



Soft Semi- I -Continuous Functions

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ABSTRACT

In this paper, we introduce and study the concept of soft semi- I -open sets with respect to an ideal. Then using these sets, we also study soft semi- I -continuous functions, soft semi- I -open functions and obtained some characterizations and several properties concerning these functions.

Keywords: Soft sets, soft ideal, soft functions, soft topology

1. INTRODUCTION

Molodtsov [1] introduced the concept of soft set theory in 1999. He applied the soft theory in several directions, such as game theory, probability, Riemann integration, operation research, etc. In recent years the development in the fields of soft set theory and its applications has been taking place in a rapid pace. The operation of the soft sets are redefined decision making method was constructed by using these new operations [2]. Topological structures of the soft set theory were first studied by Shabir and Naz [3]. They introduced and studied some concepts such as soft topological space, soft closed and soft open set, soft interior, soft closure and soft subspace, etc. Kharal and Ahmad [4] defined the notion of soft mappings on soft classes. Then Aygünoğlu and Aygün [5] introduced soft continuity of soft mappings, soft product topology and studied soft compactness. Nazmul and Samanta [6] studied the neighbourhood properties in a soft topological space.

The topic of ideals in general topological spaces is treated in the classic text by Kuratowski [7]. This topic has an excellent potential for applications in other branches of mathematics. This subject was continued to study by general topologists in recent years [7, 8, 9, 10]. In 1990, Jankovic and Hamlett [11] introduced another topology $\tau^*(I)$ by using a given topology τ , which satisfies $\tau \subset \tau^*(I)$. In 1952, Hashimoto [12, 13] introduced the notion of ideal continuity. Then Jankovic and Hamlett [8] defined I -open set in ideal topological spaces. Later, Abd El-Monsef [14] studied I -continuity for functions. Then Hatır and Noiri [15] introduced the semi- I -open set and semi- I -continuity in 2002. Kale and Guler [16] gave the definition of soft ideal and studied the properties the soft ideal topological space. Moreover they introduced the notion of soft I -regularity and soft I -normality. Then, Akdağ and Erol [23] defined soft I -open sets and soft I -continuity of functions in 2014.

The purpose of this paper is to define soft semi- I -open set, soft semi- I -continuity, soft semi- I -open function and we studied the several properties of these concepts.

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2. PRELIMINARIES

Throughout this paper, X will be a nonempty initial universal set and A will be a set of parameters. Let $P(X)$ denote the power set of X and $S(X)$ denote the set of all soft sets over X .

Definition 1. [1] A pair (F, A) is called a soft set over X , where F is a mapping from A to $P(X)$.

Definition 2. [17] Let (F_1, A) and (F_2, A) be soft sets over a common universe X . Then (F_1, A) is said to be a soft subset of (F_2, A) if $F_1(\alpha) \subset F_2(\alpha)$, for all $\alpha \in A$ and this relation is denoted by $(F_1, A) \tilde{\subset} (F_2, A)$. Also, (F_1, A) is said to be a soft equal to (F_2, A) if $F_1(\alpha) = F_2(\alpha)$, for all $\alpha \in A$ and this relation is denoted by $(F_1, A) = (F_2, A)$.

Definition 3. [18] A soft set (F, A) over X is said to be a null soft set if $F(\alpha) = \emptyset$ for all $\alpha \in A$ and this denoted by $\tilde{\emptyset}$. Also, (F, A) is said to be an absolute soft set if $F(\alpha) = X$, for all $\alpha \in A$ and this denoted by \tilde{X} .

Definition 4. [19] The complement of a soft set (F, A) is defined as $(F, A)^c = (F^c, A)$, where $F^c(\alpha) = (F(\alpha))^c = X - F(\alpha)$, for all $\alpha \in A$. Clearly, we have $(\tilde{\emptyset})^c = \tilde{X}$ and $(\tilde{X})^c = \tilde{\emptyset}$.

Definition 5. [3] The difference of two soft sets (F_1, A) and (F_2, A) is defined by $(F_1, A) - (F_2, A) = (F_1 - F_2, A)$, where $(F_1 - F_2)(\alpha) = F_1(\alpha) - F_2(\alpha)$, for all $\alpha \in A$.

Definition 6. [20] Let $\{(F_j, A): j \in J\}$ be a nonempty family of soft sets over a common universe X . The intersection of these soft sets denoted by $\cap_{j \in J}$, is defined by $\cap_{j \in J} (F_j, A) = (\cap_{j \in J} F_j, A)$, where $(\cap_{j \in J} F_j)(\alpha) = \cap_{j \in J} (F_j(\alpha))$, for all $\alpha \in A$. The union of these soft sets denoted by $\cup_{j \in J}$, is defined by $\cup_{j \in J} (F_j, A) = (\cup_{j \in J} F_j, A)$, where $(\cup_{j \in J} F_j)(\alpha) = \cup_{j \in J} (F_j(\alpha))$ for all $\alpha \in A$.

Definition 7. [6] A soft set (E, A) over X is said to be a soft element if $\exists \alpha \in A, \beta \neq \alpha$ such that $E(\alpha) = \{x\}$ and $E(\beta) = \emptyset$, for all $\beta \in A$. Such a soft element is denoted by E_α^x . The soft element E_α^x is said to be in the soft set (G, A) if $x \in G(\alpha)$ and denoted by $E_\alpha^x \tilde{\in} (G, A)$.

Definition 8. [3] Let τ be the collection of soft sets over X . Then τ is said to be a soft topology on X if,

- (i) $\tilde{\emptyset}, \tilde{X} \in \tau$.
- (ii) the intersection of any two soft sets in τ belongs to τ ,
- (iii) the union of any number of soft sets in τ belongs to τ .

