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Some Transactions Made with Hadamard Gate in Qutrit Systems

Mehpeyker KOCAKOÇ[1*], Recep TAPRAMAZ[2]

[1]Çukurova University, Computer Technologies, Vocational School of İmamoğlu,01700, Adana, Turkey

[2]Ondokuz Mayıs University, Science and Literature Faculty, Physics Department, 55110, Samsun, Turkey

Abstract

Progress in computer technology turned toward atomic-sized computer design with a rapid progress,therefore it is important to establish logic gates and algorithms for quantum computers. In this work, the Hadamart gate for qutrit systems, triplet state in quantum information theory, is discussed. Some operations were performed on the Hadamard matrix representing the qutrit systems (3x3 complex matrix). The Hadamart gate is well known for qubit systems, but is rarely used for qutrit and other systems. Performing operations on the matrix tested. Unlike qubit systems, it has been shown that monolithic SWAP gates can be formed in qutrit systems. The powers of Hadamart matrix and its inverse do not provide a separate property, they only repeat itself. Hadamard gate, which is widely used in quantum information processes, is used in many algorithms and logical information processes without attaining quantum fuzzy states. In this work, theoretically, successful results have been obtained by applying Hadamart gates for qutrit systems.

Keywords:Qutrit, qubit, hadamard gate, swap gate, quantum computing.

Kutrit Sistemlerde Hadamard Geçidi ile Yapılan Bazı İşlemler

Mehpeyker KOCAKOÇ[1]*, Recep TAPRAMAZ[2]

Özet

Bilgisayar teknolojisi, hızlı bir ilerleme ile atom boyutlu bir bilgisayar tasarımı olarak düşünülmeye başlamıştır. Bu nedenle, kuantum bilgisayarlar için mantık kapıları ve algoritmalar kurmak önemlidir. Bu çalışmada, kuantum bilgi teorisinde üçlü hali olarak bilinen kutrit sistemleri için kullanılabilen Hadamart geçişi tartışılmıştır. Konuyla bağlantılı olarak kutrit sistemler için (3x3 karmaşık matris) Hadamard matrisi üzerinde bazı işlemler yapılmıştır. Kübit sistemler için Hadamart geçidi bilinmekte ve kullanılmaktadır.Bu çalışmada da kutrit sistemlerinin Hadamart geçidini temsil eden Hadamart matrisi üzerinde bazı işlemler yapılmıştır. Kubit sistemlerden farklı olarak, monolitik SWAP kapılarının kutrit sistemlerinde oluşturulabileceği gösterilmiştir. Hadamart matrisi ve tersinin diğer kuvvetleri ayrı durumlar oluşturmak yerine birbirini tekrarlar. Kuantum bilgi işleme süreçlerinde yaygın olarak kullanılan Hadamard geçidi, birçok algoritmada ve mantıksal bilgi işleme süreçlerinde kuantum bulanık durumlara erişmeden kullanılmaktadır. Bu çalışmada teorik olarak Hadamart kapılarının kutrit sistemlerine uygulanmasıyla uyumlu sonuçlar elde edilmiştir.

Anahtar Kelimeler :Kutrit, kübit, hadamart geçidi, swap geçidi, kuantum hesaplama.

[*]*Corresponding Author,* e- mail: mkocakoc@cu.edu.tr

1. Introduction

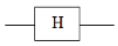
With the rapid development of computer technology and the shrinking of its dimensions, studies on quantum information theory based on the idea of atomic computer design by means of tools of quantum physics. The logic gateways and algorithms for quantum computers are expected to lead to major improvements in the field by the introduction of our lives. The idea of designing computers at the atomic scale emerged with the rapid progress of computer technology together with quantum theory. For this reason, the creation of logic gates and algorithms for quantum computers has become important. This work deals with the Hadamard gate, which can be used for the qutrit systems, that is for triplet state in quantum information theory.

2. Materials and Methods

Hadamard gate is applied for the superpositions of target states. An important feature of a quantum gate is reversibility which is applicable in classical gates is reversibility. That is, a process is reversible and recoverable in quantum information processing technique; but in classical logic gates, a transaction is only one way. This is due to the "similarity transformation" process, one of the basic operations on which quantum mechanics is based[1,2-3].

All of the classical logic gates are basically composed of different combinations of AND, OR, and NOT, and two levels, that is {0 and 1} states. Passages like quantum logic gates such as CNOT, SWAP, Hadamard do not have classical equivalents. For example, the Hadamard gate brings the state of superstructure to other situations[4,5-6]. Meanwhile the states in quantum information theory are $\{|0\rangle$ and $|1\rangle\}$ for qubit system and $\{\{|0\rangle$, $\{|1\rangle$ and $\{|2\rangle\}$ for qutrit system.

2.1.Hadamard Gate in Qubit Systems

In quantum information theory, Hadamard gate has an important place. Hadamard is indicated by$\hat{H}$the transitional letter and given 

as a symbol.

