58
November	1.41	1.74	2.37	0.27	2.39	0.54
December	1.42	1.75	2.39	0.27	2.41	0.55

The ratios of inventory holding costs to item unit costs were calculated in order to eliminate the need to make recalculations to compensate for the changing costs in the future. The ratios obtained for sunflower oil, red lentil, haricot beans, tomato paste, green lentil and black olives are 0.043, 0.048, 0.046, 0.043, 0.046 and 0.043, respectively. These values should be multiplied by the item unit costs according to the following formula to calculate the inventory holding costs easily in the future.

$$\text{Inventory holding cost } (h) = \text{Item Unit cost } (c) \times \text{Ratio of inventory holding } (i)$$

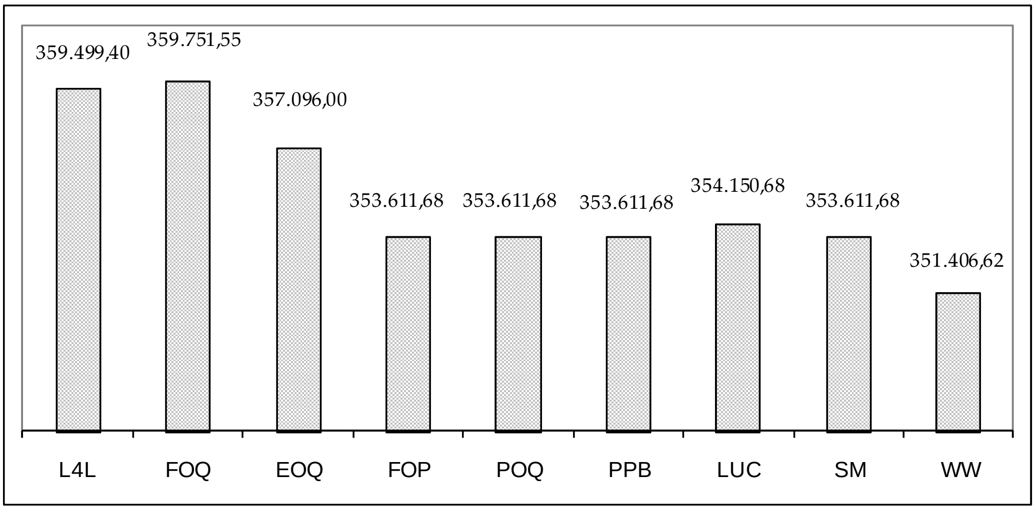

Figure 1. Total Costs Obtained from the Models for Sunflower Oil.

2.2. Model Applications and Comparisons

Using the data obtained so far, nine lot-sizing models appropriate to the problem for the items used in the study were applied on WinQSB (version 1.0) software. Least total costs in the planning horizon for each supply item was obtained by WW algorithm and order policies were determined accordingly. Total Costs Obtained from nine lot sizing Models for Sunflower Oil was shown in Figure 1 as an example.

Recommended order policy of sunflower oil obtained by WW algorithm is shown in the Table 5.

Table 5. Order Plan of Sunflower Oil Obtained from WW Algorithm.

Period	**Order Quantities (unit)**
January	1678
February	
March	1821
April	
May	5380
June	
July	
August	
September	
October	
November	1535
December	
Total Cost (Euros)	**351406.61**

Figure 2 shows mean performance values of the nine lot-sizing models applied on supplies involved in this study based on WW model.

