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A Note on Permuting Tri-Derivation In Near Ring

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ABSTRACT

In this paper, we introduce a permuting tri-(σ,τ)-derivation and permuting tri-generalized derivation in a near ring and generalize some of the results in [4], [6], [8] .

Key Words: Permuting, Tri-deviation, Near ring

1. INTRODUCTION

The concept of a permuting tri-derivation has been introduced Öztürk in [5]. Some recent results on properties of prime rings, semi-prime rings and near rings with derivations have been investigated in several ways [2-5, 8]. In [6], Kyoo-Hong Park and Yong-Soo Jung have introduced the concept of a permuting tri-derivation of a near ring and investigated the conditions for a near ring to be commutative ring.

In this note, we introduce the concepts of permuting tri-(σ,τ)-derivation and permuting tri-generalized derivation of near ring and give some properties.

Throughout this paper N will be a zero-symmetric left near ring with multiplicative center Z. Recall that a near ring N is prime $xNy=\{0\}$ implies $x=0$ or $y=0$. For $x,y\in N$, $[x,y]$, $[x,y]_{\sigma,\tau}$ and (x,y) will denote the commutator $xy-yx$, $x\sigma(y)-\tau(y)x$ and $x+y-x-y$ respectively. A mapping $D:N\times N\times N\to N$ is said to be permuting if $D(x,y,z)=D(y,x,z)=D(z,y,x)=D(x,z,y)=D(y,z,x)=D(z,x,y)$ for all $x,y,z\in N$. A mapping $d:N\to N$ defined by $d(x)=D(x,x,x)$ is called the trace of D where $D:N\times N\times N\to N$ is a permuting mapping. It is obvious that, if $D:N\times N\times N\to N$ is a permuting mapping which is also tri-additive (i.e., additive in all arguments), then the trace of D satisfies the relation $d(x+y)=d(x)+2D(x,x,y)+D(x,y,y)+D(x,x,y)+2D(x,y,y)+d(y)$ for all $x,y\in N$. A permuting tri-additive mapping $D:N\times N\times N\to N$ is called a permuting tri-derivation if $D(xw,y,z)=D(x,y,z)w+xD(w,y,z)$ is fulfilled for all $x,y,z,w\in N$. For the terminology used in near rings, see [7].

2. PERMUTING TRI-(σ,τ)DERIVATION

The following lemmas and theorems are necessary for the paper.

Lemma 1. [2, Lemma 3] *Let N be a prime near ring.*

(i) If $z\in Z-\{0\}$, then z is not a zero divisor.

(ii) If $Z-\{0\}$ contains an element z for which $z+z\in Z$, then $(N,+)$ is abelian.

Lemma 2. [6, Lemma 2.2] *Let N be a $3!$-torsion free near ring. Suppose that there exists a permuting tri-additive mapping $D:N\times N\times N\to N$ such that $d(x)=0$ for all $x\in N$, where d is the trace of D. Then we have $D=0$.*

Lemma 3. [6, Lemma 2.3] *Let N be a $3!$-torsion free prime near ring and let $x\in N$. Suppose that there exists a nonzero permuting tri-derivation*

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$D: N\times N\times N\to N$ such that $xd(y)=0$ for all $x,y\in N$, where d is the trace of D. Then we have $x=0$.

Firstly, we introduce the definition of permuting tri-(σ,τ)-derivation in a near ring.

Definition 1. *A permuting tri-additive mapping $D: N\times N\times N\to N$ is called a permuting tri-(σ,τ)-derivation if there exist functions $\sigma,\tau: N\to N$ such that $D(xw,y,z)=D(x,y,z)\sigma(w)+\tau(x)D(w,y,z)$ for all $x,y,z,w\in N$.*

Note that if $\sigma=1$ and $\tau=1$ then D is a permuting tri-derivation.