The triple (X, τ, E) is called a soft topological space over X . The members of τ are said to be τ -soft open sets (simply, soft open set in X). A soft set over X is said to be soft closed in X if its complement belongs to τ .

Definition 9. [3] Let (F, A) be a soft set over X and Y be a nonempty subset of X . Then the soft subset of (F, A) over Y denoted by $({}^Y F, A)$ is defined as $({}^Y F, A) = (Y \cap F, A)$, for each $\alpha \in A$. In other word $({}^Y F, A) = \tilde{Y} \tilde{\cap} (F, A)$.

Definition 10. [3] Let (X, τ, E) be a soft topological space over X and Y be a nonempty subset of X . Then $\tau_Y = \{({}^Y F, A) \mid (F, A) \in \tau\}$ is said to be the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E) .

Definition 11. [3] Let (X, τ, E) be a soft topological space over X and (F, A) be a soft set over X . The soft closure of (F, A) denoted by $Cl(F, A)$ is the intersection of all closed soft super sets of (F, A) . The soft interior of (F, A) denoted by $Int(F, A)$ is the union of all open soft subsets of (F, A) .

Definition 12. [6] Let (X, τ) be a soft topological space over X . A soft set (F, A) is said to be a neighbourhood of the soft set (H, A) if there exist a soft set $(G, A) \in \tau$ such that $(H, A) \tilde{\subset} (G, A) \tilde{\subset} (F, A)$. If $(H, A) = E_\alpha^x$, then (F, A) is said to be a soft neighbourhood of the soft element E_α^x . The soft neighbourhood system of soft element E_α^x , denoted by $N(E_\alpha^x)$, is the family of all its soft neighbourhood. The soft open neighbourhood system of soft element E_α^x , denoted by $V(E_\alpha^x)$, is the family of all its soft open neighbourhood.

Lemma 1. [6] A soft element $E_\alpha^x \tilde{\in} Cl(F, A)$ if and only if each soft neighbourhood of E_α^x intersects with (F, A) .

Definition 13. [21] Let (X, τ, E) be a soft topological space over X , (G, A) be a soft closed set in X and E_α^x be a soft point such that $E_\alpha^x \tilde{\in} (G, A)$. If there exist soft open sets (F_1, A) and (F_2, A) such that $E_\alpha^x \tilde{\in} (F_1, A)$, $(G, A) \tilde{\subset} (F_2, A)$ and $(F_1, A) \tilde{\cap} (F_2, A) = \tilde{\emptyset}$, then (X, τ) is called a soft regular space.

Definition 14. [21] Let (X, τ, E) be a soft topological space over X , and let (G_1, A) and (G_2, A) be two disjoint soft closed sets. If there exist two soft open sets (F_1, A) and (F_2, A) such that $(G_1, A) \tilde{\subset} (F_1, A)$, $(G_2, A) \tilde{\subset} (F_2, A)$ and $(F_1, A) \tilde{\cap} (F_2, A) = \tilde{\emptyset}$, then (X, τ) is called a soft normal space.

Definition 15. [15] A soft ideal I is a nonempty collection of soft sets over X if;

- (i) $(F, A) \tilde{\in} I, (G, A) \tilde{\subset} (F, A)$ implies $(G, A) \tilde{\in} I$.
- (ii) $(F, A) \tilde{\in} I, (G, A) \tilde{\in} I$ implies $(F, A) \tilde{\cup} (G, A) \tilde{\in} I$.

A soft topological space (X, τ, E, I) with a soft ideal I called soft ideal topological space and denoted by (X, τ, E, I) .

Definition 16. [15] Let (F, A) be a soft set in a soft ideal topological space (X, τ, E, I) and $(.)^*$ be a soft operator from $S(X)$ to $S(X)$. Then the soft local mapping of (F, A) defined by $(F, A)^*(I, \tau) = \{E_\alpha^X: (U, A) \tilde{\cap} (F, A) \tilde{\in} I \text{ for every } (U, A) \tilde{\in} V(E_\alpha^X)\}$ denoted by $(F, A)^*$ simply. Also, the soft set operator Cl^* is called a soft $*$ -closure and is defined as $Cl^*(F, A) = (F, A) \tilde{\cup} (F, A)^*$ for a soft subset (F, A) .

Lemma 2. [15] Let (X, τ, E, I) be a soft ideal topological space and $(F, A), (G, A)$ be two soft sets. Then the following are hold:

- (i) $(F, A) \tilde{\subset} (G, A)$ implies $(F, A)^* \tilde{\subset} (G, A)^*$ and $((F, A) \tilde{\cup} (G, A))^* = (F, A)^* \tilde{\cup} (G, A)^*$.
- (ii) $(F, A)^* \tilde{\subset} Cl(F, A)$ and $((F, A)^*)^* \tilde{\subset} (F, A)^*$.
- (iii) (F, A) is soft open and $(F, A) \tilde{\cap} (G, A) \in I$ implies $(F, A) \tilde{\cap} (G, A)^* = \tilde{\emptyset}$.
- (iv) $(F, A)^*$ is soft closed.
- (v) If (F, A) is soft closed then $(F, A)^* \tilde{\subset} (F, A)$.

Proposition 1. [15] Let (X, τ, E, I) be a soft ideal topological space and $(F, A), (G, A)$ be two soft sets. Then the following are hold:

- (i) $Cl^*(\tilde{\emptyset}) = \tilde{\emptyset}$ and $Cl^*(\tilde{X}) = \tilde{X}$.
- (ii) $(F, A) \tilde{\subset} Cl^*(F, A)$ and $Cl^*(Cl^*(F, A)) = Cl^*(F, A)$.
- (iii) If $(F, A) \tilde{\subset} (G, A)$ then $Cl^*(F, A) \tilde{\subset} Cl^*(G, A)$.
- (iv) $Cl^*(F, A) \tilde{\cup} Cl^*(G, A) = Cl^*((F, A) \tilde{\cup} (G, A))$.

