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Research Article

Analysis of Contact Stresses in Spur Gears by Finite Element Method

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Abstract

Gears are one of the most critical elements in power transmission because it plays a significant role in the industry. Spur gear is used to transmit power and rotary motion between parallel shafts. It is one of the simplest types of the gears. Surface failure of the gear tooth is a pitting when contact stress is exceeding the strength of the material to surface fatigue. This paper studies the contact stresses in the contact zone among the spur gear pairs by using finite element method under static conditions. Contact stress among the gear tooth pair's engagement determines the facility of the gear to transmit the power without harm. The contact stress in gears has played an important role for last years, but an extensive research is still required to understand the several parameters affecting this stress. Among these parameters, the most important factors affecting the surface contact stress are; number of teeth, module and face width. In the present study, the contact stress in spur gear is calculated by changing one of these parameters and keeping remaining constant to obtain the influence of each parameter on contact stress separately based on AGMA's equations and finite element method (FEM). A computer program is used to build up the gears by using homemade software and SolidWorks. The results of the FEM analyses from MSC Software (MARC) are presented. These results are compared with the theoretical results (AGMA's equations). The contact stress achieved by FEM is lower than the obtained results by AGMA's equations and the corresponding percent difference detected are about 8 %. The results of the contact stress analysis specify that increasing the values of geometrical parameters (number of teeth, module and face width) lead to decrease in the tooth contact stress.

Keywords: Spur Gears, Contact Stress, FEM, AGMA.

Sonlu Eleman Yöntemi İle Düz Dişlilerde Temas Gerilmelerinin İncelenmesi

Öz

Dişliler, endüstriyel uygulamalarda güç aktarmında kullanılan en kritik elemenlardan bir tanesidir. Düz dişliler ise paralel şaftlarda güç ve dönme harekerini aktarmada kullanılmaktadır. Düz dişliler en basit dişli çark çeşididir. Diş yüzeyindeki temas baskısı, malzemenin yüzey yorulmasına karşı dayanımını aştığı zaman dişli yüzeyinde karıncalanma şeklinde oluşmaktadır. Bu çalışmada statik şartlar altında, sonlu elemanlar yöntemi kullanılarak, düz dişli çarklarda diş yüzeyindeki temas gerilmeleri incelenmektedir. Dişli çiftinin çalışması esnasında oluşan yüzey temas gerilmesinin değeri, dişli çarkın istenen gücü dişli yüzeyine kalıcı bir zarar vermeden iletebilmesi için önemlidir. Dişlilerdeki temas gerilmeleri son yıllardaki çalışmalarda önemli bir yer tutmuştur, ancak yine de bu temas gerilmelerine etki eden çeşitli parametrelerin anlaşılması için kapsamlı araştırmalar gerekmektedir. Bu parametreler arasında yüzey temas gerilmesini en çok etkileyenler; diş sayısı, modül ve diş genişliği sayılabilir. Bu çalışmada, düz dişli çarkların yüzeyindeki temas gerilmeleri, farklı parametrelerin etkisini görmek için, bir tanesi değiştrilip, diğerleri sabit tutularak, AGMA denklemleri ve sonlu elemanlar metodu (SEM) kullanılarak hesaplanmıştır. Dişli geometrileri bir projede hazırlanan bilgisayar programıyla elde edilerek SolidWorks katı modellemeye aktarılıp 2B dişli yüzeyleri elde edilmiştir. Bu dişli yüzeyleri kullanarak MSC SimXpert yazılımıyla SEM analizi için ağ oluşturulmuş, SEM analizleri ise MSC Marc yazılımıyla yapılmıştır. SEM ile elde edilen yüzey temas gerilimleri, teorik denklemlerle elde edilen sonuçlardan daha düşüktür ve fark yaklaşık yüzde 8'dir. Yüzey temas gerilmesi değerleri, geometrik parametrelerin (diş sayısı, modül, diş genişliği) artmasıyla azalmıştır.

Anahtar Kelimeler: Düz dişli çark, Yüzey temas gerilmesi, SEM, AGMA.

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1. Introduction

Pitting failure of the gear tooth occurs due to misalignment, wrong viscosity oil selection, and contact stress exceeding the yield strength of the material. The material in the failure region gets removed, and a pit is formed. Consequently, higher impact load occurring from pitting may cause fracture of the already weakened tooth. The performance and life of the gear teeth are directly related to the strength of the teeth to withstand contact stresses. To increase the life, the gear analysis is crucial against the pitting failure.