Example 1. *Let $(N,+)$ be the Klein's four group with multiplication defined as following*

·	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	c
c	0	a	b	c

Define a map D on N by

$$D(x,y,z)=\begin{cases} b, & x,y,z\notin\{0,a\} \\ 0, & otherwise \end{cases}$$

If we take $\sigma(0)=\sigma(a)=0$, $\sigma(b)=b$, $\sigma(c)=c$ and $\tau(x)=0$ for all $x\in N$, D is a permuting tri-(σ,τ)-derivation on N. But, since

$$D(ba,b,c)=D(b,b,c)a+bD(a,b,c)=a$$

and

$$D(ba,b,c)=D(a,b,c)=0,$$

D is not permuting tri-derivation.

Throught this paper, σ and τ will represent automorphisms of N.

Lemma 4. *Let N be a $3!$-torsion free prime near ring, D a permuting tri-(σ,τ)-derivation of N and d the trace of D. If $xd(N)=\{0\}$ for all $x\in N$, then $x=0$ or $D=0$.*

Proof. Using same method in proof of Lemma 2, we have $xD(w,y,z)=0$ for all $x,y,z,w\in N$. Replacing z by zv, $v\in N$, to get $x\tau(z)D(w,y,v)=0$ for all $x,y,z,w,v\in N$. Since τ is an automorphism of N, we get $xND(w,y,v)=\{0\}$. Again N is prime near ring, we have $x=0$ or $D=0$.

Lemma 5. *Let N be a near ring. D is a permuting tri-(σ,τ)-derivation of N if and only if $D(xw,y,z)=\tau(x)D(w,y,z)+D(x,y,z)\sigma(w)$ for all $x,y,z,w\in N$.*

Proof. Let D be a permuting tri-(σ,τ)-derivation of N. Since σ is an automorphism, we get for all $x,y,z,w\in N$,

$$\begin{aligned} D(x(w+w),y,z) &= D(x,y,z)\sigma(w+w)+\tau(x)D(w+w,y,z) \\ &= D(x,y,z)\sigma(w)+D(x,y,z)\sigma(w)+\tau(x)D(w,y,z) \\ &\quad+\tau(x)D(w,y,z) \end{aligned}$$

and

$$\begin{aligned} D(x(w+w),y,z) &= D(xw+xw,y,z)=D(xw,y,z)+D(xw,y,z) \\ &= D(x,y,z)\sigma(w)+\tau(x)D(w,y,z)+D(x,y,z)\sigma(w) \\ &\quad+\tau(x)D(w,y,z) \end{aligned}$$

Combining the above two equality, we find that

$$D(x,y,z)\sigma(w)+\tau(x)D(w,y,z)=\tau(x)D(w,y,z)+D(x,y,z)\sigma(w)$$

Hence we have

$D(xw,y,z)=\tau(x)D(w,y,z)+D(x,y,z)\sigma(w)$ for all $x,y,z,w\in N$. Converse can be proved in a similar way.

Lemma 6. *Let N be a near ring, D a permuting tri-(σ,τ)-derivation of N. Then, for all $x,y,z,w,v\in N$,*

(i) $[D(x,y,z)\sigma(w)+\tau(x)D(w,y,z)]v = D(x,y,z)\sigma(w)v+\tau(x)D(w,y,z)v$,

(ii) $[\tau(x)D(w,y,z)+D(x,y,z)\sigma(w)]v = .$

$$\tau(x)D(w,y,z)v+D(x,y,z)\sigma(w)v$$

Proof. (i) Let D be a permuting tri-(σ,τ)-derivation of N. Since σ and τ are automorphisms, we get for all $x,y,z,t,w\in N$,

$$\begin{aligned} D((xw)t,y,z) &= D(xw,y,z)\sigma(t)+\tau(xw)D(t,y,z) \\ &= [D(x,y,z)\sigma(w)+\tau(x)D(w,y,z)]\sigma(t) \\ &\quad+\tau(x)\tau(w)D(t,y,z) \end{aligned}$$

and

$$\begin{aligned} D(x(wt),y,z) &= D(x,y,z)\sigma(wt)+\tau(x)D(wt,y,z) \\ &= D(x,y,z)\sigma(w)\sigma(t)+\tau(x)D(w,y,z)\sigma(t) \\ &\quad+\tau(x)\tau(w)D(t,y,z). \end{aligned}$$

Combining the above two equality, we find that

$$[D(x,y,z)\sigma(w)+\tau(x)D(w,y,z)]\sigma(t)=D(x,y,z)\sigma(w)\sigma(t)+\tau(x)D(w,y,z)\sigma(t)$$

Therefore, we have

$[D(x,y,z)\sigma(w)+\tau(x)D(w,y,z)]v = D(x,y,z)\sigma(w)v+\tau(x)D(w,y,z)v$

for all $x,y,z,w,v\in N$.