Definition 17. [4] Let (X, τ, E, I) be a soft ideal topological space, (Y, ϑ, K) be a soft topological space, $p: E \rightarrow K$ and $u: X \rightarrow Y$ be mappings. Then the mapping $f_{pu}: (X, \tau, E, I) \rightarrow (Y, \vartheta, K)$ is defined as follows:

- (i) The image a soft set (F, A) in (X, E) , written by $(f_{pu}(F, A), B)$, where $B = p(A) \subseteq K$ is a soft set in (Y, K) and $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set of (Y, ϑ, B) such that $f_{pu}(F, A)(\beta) = u(\cup F(\alpha))$ for $\beta \in K$. If $B = K$, then we will write $(f_{pu}(F, A), K)$ as (F, A) .
- (ii) The inverse image a soft set (G, C) in (Y, E) , written by $(f_{pu}^{-1}(G, C), D)$, where $D = p^{-1}(C)$ and $f_{pu}^{-1}(G, C)(\alpha) = u^{-1}(G(p(\alpha)))$ for $\alpha \in D \subseteq E$. Here after we will write $(f_{pu}^{-1}(G, C), E)$ as $f_{pu}^{-1}(G, C)$.

Proposition 2. [4] Let $f_{pu}: (X, \tau, E, I) \rightarrow (Y, \vartheta, K)$, $p: E \rightarrow K$ and $u: X \rightarrow Y$ be mappings. Let $(F, A), (G, B)$ are soft sets and (F_i, A_i) be family of soft sets in the soft class (X, E) ; $(H, C), (T, D)$ are soft sets and (H_i, C_i) be family of soft sets in the soft class (Y, K) , then the follows are hold:

- (1) $f_{pu}(\tilde{\emptyset}) = \tilde{\emptyset}$.
- (2) $f_{pu}(\tilde{X}) = \tilde{Y}$.
- (3) $f_{pu}((F, A) \tilde{\cup} (G, B)) = f_{pu}(F, A) \tilde{\cup} f_{pu}(G, B)$ and in general $f_{pu}(\cup_i (F_i, A_i)) = \cup_i (f_{pu}(F_i, A_i))$.
- (4) $f_{pu}((F, A) \tilde{\cap} (G, B)) \tilde{\subset} f_{pu}(F, A) \tilde{\cap} f_{pu}(G, B)$ and in general $f_{pu}(\cap_i (F_i, A_i)) \tilde{\subset} \cap_i (f_{pu}(F_i, A_i))$.
- (5) If $(F, A) \tilde{\subset} (G, B)$ then $f_{pu}(F, A) \tilde{\subset} f_{pu}(G, B)$.
- (6) $f_{pu}^{-1}(\tilde{\emptyset}) = \tilde{\emptyset}$.
- (7) $f_{pu}^{-1}(\tilde{Y}) = \tilde{X}$.
- (8) $f_{pu}^{-1}((H, C) \tilde{\cup} (T, D)) = f_{pu}^{-1}(H, C) \tilde{\cup} f_{pu}^{-1}(T, D)$ and in general $f_{pu}^{-1}(\cup_i (H_i, C_i)) = \cup_i (f_{pu}^{-1}(H_i, C_i))$.
- (9) $f_{pu}^{-1}((H, C) \tilde{\cap} (T, D)) = f_{pu}^{-1}(H, C) \tilde{\cap} f_{pu}^{-1}(T, D)$ and in general $f_{pu}^{-1}(\cap_i (H_i, C_i)) = \cap_i (f_{pu}^{-1}(H_i, C_i))$.
- (10) If $(H, C) \tilde{\subset} (T, D)$ then $f_{pu}^{-1}(H, C) \tilde{\subset} f_{pu}^{-1}(T, D)$.

3. SOFT SEMI-I-OPEN SETS

Definition 18. A subset S of an ideal topological space (X, τ, E, I) is said to be

- (i) [9] I -open if $S \subset Int(S^*)$
- (ii) [10] pre- I -open if $S \subset Int(Cl^*(S))$
- (iii) [22] semi- I -open if $S \subset Cl^*(Int(S))$.

Definition 19. A soft subset (F, A) of an soft ideal topological space (X, τ, E, I) is said soft semi- I -open if $(F, A) \tilde{\subset} Cl^*(Int(F, A))$.

By $SSIO(X, \tau, E, I)$ we denote the family of all soft semi- I -open sets of a soft topological space (X, τ, E, I) .

Conclusion 1. A soft subset (F, A) of an soft ideal topological space (X, τ, E, I) is soft semi- I -open if and only if $Cl^*(F, A) = Cl^*(Int(F, A))$.

Proof. Let (F, A) be soft-semi- I -open set, we have $(F, A) \subseteq Cl^*(Int(F, A))$. Then $Cl^*(F, A) \subseteq Cl^*(Cl^*(Int(F, A))) = Cl^*(Int(F, A))$, hence $Cl^*(F, A) \subseteq Cl^*(Int(F, A))$. Also, $Cl^*(Int(F, A)) \subseteq Cl^*(F, A)$. Therefore $Cl^*(Int(F, A)) = Cl^*(F, A)$. The converse is obvious.

Definition 20. [23] A soft subset (F, A) of an soft ideal topological space (X, τ, E, I) is said soft I -open if $(F, A) \subseteq Int(F, A)^*$.

Conclusion 2. Every soft I -open set is soft semi- I -open but the inverse is not true in general.

Theorem 1. A soft subset (F, A) of an soft ideal topological space (X, τ, E, I) is soft semi- I -open if and only if there exists $(G, A) \in \tau$ such that $(G, A) \subseteq (F, A) \subseteq Cl^*(G, A)$.