Contact stress may cause pitting on the tooth; therefore, the contact stress must be within the allowable limits. In order to explain the behavior of the contact stress, the stress analysis needs to be carried out. Among the main influencing factors; the geometric profile of the tooth (number of teeth, modules and face width) can be discussed.

(Marimuthu and Muthuveerappan, 2016:1) included coupling between load sharing ratio (LSR), maximum fillet and contact stresses. Reduction of stress is obtained due to increase in teeth number. (Zhan, Fard and Jazar, 2015:2) developed quasi-static FEM model newly based on ANSYS Workbench for performing the analysis of time-varying load capacity. The results were compared with analytical calculations based on AGMA equations. Their results are acceptable and accurate according to AGMA results. (Farhan, Karuppanan and Patil, 2015:3) evaluated a contact stress of spur gear with and without friction by using the finite element method (FEM), Hertzian theory and AGMA standard. They showed an inversely proportional relation between contact stresses and face width (the contact stress decreased with the increase of face width). (Sankpal and Mirza, 2014:4) find out contact stresses by using the polariscope. By using methods of FEM and experiment, they evaluated and compared the results. Calculated values are close for both methods. They also found that selection of module size is an important factor before designing gear and contact stress decreases with increasing module.

(Patil, Karuppanan, Atanasovska and Wahab, 2014:5) also used different ways to analyze gears. Hence, a gear model was prepared to study the relationship between spur gear contact stress and friction. The analysis was performed by finite element method and complimented by two theoretical methods. (Karaveer, Mogrekar, and Joseph, 2013:6) presented the stress analysis of spur gear pairs to find contact stress on the gear teeth. The results were obtained in Finite Element Analysis (FEA) and compared with theoretical values (Hertzian equation). The spur gear was sketched and modeled in ANSYS Design Modeller and analysed in ANSYS. The results show that the difference between maximum contact stresses obtained from Hertz equation and Finite Element Analysis is acceptable and it is very low. (Hwang, Lee, J. H., Lee, D. H., Han, Lee, K. H. 2013:7) used 2D model to analyse the contact stress of a pair of mating gears in static FEM and compared these results with the AGMA standard, and they found the results obtained by FEM were more severe than that of the AGMA standard. (Patil, Karuppanan and Wahab, 2013:8) determined the contact pressure by using ANSYS software for a pair of mating spur gears by Finite Element Analysis (FEA). They showed that the FEA results for the contact stress gives the similar trend with the twin-disc test results, and the FEA model gives good results. Also, they observed the maximum variation between these two results was about 10.98 %. (Gupta, Choubey and Varde, 2012:9) presented a paper that provides the detailed study of contact stress established between different mating gears is mostly important for the gear design. They used Hertzian equations for calculating gear contact stresses. So for contact stress, they recognized and determined suitable models of contact elements, and measured contact stresses by using ANSYS and compared the results with Hertz theory. Conclusions suggest that to decrease contact stresses, the module must be increased in a pair of spur gears. (Shinde, Nikam and Mulla, 2010:10) also recommended that the stress distribution evaluation applying the FE method is good and is comparable with theoretical values. (Tesfahunegn, Rosa and Gorla, 2010:11) investigated the influence of the shape of profile modifications on contact stress through nonlinear finite element approach.

2. Methodology

2.1 FEA of Gears

In this paper, in order to study the influence of number of teeth, module and face width on contact stresses of spur gear pairs, different gears are modelled. Spur gear pairs with different number of teeth (20, 40, 60, 100 and 120), modules (2, 3, 4, 6 and 10 mm) and face width (20, 30, 40, 50 and 75 mm) are selected. Specifications of these five gear sets are represented in Tables 1, 2 and 3.