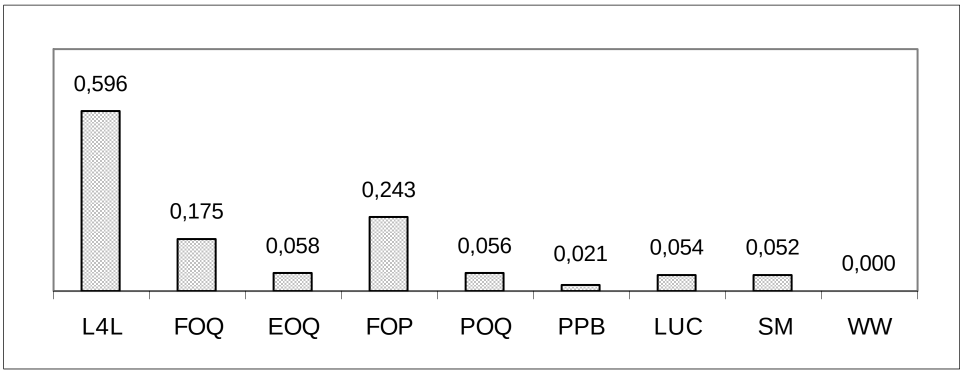

Figure 2. Mean performance values of models based on WW algorithm.

As a result of this study, mean model performance values for six supply items were found to be in the order of WW, PPB, SM, LUC, POQ, EOQ, FOQ, FOP, L4L beginning with the best. Results obtained in this study are similar to the results in literature. Therefore, when making a choice, priority of the models for class I supply items should be according to these results.

In order to clearly demonstrate the savings that may be obtained by a optimum order/inventory policy, WW algorithm was applied to each supply item included in the study, taking into consideration the inventory at the beginning and at the end of the year, and the results were compared to the real total costs.

Accordingly, for each supply item chosen as sampling, the savings were found to be between 5377.00 and 17888.04 Euros (Table 6). When the supply items used in the study considered all together, a total of 57541.21 Euros was saved. [16]. As a result it can be concluded that order plans obtained through application of the models suggested here would provide a total saving of 1 million Euros annually (partially opportunity cost) for 100 different class I supplies.

Among the applied models, WW algorithm offers the optimum solution. However, in reality there are some environmental variables that cannot be incorporated into these models. Due to the reason that there are a lot of environmental variables that it is impossible to take all of them into consideration and that these cannot be predetermined, the results obtained do not come true completely. Considering this aspect of the problem, fuzzy WW algorithm has been built into system.

Table 6. Cost comparison for real total costs and WW algorithm (Euros).

	Real Total Costs	**WW Algorithm's**	**Savings**
Sunflower Oil	737013.84	719125.81	17888.04
Red Lentil	38712.86	33267.70	5445.16
Haricot Beans	120406.36	112730.05	7676.31
Tomato Paste	86508.98	81131.99	5377.00
Green Lentil	67852.13	56585.94	11266.19
Black Olives	110205.22	100316.72	9888.51

3. FUZZY SETS

In many real-world applications, demands and costs may vary dynamically with the situation. If the demands and costs are uncertain, the lot sizing and total cost are apparently also uncertain.

Fuzzy set theory provides a means for representing uncertainties. The nature of the uncertainty in a problem is very important point. Uncertain information can take on many different forms. There is uncertainty that arises because of complexity, from ignorance, from chance, from various classes of randomness, from imprecision, from the inability to perform adequate measurements, from lack of knowledge or from vagueness. Fuzzy sets theory is used as an appropriate method to express the uncertainty and provide a mathematical way to represent vagueness in humanistic systems. All information contained in a fuzzy set is described by its membership function. Membership functions may take on several common forms like triangular, trapezoidal, bell-curve and so on. For the sake of computational efficiency and ease of data acquisition triangular membership functions are often used.

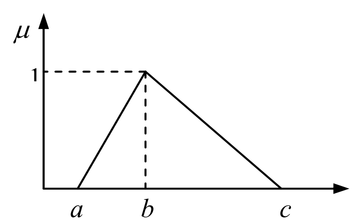

Figure 3. Triangular distribution function.

Although fuzzy set concepts are mainly used in linguistic domains, they are also used in numerical domains. A convex and normalized fuzzy set defined on ***R*** whose membership function is piecewise continuous is called a fuzzy number. It is especially useful in solving possibilistic mathematical programming.

A triangular fuzzy number A is characterized by a triangular distribution function parameterized by a triplet (a,b,c) as shown in Figure 3.