Proof. Let (F, A) be soft semi- I -open set, then $(F, A) \subseteq Cl^*(Int(F, A))$. Take $Int(F, A) = (G, A)$, then we have $(G, A) \subseteq (F, A) \subseteq Cl^*(G, A)$.

Conversely, let $(G, A) \subseteq (F, A) \subseteq Cl^*(G, A)$ for some $(G, A) \in \tau$. Since $(G, A) \subseteq (F, A)$ we have $(G, A) = Int(G, A) \subseteq Int(F, A)$ and hence $Cl^*(G, A) \subseteq Cl^*(Int(F, A))$. Thus we obtain $(F, A) \subseteq Cl^*(Int(F, A))$. This shows that (F, A) is soft-semi- I -open set.

Theorem 2. If (F, A) is a soft semi- I -open set in a soft ideal topological space (X, τ, E, I) and $(F, A) \subseteq (G, A) \subseteq Cl^*(F, A)$ then (G, A) soft semi- I -open set in (X, τ, E, I) .

Proof. Since (F, A) is soft semi- I -open set, there exists a soft open set (H, A) such that $(H, A) \subseteq (F, A) \subseteq Cl^*(H, A)$. Then we have $(H, A) \subseteq (F, A) \subseteq (G, A) \subseteq Cl^*(F, A) \subseteq Cl^*(Cl^*(H, A)) = Cl^*(H, A)$ and hence $(H, A) \subseteq (G, A) \subseteq Cl^*(H, A)$. By Theorem 1, we obtain $(G, A) \in SSIO(X, \tau, E, I)$.

Lemma 3. Let (F, A) be a soft subset of an soft ideal topological space (X, τ, E, I) . If $(H, A) \in \tau$, then $(H, A) \cap (F, A)^* \subseteq ((H, A) \cap (F, A))^*$.

Proof. Let $E_\alpha^x \in (H, A) \cap (F, A)^*$, then $E_\alpha^x \in (H, A)$, $(H, A) \in \tau$ and $E_\alpha^x \in (F, A)^*$. Then $(U, A) \cap (F, A) \notin I$, for all $(U, A) \in \nu(E_\alpha^x)$. Since $(H, A) \in \nu(E_\alpha^x)$ then $(H, A) \cap (F, A) \notin I$. Thus $(H, A) \cap ((U, A) \cap (F, A)) \notin I$, for all $(U, A) \in \nu(E_\alpha^x)$. Hence, $E_\alpha^x \in ((H, A) \cap (F, A))^*$. This shows that $(H, A) \cap (F, A)^* \subseteq ((H, A) \cap (F, A))^*$.

Theorem 3. Let (X, τ, E, I) be a soft ideal topological space, $(F, A), (G, A)$ be soft subset of X and $\{(F_i, A_i) : i \in \Delta\}$ be a soft family of X .

- (1) if $(F_i, A_i) \in SSIO(X, \tau, E, I)$ for each $i \in \Delta$, then $\cup \{(F_i, A_i) : i \in \Delta\} \in SSIO(X, \tau, E, I)$.
- (2) if $(F, A) \in SSIO(X, \tau, E, I)$ and $(G, A) \in \tau$, then $(F, A) \cap (G, A) \in SSIO(X, \tau, E, I)$.

Proof. (1) Since $(F_i, A_i) \in SSIO(X, \tau, E, I)$, we have $(F_i, A_i) \subseteq Cl^*(Int(F_i, A_i))$ for each $i \in \Delta$. Thus by using Lemma 3, we obtain $\cup (F_i, A_i) \subseteq \cup Cl^*(Int(F_i, A_i)) \subseteq \cup \{(Int(F_i, A_i))^* \cup (Int((F_i, A_i)))\} \subseteq (\cup Int(F_i, A_i))^* \cup Int(\cup (F_i, A_i)) \subseteq (Int \cup (F_i, A_i))^* \cup Int(\cup (F_i, A_i)) = Cl^*(Int(\cup (F_i, A_i)))$. This shows that $\cup \{(F_i, A_i) : i \in \Delta\} \in SSIO(X, \tau, E, I)$.
 (2) Let $(F, A) \in SSIO(X, \tau, E, I)$ and $(G, A) \in \tau$. Then $(F, A) \subseteq Cl^*(Int(F, A))$ and by using Lemma 3, we obtain $(F, A) \cap (G, A) \subseteq Cl^*(Int((F, A) \cap (G, A))) = ((Int(F, A))^* \cup Int(F, A)) \cap (G, A) = ((Int(F, A))^* \cap (G, A)) \cup (Int(F, A) \cap (G, A)) \subseteq (Int(F, A) \cap (G, A))^* \cup Int((F, A) \cap (G, A)) = Int((F, A) \cap (G, A))^* \cup Int((F, A) \cap (G, A)) = Cl^*(Int((F, A) \cap (G, A)))$. This shows that $(F, A) \cap (G, A) \in SSIO(X, \tau, E, I)$.

Definition 21. A soft subset (F, A) of a soft ideal topological space (X, τ, E, I) is said to be soft semi- I -closed if its complement is soft semi- I -open.

By $SSIC(X, \tau, E, I)$ we denote the family of all soft semi- I -closed sets of a soft topological space (X, τ, E, I) .

Remark 1. For a soft subset (F, A) of a soft ideal topological space (X, τ, E, I) we have $\tilde{X} - Int(Cl^*(F, A)) \neq Cl^*(Int(\tilde{X} - (F, A)))$.

Lemma 4. If a soft subset (F, A) of a soft ideal topological space (X, τ, E, I) is soft semi- I -closed, then $Int(Cl^*(F, A)) \subseteq (F, A)$.