(ii) It can be proved in a similar way.

Lemma 7. *Let N be a prime near-ring, D a nonzero permuting tri-(σ,τ)-derivation of N. Then $D(N,N,N)x=\{0\}$ for all $x\in N$ implies $x=0$.*

Proof. Suppose $D(y,z,w)x=0$ for all $x,y,z,w\in N$ Then taking yv instead of y, we have

$$0 = D(y,z,w)\sigma(v)x+\tau(y)D(v,z,w)x = D(y,z,w)\sigma(v)x$$

Since σ is an automorphism, we have $D(y,z,w)Nx=\{0\}$ for all $x,y,z,w\in N$. Since N is prime near ring and D is nonzero, this implies $x=0$.

Theorem 1. *Let N be a prime near ring, D is a nonzero permuting tri-(σ,τ)-derivation of N. If $D(N,N,N)\subseteq Z$, then N is a commutative ring.*

Proof. Since $D(N,N,N)\subseteq Z$ and D is a nonzero permuting tri-(σ,τ)-derivation of N, then there exist nonzero elements $x,y,z\in N$ such that $D(x,y,z)\in Z-\{0\}$.Then $D(x+x,y,z)=D(x,y,z)+D(x,y,z)\in Z$ and hence $(N,+)$ is abelian by Lemma 1. $D(x,y,z)\in Z$ for $x,y,z\in N$, implies that $D(x,y,z)w = wD(x,y,z)$ for all $w\in N$. Hence replace xv with x to get

(2.1) $D(x,y,z)\sigma(v)w+\tau(x)D(v,y,z)w = wD(x,y,z)\sigma(v)+w\tau(x)D(v,y,z)$

Taking $\sigma(v)$ instead of w, we have

$$D(x,y,z)\sigma(v)\sigma(v)+\tau(x)D(v,y,z)\sigma(v) = \sigma(v)D(x,y,z)\sigma(v) +\sigma(v)\tau(x)D(v,y,z)$$

Since $D(N,N,N)\subseteq Z$, we get

$$D(v,y,z)[\tau(x),\sigma(v)]=0$$

for all $x,y,z,v\in N$.

Since Z contains no nonzero divisors of zero, we see that for each $v\in N$, either $D(v,y,z)=0$ or $[\tau(x),\sigma(v)]=0$ for all $x,y,z\in N$. If $D(v,y,z)=0$, then $D(x,y,z)[w,\sigma(v)]=0$ from (2.1). Since N is prime near ring and $D\neq 0$, $[w,\sigma(v)]=0$. Since σ is an automorphism, N is commutative near ring. If $[\tau(x),\sigma(v)]=0$ for all $x,v\in N$, then N is commutative ring, since σ and τ are automorphisms.

Theorem 2. *Let N be a $3!$-torsion free prime near-ring, D a permuting tri-(σ,τ)-derivation of N and d the trace of D. If $d(x)\sigma(y)=\tau(x)d(y)$ for all $x,y\in N$, then $d=0$.*

Proof. Assume that $d(x)\sigma(y)=\tau(x)d(y)$ *for all* $x,y\in N$. Replacing y by $y+z$, $z\in N$ in hypothesis, we get

$$d(x)\sigma(y+z)=\tau(x)d(y+z)$$

$$d(x)\sigma(y)+d(x)\sigma(z) = \tau(x)d(y)+2\tau(x)D(y,y,z)+\tau(x)D(y,z,z) +\tau(x)D(y,y,z)+2\tau(x)D(y,z,z)+\tau(x)d(z)$$

and so

(2.2) $2D(y,y,z)+D(y,z,z)+D(y,y,z)+2D(y,z,z)=0$

since τ is an automorphism and N is prime near ring.