Among the quantum logic gateways, the Hadamard gate, one of the most common gates, introduces the quantum information system into superposition[5]. The matrix representation of the Hadamard gate for a qubit system is given in Eqn. 1.

$$\hat{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{1}$$

When the Hadamard gate is applied to a single qubit system, it is shown in Eqs. 2 and 3 for the $|0\rangle$state and $|1\rangle$ state.

$$\hat{H}|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \tag{2}$$

$$\hat{H}|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \quad (3)$$

This means that Hadamart gateway when applied to $|0\rangle$ state produces$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$state and to $|1\rangle$stateproduces $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$state. The Hadamard transition has two important features: it maps *n* different qubits initialized with$|0\rangle$ to a superposition of all 2*n* orthogonal states in the$|0\rangle$, $|1\rangle$basis with equal weight. Second property is that Hadamart is a self inverse processor.

$\hat{H}$operator can be written as shown in Eqn. 4 using Pauli-X, $(\hat{X})$, and Pauli-Z,$(\hat{Z})$, matrices.

$$\hat{H} = \frac{1}{\sqrt{2}}(\hat{X} + \hat{Z})\text{where}(\hat{X}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } \hat{Z} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (4)$$

3. Results and Discussion

3.1. Some Operations on The Hadamard Gate in The Qutrit Systems

Hadamard gate is the basic gate that superposes pure states. It is symmetric and orthogonal matrix. It can be applies to single-qubit systems and multi-qubit systems. The Hadamard matrix is somehow continuation of qubit matrix in qutrit systems. It produces similar results as in the qubit system.

In qubit systems, Hadamard matrix for qubit system is shown in Eqn. 5 where$n = 1$ and $\omega = e^{2i\pi/n} = -1$.

$$\hat{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & \omega \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (5)$$

The Hadamard matrix for qutrit systems is also shown in a complex form in Equation 6 with a similar form $\omega = e^{2i\pi/n}$where*n*=3 representing qutrit system;

$$\hat{H} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \quad (6)$$

The square of ω in matrix elements in Eq. 6 is defined as $\omega^2 = \omega^*$,more precisely,

$$\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \omega^2 = \omega^* = -\frac{1}{2} - i\frac{\sqrt{3}}{2} \quad (7)$$

The matrix is orthogonal and symmetric to these values, but not Hermitian. When the Hadamart matrix is applied to state functions for qutrit systems, it mixes or superpozes pure states,

$$\hat{H}|0\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle) \quad (8)$$

$$\hat{H}|1\rangle = \frac{1}{\sqrt{3}}(|0\rangle + \omega|1\rangle + \omega^2|2\rangle) \quad (9)$$

$$\hat{H}|2\rangle = \frac{1}{\sqrt{3}}(|0\rangle + \omega^2|1\rangle + \omega|2\rangle) \quad (10)$$

The square of Hadamart gate for qutrit systems is shown in Eqn.(11).

$$\hat{H}^2 = \hat{H} \cdot \hat{H} = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

$$\hat{H}^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad (11)$$

If the square of the Hadamard gate is taken, the term in Eqn. (11) are found using the definitions given in Eq.12.

$$\omega = e^{2\pi i/3} = \cos 120 + i\sin 120 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\omega^2 = e^{4\pi i/3} = \cos 240 + i\sin 240 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i \quad (12)$$

$$\omega^3 = e^{2\pi i} = \cos 360 + i\sin 360 = 1$$

$$\omega^4 = \omega$$

In qutrit systems, it is possible to generate swap gateway as in the qubit systems. The matrix $\hat{H}^2$given in Eqn. (11) is actually a SWAP gate with mono qutrit. The process given in Eq. (13) proves this.

$$\hat{H}^2|0\rangle = |0\rangle, \hat{H}^2|1\rangle = |2\rangle, \hat{H}^2|2\rangle = |1\rangle \quad (13)$$

The third power of the Hadamard matrix shown in Eqn. (14) gives the inverse of the Hadamard matrix.

$$\hat{H}^3 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix} = \hat{H}^{-1} \quad (14)$$

The matrix in Eqn.(15) has the same properties as Hadamard. The other forces of the matrix and the inverse repeat.

$$\hat{H}^4 = I,\ \hat{H}^5 = H,\ \hat{H}^6 = \hat{H}^2,\ \hat{H}^3 = \hat{H}^{-1},\ \hat{H}^7 = \hat{H}^3, \hat{H}^8 = I \ldots \quad (15)$$

Some use of Hadamart gate in quantum information processing are in quantum entanglement theory, in Grover's search algorithm, in Peter Shor's factorization algorithm and in many other algorithms and in logical information processing. Hadamard gate can be used in qutrit systems also in both single-qubit and multi-qubit structures. Theoretically successful results have been obtained on algorithms for quantum computers and the studies in this area are continuing. This work also discusses the situations that arise when the Hadamard gates are applied on the qutrit systems.

4. Conclusions

In this work, it is shown that, when Hadamart matrix representing Hadamart gate is applied to qutrit systems it carries pure states into mixed or superposed system. The matrix $\widehat{H}^2$is actually a SWAP gate with one qutrit. The third power of the Hadamard matrix is the inverse of the Hadamard matrix. Invers Hadamart matrix has the same properties as itself. It is also shown that the powers of the Hadamard matrix and its inverse repeat itself. The Hadamard gate from quantum logic gates brings the quantum information system into superposition..

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