Proof. Since (F, A) is soft semi- I -closed, then $\tilde{X} - (F, A) \in SSIO(X, \tau, E, I)$. Since $\tau^*(I)$ is finer than τ , we have $\tilde{X} - (F, A) \subseteq Cl^*(Int(\tilde{X} - (F, A))) \subseteq Cl(Int(\tilde{X} - (F, A))) = \tilde{X} - Int(Cl(F, A)) \subseteq \tilde{X} - Int(Cl^*(F, A))$. Therefore we obtain $Int(Cl^*(F, A)) \subseteq (F, A)$.

Conclusion 3. Let (F, A) be a soft subset of a soft ideal topological space (X, τ, E, I) such that $\tilde{X} - Int(Cl^*(F, A)) = Cl^*(Int(\tilde{X} - (F, A)))$. Then (F, A) is soft semi- I -closed if and only if $Int(Cl^*(F, A)) \subseteq (F, A)$.

4. SOFT SEMI-I-CONTINUOUS FUNCTIONS

Definition 22. A function $f: (X, \tau, E, I) \rightarrow (Y, \sigma, K)$ is said to be soft semi-I-continuous if $f^{-1}(F, A)$ is soft semi-I-open in (X, τ, E, I) for each soft open set (F, A) of (Y, σ, K) .

Remark 2. It is obvious that soft continuity implies soft semi-I-continuity and soft semi-I-continuity implies soft semi-continuity. But these converse is not true in general.

Example 1. Let a soft ideal topological space (X, τ, E, I) and a soft topological space (Y, ϑ, K) given as follows:
 $X = \{h_1, h_2\}, E = \{e_1, e_2\}, \tau = \{\tilde{\emptyset}, \tilde{X}, \{(e_1, \{h_1\})\}, \{(e_2, \{h_2\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\},$
 $I = \{\tilde{\emptyset}, \{(e_1, \{h_1\})\}, \{(e_1, \{h_2\})\}, \{(e_1, X)\}\}, Y = \{y_1, y_2\}, K = \{k_1, k_2\}, \vartheta = \{\tilde{\emptyset}, \tilde{Y}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}\}$. Also, let $u: X \rightarrow Y, u(h_1) = y_1, u(h_2) = y_2, p: E \rightarrow K, p(e_1) = k_2, p(e_2) = k_1$. Then the soft function $f_{up}: (X, \tau, E, I) \rightarrow (Y, \vartheta, K)$ is soft semi-I-continuous but is not soft continuous.

Example 2. Let a soft ideal topological space (X, τ, E, I) and a soft topological space (Y, ϑ, K) given as follows:
 $X = \{h_1, h_2\}, E = \{e_1, e_2\}, \tau = \{\tilde{\emptyset}, \tilde{X}, \{(e_1, \{h_1\})\}, \{(e_2, \{h_2\})\}, \{(e_1, \{h_1\}), (e_2, \{h_2\})\}\},$
 $I = \{\tilde{\emptyset}, \{(e_1, \{h_1\})\}, \{(e_1, \{h_2\})\}, \{(e_1, X)\}\}, Y = \{y_1, y_2\}, K = \{k_1, k_2\}, \vartheta = \{\tilde{\emptyset}, \tilde{Y}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}\}$. Also, let $u: X \rightarrow Y, u(h_1) = y_1, u(h_2) = y_2, p: E \rightarrow K, p(e_1) = k_2, p(e_2) = k_1$. Then the soft function $f_{up}: (X, \tau, E, I) \rightarrow (Y, \vartheta, K)$ is soft semi-continuous but is not soft semi-I-continuous.

Theorem 4. For a function $f: (X, \tau, E, I) \rightarrow (Y, \sigma, K)$ the following are equivalent:

- (1) f is soft semi-I-continuous
- (2) for each $E_\alpha^x \in (X, A)$ and each $(V, B) \in \sigma$ containing $f(E_\alpha^x)$, there exists $(U, A) \in SSIO(X, \tau, E, I)$ containing E_α^x such that $f(U, A) \tilde{\subseteq} (V, B)$.
- (3) the inverse image of each soft closed set in Y is soft semi-I-closed.

Proof. (1) \Leftrightarrow (3) Let (F, B) be a soft closed subset in Y . Then $\tilde{Y} - (F, B)$ is soft open in Y . Since f is soft semi-I-continuous, then $f^{-1}(\tilde{Y} - (F, B)) = \tilde{X} - f^{-1}(F, B)$ is soft I-open in X . Thus $f^{-1}(F, B)$ is soft I-closed in X .

(1) \Rightarrow (2) For each $E_\alpha^x \in \tilde{X}$ and each $(V, B) \in \sigma$ containing $f(E_\alpha^x)$ then $E_\alpha^x \in f^{-1}(V, B)$ and $f^{-1}(V, B) \in SSIO(X, \tau, E, I)$. If we take $(U, A) = f^{-1}(V, B)$ then we obtain $(U, A) \in SSIO(X, \tau, E, I)$ containing E_α^x and $f(U, A) \subset (V, B)$. The implication (2) \Rightarrow (1) follows from Theorem 3.

Definition 23. Let (X, τ, E, I) be a soft ideal topological space and (G, A) be a soft set of X . The soft semi-I-closure of (G, A) is a soft set $SSCl^*(G, A) = \{ \cap (F, A) : (G, A) \subset (F, A) \text{ and } (F, A) \in SSIC(X, \tau, E, I) \}$.

Theorem 5. $f: (X, \tau, E, I) \rightarrow (Y, \sigma, K)$ is soft semi-I-continuous function if and only if $f(SSCl^*(F, A)) \tilde{\subseteq} Cl(f(F, A))$ for every soft set (F, A) of X .