Replacing y by $-y$ in (2.2), it follows that

(2.3) $2D(y,y,z)-D(y,z,z)+D(y,y,z)-2D(y,z,z)=0$

On the other hand, replacing y by $z+y$, $z\in N$ in hypothesis, we get

$$d(x)\sigma(z+y)=\tau(x)d(z+y)$$

$$d(x)\sigma(z)+d(x)\sigma(y) = \tau(x)d(z)+2\tau(x)D(z,z,y)+\tau(x)D(z,y,y) +\tau(x)D(z,z,y)+2\tau(x)D(z,y,y)+\tau(x)d(y)$$

and so

(2.4) $2D(z,z,y)+D(z,y,y)+D(z,z,y)+2D(z,y,y)=0$

since τ is an automorphism and N is prime near ring. Comparing (2.2) with (2.3), we get

$$2D(y,z,z)+D(y,y,z)+D(y,z,z)+2D(y,y,z) = D(y,y,z) -3D(y,z,z)+2D(y,y,z)$$

for all $y,z\in N$. From (2.4), we have

(2.5) $D(y,y,z)-3D(y,z,z)+2D(y,y,z)=0$

Replacing y by $-y$ in (2.5), we get

(2.6) $D(y,y,z)+3D(y,z,z)+2D(y,y,z)=0$

Combining (2.5) and (2.6), we obtain $D(y,z,z)=0$ for all $y,z\in N$. Replacing z by $z+x$, $x\in N$, we get $D(x,y,z)=0$ for all $x,y,z\in N$. Thus, $D=0$. That is, $d=0$.

Theorem 3. *Let N be a $3!$-torsion free prime near ring, D a permuting tri-(σ,τ)-derivation of N and d the trace of D. If $d(x)$, $d(x)+d(x)\in C(D(N,N,N))$ for all $x\in N$, then $(N,+)$ is abelian.*

Proof. Assume that

(2.7) $d(x), d(x)+d(x)\in C(D(y,v,w))$

for all $y,v,w\in N$.

From (2.7), we get

$$\begin{aligned} D(y+t,v,w)(d(x)+d(x)) &= (d(x)+d(x))D(y+t,v,w) \\ &= (d(x)+d(x))D(y,v,w)+(d(x)+d(x))D(t,v,w) \\ &= d(x)D(y,v,w)+d(x)D(y,v,w)+d(x)D(t,v,w) \\ &+ d(x)D(t,v,w) \\ &= d(x)[D(y,v,w)+D(y,v,w)+D(t,v,w)+D(t,v,w)] \\ &= [D(y,v,w)+D(y,v,w)+D(t,v,w)+D(t,v,w)]d(x) \end{aligned}$$

and

$$\begin{aligned} D(y+t,v,w)(d(x)+d(x)) &= D(y+t,v,w)d(x)+D(y+t,v,w)d(x) \\ &= D(y,v,w)d(x)+D(t,v,w)d(x)+D(y,v,w)d(x) \\ &+ D(t,v,w)d(x) \\ &= [D(y,v,w)+D(t,v,w)+D(y,v,w)+D(t,v,w)]d(x) \end{aligned}$$

for all $x,y,t,v,w \in N$. Comparing last two equations, $D((y,t),v,w)d(x)=0$,for $x,y,t,v,w \in N$.

Replacing y by yz and t by yt, $z \in N$, we get

$$\begin{aligned} 0 &= D(y(z,t),v,w) \\ &= D(y,v,w)\sigma(z,t)+\tau(y)D((z,t),v,w) \\ &= D(y,v,w)\sigma(z,t). \end{aligned}$$

Since D is a nonzero permuting tri-(σ,τ)-derivation and N is prime ring, we have $\sigma(z,t)=0$. Since σ is an automorphism, $(z,t)=0$ for all $z,t \in N$. Thus, $(N,+)$ is abelian.