3.1. Fuzzy Arithmetic

Fuzzy arithmetic is a direct application of the extension principle and used on fuzzy numbers. The term fuzzy number is used to handle imprecise numerical quantities.

Applying some fuzzy arithmetical operations on the triangular fuzzy numbers are as follow. Let A and B be two triangular fuzzy numbers defined by the triplets (a,b,c) and (d,e,f).

The addition of *A* and *B* is $A \oplus B = (a+d, b+e, c+f)$, where *a,b,c,d* and *f* are any real numbers. Addition is an increasing operation. Hence, the extended addition of fuzzy numbers gives a fuzzy numbers.

Multiplication of fuzzy numbers is a bit complicated because the signs of fuzzy numbers must be considered. Since fuzzy demands and fuzzy costs are positive fuzzy number, we shall consider the case in which both fuzzy numbers are positive fuzzy numbers. The multiplication of *A* and *B* is $A \otimes B = (a \times d, b \times e, c \times f)$, where a,b,c,d,e and f are all nonzero positive real numbers.

The scalar multiplication is $kA = (ka,kb,kc)$ *where* $\forall k > 0, k \in R$ or

$kA = (kc,kb,ka)$ *where* $\forall k < 0, k \in R$ [17]

3.2. Fuzzy Ranking Methods

To resolve this problem, many fuzzy ranking methods that can be used to compare fuzzy numbers have been proposed in the literature [17]. In this study the Lee and Li's [18] method is used to compare fuzzy numbers. Fuzzy numbers are ranked based on the fuzzy mean and fuzzy spread of the fuzzy number. A fuzzy number with a higher mean value and a lower spread would be favor.

3.2.1. Fuzzy Mean and Spread

Lee and Li [18] assume uniform and proportional distributions for fuzzy events. For simplicity, uniform case is used in this study for fuzzy events. The probability density function for uniform distribution is as follows.

$$f(x) = \frac{1}{|A|} \tag{1}$$

In the case of triangular fuzzy numbers, the mean and standard deviation of a fuzzy number *A* are calculated by using Equation 2 and Equation 3 [18].

$$m_U = \frac{1}{3}(a_1 + a_2 + a_3) \tag{2}$$

$$\sigma_U = \frac{1}{18}\left(a_1^{\,2} + a_2^{\,2} + a_3^{\,2} - a_1a_2 - a_1a_3 - a_2a_3\right) \tag{3}$$

The fuzzy number with the higher mean value is then ranked higher than the fuzzy number with lower mean value. If the mean values happen to be equal, the standard deviation is used. Fuzzy number with the smaller standard deviation is judged the smaller.

4. FUZZY WAGNER WHITIN ALGORITHM

Deterministic models are convenient to obtain precise results provided that the data are known. However, there is the possibility that data which were used in the model may change in the future. Data related to these variables serving a base for future plans, may only be obtained by adapting previous changes for the future. A complete correlation of the data obtained this way is nearly impossible.

When characteristics of Wagner Whitin model are investigated, it appears that changes in any of the data used in the model would change the whole order planning. In such cases, results obtained through application of Wagner Whitin model will not be optimum.

To prevent from such scenery and to give a form to WW model that is feasible and sensitive to changes is possible through the use of fuzzy numbers.

In this study demand, purchasing cost, ordering cost, and inventory holding cost data used in the WW algorithm were converted into triangular fuzzy numbers for once, using our "generator" software encoded in C language in order to generate fuzzy numbers.

Because it was taken into consideration that the estimated values of costs and demands are more likely to be higher than being lower in reality; while converting these data into fuzzy form, δ = 0,85 is used for calculating optimistic and δ = 1,30 is for pessimistic values.

Having done this conversion, instead of single values for each variable, fuzzy number sets consisting of three values; optimistic, expected, and pessimistic values entered for Wagner Whitin algorithm. The procedure for determination of minimum costs among alternative order plans is performed according to fuzzy mean and separation method.