Proof. Let f is soft semi-I-continuous. $Cl(f(F, A))$ is soft closed set, so by soft semi-I-continuity, $f^{-1}(Cl(f(F, A)))$ is soft semi-I-closed set and $(F, A) \tilde{\subseteq} f^{-1}(Cl(f(F, A)))$. But $SSCl^*(F, A)$ is the smallest semi-I-closed set containing (F, A) , hence $SSCl^*(F, A) \tilde{\subseteq} f^{-1}(Cl(f(F, A)))$. Thus, $f(SSCl^*(F, A)) \tilde{\subseteq} Cl(f(F, A))$.

Conversely, let (F, B) be any soft closed set, then $f^{-1}(F, B)$ a soft set in X , and by hypothesis $f(SSCl^*(f^{-1}(F, B))) \tilde{\subseteq} Cl(f(f^{-1}(F, B)))$. Then $f(SSCl^*(f^{-1}(F, B))) \tilde{\subseteq} Cl(F, B) = (F, B)$. Thus $SSCl^*(f^{-1}(F, B)) \tilde{\subseteq} f^{-1}(F, B)$. Hence $f^{-1}(F, B)$ is semi-I-closed set. This shows that f is soft semi-I-continuous.

Lemma 6. Let (X, τ, E, I) be a soft ideal topological space. If $(U, A) \in \tau$ and $(G, A) \in SSIO(X, \tau)$, then $(U, A) \tilde{\cap} (G, A) \in SSIO((U, A), \tau \upharpoonright_{(U, A)}, I \upharpoonright_{(U, A)})$

Proof. The proof can be easily.

Theorem 6. Let $f: (X, \tau, E, I) \rightarrow (Y, \sigma, K)$ be soft semi-I-continuous function and $(U, A) \in \tau$. Then the restriction $(f \upharpoonright_{(U, A)}): ((U, A), \tau \upharpoonright_{(U, A)}, I \upharpoonright_{(U, A)}) \rightarrow (Y, \sigma, K)$ is soft semi-I-continuous.

Proof. Let (V, B) be any soft open set of (Y, σ, K) . Since f is soft semi-I-continuous function, then $f^{-1}(V, B) \in SSIO(X, \tau, E, I)$ and by Lemma 6, $(f \upharpoonright_{(U, A)})^{-1}(V, B) = f^{-1}(V, B) \tilde{\cap} (U, A) \in SSIO((U, A), \tau \upharpoonright_{(U, A)})$. This shows that $f \upharpoonright_{(U, A)}$ is soft semi-I-continuous.

Definition 24. A function $f: (X, \tau, E, I) \rightarrow (Y, \sigma, K, J)$ is said to be soft I-irresolute if $f^{-1}(F, A)$ is soft semi-I-open in (X, τ, E, I) for each soft semi-J-open set (F, A) of (Y, σ, K, J) .

Theorem 7. Let $f: (X, \tau, E, I) \rightarrow (Y, \sigma, K, J)$ and $g: (Y, \sigma, K, J) \rightarrow (Z, \eta, C)$ are soft continuous functions. Then the following are hold:

- (1) if f is soft semi-I-continuous and g is soft continuous then gof is soft semi-I-continuous.
- (2) if f is soft I-irresolute and g is soft semi-I-continuous then gof is soft semi-I-continuous.

Proof. (1) Let (H, C) be a soft open subset of Z . Since g is soft continuous then $g^{-1}(H, C)$ is soft open in Y . Since f is soft semi- I -continuous then $f^{-1}(g^{-1}(H, C)) = (gof)^{-1}(H, C)$ is soft semi- I -open in X . Thus gof is soft semi- I -continuous.

(2) Let (H, C) be a soft open subset of Z . Since g is soft semi- I -continuous then $g^{-1}(H, C)$ is soft semi- I -open set in Y . Since f is soft I -irresolute then $f^{-1}(g^{-1}(H, C)) = (gof)^{-1}(H, C)$ is soft semi- I -open in X . Thus gof is soft semi- I -continuous.

Theorem 8. A soft function $f: (X, \tau, E, I) \rightarrow (Y, \sigma, K, J)$ is soft semi- I -continuous if and only if the graph soft function $g: X \rightarrow X \times Y$, defined by $g(E_{\alpha}^x) = (E_{\alpha}^x, f(E_{\alpha}^x))$ for each $E_{\alpha}^x \in \tilde{X}$, is soft semi- I -continuous.

Proof. (Necessity) Suppose that f is soft semi- I -continuous. Let $E_{\alpha}^x \in \tilde{X}$ and (W, A) be any soft open set of $X \times Y$ containing $g(E_{\alpha}^x)$. Then there exists a basic soft open set $(U, A) \times (V, A)$ such that $g(E_{\alpha}^x) = (E_{\alpha}^x, f(E_{\alpha}^x)) \in (U, A) \times (V, A) \subseteq (W, A)$. Since f is soft semi- I -continuous, there exists a soft semi- I -open set (U_0, A_0) of X containing E_{α}^x such that $f(U_0, A_0) \subseteq (V, A)$. By Theorem 3, $(U_0, A_0) \tilde{\cap} (U, A) \in SSIO(X, \tau, E, I)$ and $g((U_0, A_0) \tilde{\cap} (U, A)) \subseteq (U, A) \times (V, A) \subseteq (W, A)$. This shows that g is soft semi- I -continuous.

(Sufficiency) Suppose that g is soft semi- I -continuous. Let $E_{\alpha}^x \in \tilde{X}$ and (V, A) be any soft open set of Y containing $f(E_{\alpha}^x)$. Then $\tilde{X} \times (V, A)$ is open in $X \times Y$. Since g is soft semi- I -continuous, there exists $(U, A) \in SSIO(X, \tau, E, I)$ containing E_{α}^x such that $g(U, A) \subseteq \tilde{X} \times (V, A)$. Therefore, we obtain $f(U, A) \subseteq (V, A)$. This shows that f is soft semi- I -continuous.