3.PERMUTING TRI-GENERALIZED DERIVATION

Definition 2. *Let N be a near ring and $D: N\times N\times N \to N$ a permuting tri-derivation of N. A permuting tri-addiive map $F: N\times N\times N \to N$ is said to be a permuting tri-right (resp. left) generalized derivation of N associated with D if $F(xw,y,z)=F(x,y,z)w+xD(w,y,z)$ $(resp. F(xw,y,z)=D(x,y,z)w+xF(w,y,z))$ for all $x,y,z,w \in N$.*

Also, F is said to be a permuting tri-generalized derivation of N with D if it is both a permuting tri-right and permuting tri-left generalized derivation of N associated with D.

Lemma 8. *Let N be a $3!$-torsion free prime near ring, D a permuting tri-derivation of N, F a permuting tri-additive mapping of N and f the trace of F. Then*

(i) If F is a permuting tri-right generalized derivation of N associated with D and $xf(y)=0$ for all $y \in N$, then $x=0$ or $D=0$.

(ii) If F is a permuting tri-generalized derivation of N associated with D and $xf(y)=0$ for all $y \in N$, then $x=0$ or $F=0$.

Proof. (i) Using the same method in proof of Lemma 2, we have

(3.1) $xF(y,z,w)=0$

for all $y,z,w \in N$. Hence replace y by yv, $v \in N$, to get $xyD(v,z,w)=0$ for all $y,z,v,w \in N$. Since N is a prime near ring,we have $x=0$ or $D=0$.

(ii) Let x be a nonzero element of N. Thus from (i), we obtain $D=0$. Replacing y by yv, $v \in N$ in (3.1) and from $D=0$, we get $xyF(v,z,w)=0$ for all $y,z,v,w \in N$. Since N is prime and $x \neq 0$, we have that $F=0$.

Lemma 9. *Let N be a near ring, D a permuting tri-derivation of N and F a permuting tri-additive mapping of N. Then, for all $y,z,x,w \in N$, the following are equivalent,*

(i) $F(xw,y,z)=F(x,y,z)w+xD(w,y,z)$

(ii) $F(xw,y,z)=xD(w,y,z)+F(x,y,z)w$

Proof. $(i)\Rightarrow(ii)$ Assume that $F(xw,y,z)=F(x,y,z)w+xD(w,y,z)$ for all $y,z,x,w \in N$. Then

$$\begin{aligned} F(x(w+w),y,z) &= F(x,y,z)(w+w)+xD(w+w,y,z) \\ &= F(x,y,z)w+F(x,y,z)w+xD(w,y,z) \\ &+ xD(w,y,z) \end{aligned}$$

and

$$\begin{aligned} F(x(w+w),y,z) &= F(xw+xw,y,z)=F(xw,y,z)+F(xw,y,z) \\ &= F(x,y,z)w+xD(w,y,z)+F(x,y,z)w \\ &+ xD(w,y,z) \end{aligned}$$

Combining the above two equality, we find that

$$F(x,y,z)w+xD(w,y,z)=xD(w,y,z)+F(x,y,z)w$$

Hence we have $F(xw,y,z)=xD(w,y,z)+F(x,y,z)w$ for all $x,y,z,w \in N$.

$(ii)\Rightarrow(i)$ This is proved in similar way.

Lemma 10. *Let N be a near ring, D a permuting tri-derivation of N, F a permuting tri-additive mapping of N and f the trace of F. Then for all $x,y,z,w,v \in N$,*

(i) $[F(x,y,z)v+xD(v,y,z)]w =$

$$F(x,y,z)vw+xD(v,y,z)w,$$

(ii) $[f(x)v + xD(x,x,v)]w = f(x)vw + xD(x,x,v)w,$

(iii) $[xD(v,y,z) + F(x,y,z)v]w =$

$xD(v,y,z)w + F(x,y,z)vw,$

(iv) $[xD(x,x,v) + f(x)v]w = .$

$xD(x,x,v)w + f(x)vw$

Proof. (i) Supposethat $F(xv,y,z) = F(x,y,z)v + xD(v,y,z)$ for all $x,y,z,v \in N$. From the asssociative law,

$$\begin{aligned} F((xv)w,y,z) &= F(xv,y,z)w + xvD(w,y,z) \\ &= [F(x,y,z)v + xD(v,y,z)]w \\ &\quad + xvD(w,y,z) \end{aligned}$$

and

$$\begin{aligned} F(x(vw),y,z) &= F(x,y,z)vw + xD(vw,y,z) \\ &= F(x,y,z)vw + xD(v,y,z)w \\ &\quad + xvD(w,y,z). \end{aligned}$$