Since there is no software capable of solving Wagner Whitin algorithm with fuzzy numbers, the algorithm was developed in Excel as software capable of solving a 12-period problem using fuzzy numbers. In the fuzzy WW program all the data or a part of them anticipated to vary can be entered as fuzzy sets. Hence Fuzzy WW algorithm is able to solve the problem when every parameter is converted to fuzzy form.

As a result, total cost and order quantities in each period are obtained as an interval of values instead of a single value, and decision maker is informed about what order quantity interval and what costs to study. The structure of 2-period fuzzy WW model is given in Table 7 below.

Table 7. The structure of 2-period fuzzy WW model.

k=1	**j=0**		**F0**						
		0	0	0					
			c1(D1)						
		23008.74	29134.38	35425.99					
			A1						
		1161.37	1341.15	1583.62					
			Sum					**F1***	
		24170.11	30475.53	37009.61		**j=0**	24170.11	30475.53	37009.61
k=2	**j=0**		**F0**						
		0	0	0					
			c1(D1+D2)						
		38537.78	48117.60	57646.59					
			A1						
		1161.37	1341.15	1583.62					
			H1(D2)						
		859.72	943.61	1087.18					
			Sum						
		40558.88	50402.37	60317.39					
			Average						
			50426.21						
	j=1		**F1**						
		24170.11	30475.53	37009.61					
			C2(D2)						
		16652.87	19078.14	25691.23					
			A2						
		1313.88	1345.70	1706.93					
			Sum					**Min(Average)**	
		42136.86	50899.37	64407.76				50426210485	
			Average					**F2***	
			52481.33			**j=0**	40558.88	50402.37	60317.39
						j=1	HIGH	HIGH	HIGH

Order planning for class I supplies of Turkish military system dealt in this study was initially calculated according to the normal Wagner Whitin method and then another order planning for the same supply items was built using WW algorithm.

Comparative results obtained from both methods are given in the Table 8 below for tomato paste.

Table 8. Comparison of WW and fuzzy WW models for tomato paste.

Period	**Order Quantity (unit)**			
	WW Model	***Fuzzy WW Model***		
January	2617	1640	1701	1777
February				
March		4308	4563	5004
April	3647			
May				
June				
July				
August				
September				
October				
November				
December				
Total Cost (Euros)	41814.87	35010.55	41860.84	48928.42

The order planning of fuzzy WW is different than WW. The advantage of fuzzy WW algorithm is that order planning for the planning horizon is not affected by the changes in demand and costs and the algorithm offers flexibility for quantities to be ordered in predefined optimistic and pessimistic intervals. As a result of this the total cost will be in optimistic and pessimistic intervals obtained from model, however the costs and order quantities will be at the expected prices in case of no variations in the data used at the beginning. Especially if costs or demands are lower than expected then total cost will be lower in fuzzy WW. Best before date of every class I item in stock is later than 12 months; hence, there is no restriction for 12-period order planning if every period is considered as month.

5. CONCLUSION

The advantage of designating an order policy for each item purchased and stored regularly cannot be neglected in means of cost. The savings for six class I supplies used in this study, which is 57.541,21 Euros annual, was obtained only by the proper ordering policy. The Wagner Within algorithm, which gives the optimum result among nine models, is the most appropriate solution that should be preferred for order planning of class I supplies. However, WW algorithm does not have enough dynamism for considering all the variations. The Fuzzy WW algorithm was developed in this study to take into consideration also the variations occurring within this dynamism.

In Fuzzy Wagner Within algorithm, decision maker is informed about the variable interval of total costs based on the changes in demand, costs, and order quantity intervals. In case of variable demands and costs, fuzzy WW algorithm also offers flexibility in means of changing the order quantities in certain limits without having to change the order planning suggested. Therefore, fuzzy WW algorithm is more sensitive to the variations compared to the classical WW algorithm; permitting flexibility in certain limits and maintaining order plans initially suggested with a more robust structure. Variations, which are probable to be faced, can be realized earlier and more successful results can be obtained in practice by using fuzzy WW algorithm. The results can then help a manager have a wide overview of order quantity and total cost.

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