Theorem 9. Let $f: (X, \tau, E, I) \rightarrow (Y, \sigma, K)$, be soft semi- I -continuous function and $f^{-1}((V, B)^*) \subseteq (f^{-1}(V, B))^*$ for each $(V, B) \in \sigma$. Then f is soft I -irresolute.

Proof Let (H, B) be any soft semi- J -open set of (Y, σ, K, J) . By Theorem 1, there exists $(V, B) \in \sigma$ such that $(V, B) \subseteq (H, B) \subseteq Cl^*(V, B)$. Therefore, we have $f^{-1}(V, B) \subseteq f^{-1}(H, B) \subseteq f^{-1}(Cl^*(V, B)) \subseteq (Cl^*(f^{-1}(V, B)))$. Since f is soft semi- I -continuous and $(V, B) \in \sigma$, then $f^{-1}(V, B) \in SSIO(X, \tau, E, I)$ and hence by Theorem 2, $f^{-1}(H, B)$ soft semi- I -open set in (X, τ, E, I) . This shows that f is soft I -irresolute.

Lemma 6. For any soft function $f: (X, \tau, E, I) \rightarrow (Y, \sigma, K)$, $f(I)$ is a soft ideal on Y .

Proof. i. Let $f(F, A) \in f(I)$ and $f(G, A) \subseteq f(F, A)$. Then $(F, A) \in I$ and $(G, A) \subseteq (F, A)$. Since I is soft ideal then $(G, A) \in I$. Thus $f(G, A) \in f(I)$.

ii. Let $f(F, A) \in f(I)$ and $f(G, A) \in f(I)$. Then $(F, A) \in I$ and $(G, A) \in I$. Since I is soft ideal, then $(F, A) \tilde{\cup} (G, A) \in I$. Thus $f((F, A) \tilde{\cup} (G, A)) = f(F, A) \tilde{\cup} f(G, A) \in f(I)$. Therefore $f(I)$ is soft ideal.

An soft ideal topological space (X, τ, E, I) is said to be soft I -compact if for every soft open cover $\{(W_i, A_i): i \in \Delta\}$ of X , there exists a finite subset Δ_0 of Δ such that $\tilde{X} - \sqcup \{(W_i, A_i): i \in \Delta_0\} \in I$.

Definition 25. An soft ideal topological space (X, τ, E, I) is said to be soft semi- I -compact if for every soft semi- I -open cover $\{(W_i, A_i): i \in \Delta\}$ of X , there exists a finite subset Δ_0 of Δ such that $\tilde{X} - \sqcup \{(W_i, A_i): i \in \Delta_0\} \in I$.

Theorem 10. The image of a soft semi- I -compact space under a semi- I -continuous surjective function is $f(I)$ -compact.

Proof. Let $f: (X, \tau, E, I) \rightarrow (Y, \sigma, K)$ be a soft semi- I -continuous surjection function and $\{(W_i, A_i): i \in \Delta\}$ an open cover of Y . Then $f^{-1}\{(W_i, A_i): i \in \Delta\}$ is a semi- I -open cover of X . By the hypothesis, there exists a finite subset Δ_0 of Δ such that $\tilde{X} - \sqcup \{f^{-1}(W_i, A_i): i \in \Delta_0\} \in I$. Therefore, $f(\tilde{X} - \sqcup \{f^{-1}(W_i, A_i): i \in \Delta_0\}) = \tilde{Y} - \sqcup \{(W_i, A_i): i \in \Delta_0\} \in f(I)$ which shows that is $f(I)$ -compact.

5. SOFT SEMI- I -OPEN AND SOFT SEMI- I -CLOSED FUNCTIONS

Definition 26. A soft function $f: (X, \tau, E) \rightarrow (Y, \sigma, K, J)$ is called soft semi- I -open if $f(U, A) \in SSIO(Y, \sigma, K, J)$ for each soft open set (U, A) in X .

Definition 27. A soft function $f: (X, \tau, E) \rightarrow (Y, \sigma, K, J)$ is called soft semi- I -closed if $f(U, A) \in SSIC(Y, \sigma, K, J)$ for each soft closed set (U, A) in X .

Remark 3. (1) Every soft semi- I -open function is soft semi-open.

(2) Every soft semi- I -closed function is soft semi-closed.

(3) Every soft open function is soft semi- I -open.

These converses is not true in general as shown in following examples.

Example 3. Let a soft topological space (X, τ, E) and a soft ideal topological space (Y, ϑ, K, J) given as follows:

$X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$, $\tau = \{\tilde{\emptyset}, \tilde{X}, \{(e_1, \{h_1\})\}\}$, $Y = \{y_1, y_2\}$, $K = \{k_1, k_2\}$,
 $\vartheta = \{\tilde{\emptyset}, \tilde{Y}, \{(k_1, \{y_1\})\}, \{(k_2, \{y_2\})\}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}\}$, $J = \{\tilde{\emptyset}, \{(k_2, \{y_2\})\}\}$. Also, let $u: X \rightarrow Y$, $u(h_1) = y_1$,

$u(h_2) = y_2, p: E \rightarrow K, p(e_1) = k_2, p(e_2) = k_1$. Then the soft function $f_{up}: (X, \tau, E) \rightarrow (Y, \vartheta, K, J)$ is soft semi-open but is not soft semi- I -open.