Combining he above two equality, we find that

$$[F(x,y,z)v + xD(v,y,z)]w = F(x,y,z)vw + xD(v,y,z)w$$

(ii) Substituting x for y and z in (i), we obtain that

$$[f(x)v + xD(x,x,v)]w = f(x)vw + xD(x,x,v)w.$$

The proof of (iii) and (iv) are straight forward from Lemma 9.

Theorem 4. *Let N be a $2,3$-torsion free prime near ring, D a nonzero permuting tri-derivation and F a nonzero permuting tri-right generalized derivation of N associated with D. If $F(N,N,N) \subseteq Z$, then N is a commutative ring.*

Proof. Since $F(N,N,N) \subseteq Z$ and F is nonzero, there exist nonzero elements $x,y,z \in N$ such that $F(x,y,z) \in Z - \{0\}$.Then $F(x+x,y,z) = F(x,y,z) + F(x,y,z) \in Z$ and hence $(N,+)$ is abelian by Lemma 1. $F(x,y,z) \in Z$ for $x,y,z \in N$, implies that $F(x,y,z)w = wF(x,y,z)$ for all $w \in N$. Hence replace xv with x to get

$$(3.2) \quad F(x,y,z)vw + xD(v,y,z)w = wF(x,y,z)v + wxD(v,y,z)$$

Replacing x by $rf(x)$ in (3.2) and from (3.2), we have

$$(3.3) \quad rD(f(x),y,z)[v,w] = 0$$

for all $x,y,z,v,r,w \in N$.

Now, suppose that N is not commutative. In this case, N is a prime near ring and (3.3), we obtain

$$(3.4) \quad D(f(x),y,z) = 0$$

for all $x,y,z \in N$. Substituting $x+v$ for x in (3.4), since N is $2,3$-torsion free and from (3.4), we get that

$$(3.5) \quad D(F(x,v,v),y,z) = 0$$

Taking $v+r$ instead of v in (3.5) and from (3.5), we have

$$(3.6) \quad D(F(x,v,r),y,z) = 0$$

since N is 2-torsion free.

Taking xw instead of x in (3.6) and from (3.6), we have

$$(3.7) \quad F(x,v,r)D(w,y,z) + D(x,y,z)D(w,v,r) + xD(D(w,v,r),y,z) = 0$$

Substituting $f(r)$ for r in (3.7) and from (3.7), we get

$$(3.8) \quad F(f(r),x,v) = 0$$

Taking $r+s$, $s \in N$ instead of r in (3.8) and from (3.8), we obtain

$$(3.9) \quad F(F(r,r,s),x,v) = 0$$

since N is $2,3$-torsion free. Substituting $r+z$, $z \in N$ for r in (3.9) and from (3.9), we have

$$(3.10) \quad F(F(r,z,s),x,v) = 0$$

since N is 2-torsion free.

Replacing r by rw in (3.10) and from (3.10), we get for all $r,z,s,x,v,w \in N$,

$$\begin{aligned} (3.11) \quad & F(r,z,s)D(w,z,v) + F(r,x,v)D(w,z,s) \\ & + rD(D(w,z,s),x,v) = 0 \end{aligned}$$

Substituting $f(r)$ for r in (3.11) and from (3.8), we get

$$f(r)D(D(w,z,s),x,v) = 0.$$

Since N is a prime near ring and $F \neq 0$, we have

$$(3.12) \quad D(D(w,z,s),x,v) = 0$$

Taking wy instead of w in (3.12) and from (3.12), we get

$$(3.13) \quad D(w,z,s)D(y,x,v) + D(w,x,v)D(y,z,s) = 0$$

for all $x,y,s,z,w,v \in N$.

Replacing x,z,s,v by w in (3.13), since N is 2-torsion free prime near ring, we get

$$(3.14) \quad d(w)D(y,w,w) = 0$$

for all $y, w \in N$.

Taking yr instead of y in (3.14) and from (3.14), we get

$$d(w)yD(r,w,w)=0$$

and so

$$d(w)yd(w)=0$$

for all $y, w \in N$. Since N is prime near ring, we get $D=0$. But this is a contradiction. Consequently, N is commutative ring.

Theorem 5. *Let N be a $2,3$-torsion free prime near ring, D a nonzero permuting tri-derivation of N and F a nonzero permuting tri-right generalized derivation of N associated with D. If $f(x)$, $f(x)+f(x)\in C(D(y,z,w))$ for all $x,y,z,w\in N$, then $(N,+)$ abelian and $f(x)\in Z$ for all $x\in N$.*

Proof. Since $f(x)$, $f(x)+f(x)\in C(D(y,z,w))$ for all $x,y,z,w\in N$, if both u and $u+u$ commute elementwise with $D(y,z,w)$ for all $y,z,w\in N$, then

$$[D(y,z,w)+D(v,z,w)](u+u)$$
$$=D(y,z,w)u+D(y,z,w)u+D(v,z,w)u+D(v,z,w)u$$

and on the other hand

$$[D(y,z,w)+D(v,z,w)](u+u)$$
$$=D(y,z,w)u+D(v,z,w)u+D(y,z,w)u+D(v,z,w)u.$$

Comparing the two expression, we get

(3.15) $D((y,v),z,w)u=0$

Taking $f(t)$ instead of u in (3.15), we get $D((y,v),z,w)=0$ for all $y,v,z,w\in N$. Since $s(y,v)$ is also an additive-group commutator for any $s\in N$, replacing (y,v) by $s(y,v)$ in the last equation, we obtain

$$D(s,z,w)(y,v)=0$$

for all $s,z,w,y,v\in N$. Since N is a prime near ring and $D\neq 0$, we have $(y,v)=0$ for all $y,v\in N$, that is $(N,+)$ abelian.

Since $f(x)\in C(D(y,z,w))$, $f(x)D(y,z,w)=D(y,z,w)f(x)$, for all $x,y,z,w\in N$. Taking yv instead of y in this equation, we obtain that

(3.16) $f(x)D(y,z,w)v+f(x)yD(v,z,w)$
$=D(y,z,w)vf(x)+yD(v,z,w)f(x)$

Replacing y by $d(y)$ in (3.16) and from hypothesis, we get

(3.17) $D(d(y),z,w)[v,f(x)]=0$

for all $x,y,z,w,v\in N$.

Substituting zt for z in (3.17), we get

$$D(d(y),z,w)t[v,f(x)]=0.$$

Since N is a prime near ring, $D(d(y),z,w)=0$ or $[v,f(x)]=0$ for all $x,y,z,w,v\in N$.

Suppose that $D(d(y),z,w)=0$ for all $y,z,w\in N$. Taking $y+v$ instead of y in the last equation, we get

$$D(d(y),z,w)+D(d(v),z,w)+3D(D(y,y,v),z,w)+3D(D(y,v,v),z,w)=0.$$

Since $D(d(y),z,w)=0$, we get

(3.18) $D(D(y,y,v),z,w)+D(D(y,v,v),z,w)=0$

since N is 3-torsion free.

Replacing y by $-y$ in (3.18), we have

(3.19) $D(D(y,y,v),z,w)-D(D(y,v,v),z,w)=0.$

Combining (3.18) and (3.19), we obtain

(3.20) $D(D(y,v,v),z,w)=0,$

since N is 2-torsion free.

Replacing y by yx, $x\in N$ in (3.20) and from (3.20), we have

(3.21) $D(y,v,v)D(x,z,w)+D(y,z,w)D(x,v,v)=0$

Taking xt instead of x, $t\in N$ in (3.21), we get

$$D(y,v,v)xD(t,z,w)+D(y,z,w)xD(t,v,v)=0$$

Replacing v by t,z,w,y in the last equation we get $d(v)xd(v)=0$ for all $x,v\in N$. Since N is prime near ring $d(v)=0$, and so $D=0$. But, since $D\neq 0$, we get $f(x)\in Z$.

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