Example 4. Let a soft topological space (X, τ, E) and a soft ideal topological space (Y, ϑ, K, J) given as follows:
 $X = \{h_1, h_2\}, E = \{e_1, e_2\}, \tau = \{\tilde{\emptyset}, \tilde{X}, \{(e_1, \{h_2\}), (e_2, X)\}\}, Y = \{y_1, y_2\}, K = \{k_1, k_2\},$
 $\vartheta = \{\tilde{\emptyset}, \tilde{Y}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}\}, J = \{\tilde{\emptyset}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}\}$. Also, let $u: X \rightarrow Y,$
 $u(h_1) = y_1, u(h_2) = y_2, p: E \rightarrow K, p(e_1) = k_2, p(e_2) = k_1$. Then the soft function $f_{up}: (X, \tau, E) \rightarrow (Y, \vartheta, K, J)$ is soft semi-closed but is not soft semi- I -closed.

Example 5. Let a soft topological space (X, τ, E) and a soft ideal topological space (Y, ϑ, K, J) given as follows:
 $X = \{h_1, h_2\}, E = \{e_1, e_2\}, \tau = \{\tilde{\emptyset}, \tilde{X}, \{(e_1, \{h_1\})\}\}, Y = \{y_1, y_2\}, K = \{k_1, k_2\},$
 $\vartheta = \{\tilde{\emptyset}, \tilde{Y}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}, \{(k_1, \{y_1\}), (k_2, \{y_2\})\}\}, J = \{\tilde{\emptyset}, \{(k_2, \{y_2\})\}\}$. Also, let $u: X \rightarrow Y, u(h_1) = y_1,$
 $u(h_2) = y_2, p: E \rightarrow K, p(e_1) = k_2, p(e_2) = k_1$. Then the soft function $f_{up}: (X, \tau, E) \rightarrow (Y, \vartheta, K, J)$ is soft semi- I -open but is not soft semi-open.

Theorem 11. A soft function $f: (X, \tau, E) \rightarrow (Y, \sigma, K, J)$ is soft semi- I -open if and only if for each $E_\alpha^x \in X$ and each soft neighborhood (U, A) of E_α^x , there exists $(V, B) \in SSIO(Y, \sigma, K)$ containing $f(E_\alpha^x)$ such that $(V, B) \tilde{\subset} f(U, A)$.

Proof. Suppose that f is a soft semi- I -open function. For each $E_\alpha^x \in X$ and each soft neighborhood (U, A) of E_α^x , there exists $(U_0, A_0) \in \tau$ such that $E_\alpha^x \in (U_0, A_0) \tilde{\subset} (U, A)$. Since f is soft semi- I -open, $(V, B) = f(U_0, A_0) \in SSIO(Y, \sigma, K)$ and $f(E_\alpha^x) \in (V, B) \tilde{\subset} f(U, A)$.
 Conversely, let (U, A) be an soft open set of (X, τ, E, I) . For each $E_\alpha^x \in (U, A)$, there exists $(V, B) \in SSIO(Y, \sigma, K)$ such that $f(E_\alpha^x) \in (V, B) \tilde{\subset} f(U, A)$. Therefore, we obtain $f(U, A) = \sqcup \{(V, B): E_\alpha^x \in (U, A)\}$ and by Theorem 3, $f(U, A) \in SSIO(Y, \sigma, K)$. This shows that f is soft semi- I -open function.

Theorem 12. Let $f: (X, \tau, E) \rightarrow (Y, \sigma, K, J)$ is soft semi- I -open (resp. soft semi- I -closed) function. If (W, A) be any soft subset of Y and (F, A) a soft closed (resp. soft open) subset of X containing $f^{-1}(W, A)$, then there exists a soft semi- I -closed (soft semi- I -open) subset (H, A) of Y containing (W, A) such that $f^{-1}(H, A) \tilde{\subset} (F, A)$.

Proof. Suppose that f is a soft semi- I -open function. Let (W, A) be any soft subset of Y and (F, A) a soft closed subset of X containing $f^{-1}(W, A)$. Then $\tilde{X} - (F, A)$ is soft open and since f is soft semi- I -open, $f(\tilde{X} - (F, A))$ is soft semi- I -open. Hence $(H, A) = \tilde{Y} - f(\tilde{X} - (F, A))$ is soft semi- I -closed. It follows from $f^{-1}(W, A) \tilde{\subset} (F, A)$ that $(W, A) \tilde{\subset} (H, A)$. Moreover, we obtain $f^{-1}(H, A) \tilde{\subset} (F, A)$.
 For a soft semi- I -closed function, we can prove similarly.

Theorem 13. For any soft bijective function $f: (X, \tau, E) \rightarrow (Y, \sigma, K, J)$ the following are equivalent:

- (1) $f^{-1}: (Y, \sigma, K, J) \rightarrow (X, \tau, E)$ is soft semi- I -continuous,
- (2) f is soft semi- I -open,
- (3) f is soft semi- I -closed.

Proof (1) \Rightarrow (2) Let (F, A) be a soft open subset in X . Since f^{-1} is soft semi- I -continuous, then $(f^{-1})^{-1}(F, A) = f(F, A)$ is soft semi- I -open in Y . Then f is soft semi- I -open.
 (2) \Rightarrow (3) Let (F, A) be a soft closed subset in X , then $\tilde{X} - (F, A)$ is soft open set and since f is soft semi- I -open, $f(\tilde{X} - (F, A)) = \tilde{Y} - f(F, A)$ is soft closed set, then $f(F, A)$ is soft open set. Thus f is soft semi- I -closed.
 (3) \Rightarrow (1) Let (F, A) be a soft open subset in X . Then $\tilde{Y} - f(F, A)$ is soft closed set, and since f is soft semi- I -closed, then $f(\tilde{X} - (F, A)) = \tilde{Y} - f(F, A)$ soft semi- I -closed. Thus $f(F, A) = (f^{-1})^{-1}(F, A)$ is soft semi- I -open. Therefore f^{-1} is soft semi- I -continuous.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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