Table 1. Spur Gear Specifications (Case1)

Gear Properties	**Case 1**				
Teeth Number	20	40	60	100	120
Module (mm)	2				
Face Width (mm)	20				
Pressure Angle (°)	20				
Gear Ratio	1				

Table 2. Spur Gear Specifications (Case2)

Gear Properties	Case 2				
Teeth Number	20				
Module (mm)	2	3	4	6	10
Face Width (mm)	20				
Pressure Angle (°)	20				
Gear Ratio	1				

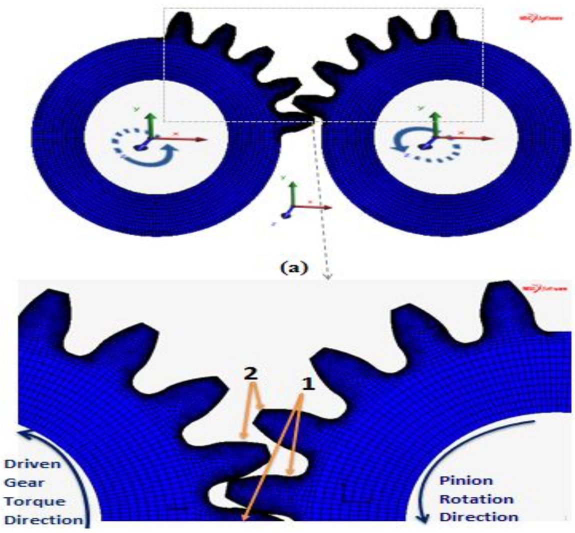
Table 3. Spur Gear Specifications (Case3)

Gear Properties	Case 3				
Teeth Number	20				
Module (m)	2				
Face Width (mm)	20	30	40	50	75
Pressure Angle (°)	20				
Gear Ratio	1				

The selected material of all gear sets is steel with Young's modulus of $21x10^4$ N/mm^2 and Poisson's ratio of 0.3 for analysis. X and Y coordinates of involute and trochoid portions of the standard gear profile are generated by using homemade software which was developed in the scope of the following two studies: (Şahin, Akpolat, Yildirim, Uctu and Ersoz, 2014:12) and (Şahin B. 2015:13). Then these points are converted to curves by using Solidworks. Different region are generated on whole gear surface to provide fine or coarse mesh areas depending on critical stress location points. The final profile (ready to mesh) is then imported to SimXpert in Parasolid (x_t) format without any data loss for meshing of the gear. Finally, it is analyzed by MARC Mentat of MSC.

Element types are triangular and quadratic (quadratic dominant) for the meshing of the gears in this study. It is applied to 2-D solid models successfully. The smallest size of the mesh in the contact region is selected as 50 micron.

FEM analysis is then carried out by using static structural analysis in MARC FEA Software of MSC. Results of the FE analysis are compared with results of the theoretical calculations.
Boundary conditions are defined to specify the torque, rotation and supports which allows gear pair rotation about the Z-axis in the counter clockwise direction. Therefore, the torque of 160.428 Nm is applied at the gear shaft at Z axis in that direction. Figure 1 indicates the rotational direction and the torque applied.



(b)

Figure.1. Spur gear pair: (a) torque applied and rotational direction on hubs at Z axis, (b) enlarged teeth pairs.

2.2 Theoretical calculation of contact stresses by the analytical method (AGMA standard)

Based on the Hertzian theory, the contact stress equation by AGMA standard (AGMA, 2010:14) is given as in Eqn. 1:

$$\sigma_c = Z_E \sqrt{W^t K_o K_v K_s \frac{K_H Z_R}{d_{W1} b Z_I}} \tag{1}$$

W^t is the transmitted tangential load applied on gear teeth. Where K_o, K_v, K_s and K_H are the overload factor, dynamic factor, size factor and the load distribution factor, respectively. d_{W1} is the pitch diameter of the pinion and b is the face width. Surface condition factor for pitting resistance (Z_R) is taken as unity because appropriate surface condition is assumed in accordance with suggestion of AGMA standard. Table 4 shows the values of the parameters used in AGMA equations (AGMA, 2010:14):

Table 4. Values of parameters used in AGMA equations

Parameter	Value
Z_E	$191\sqrt{MPa}$
K_o	1.0
K_v	1.0
K_s	1.0
K_H	1.0
Z_I	0.08

Z_E is the elasticity coefficient can be written as in Eqn. 2:

$$Z_E = \left[\frac{1}{\pi\left(\frac{1-\upsilon_P^2}{E_P} + \frac{1-\upsilon_G^2}{E_G} \right)} \right]^{1/2} \tag{2}$$

Where ν and E are the Poisson's ratio and modulus of elasticity, respectively. Suffix G represents the gear and P stands for pinion.
Z_I is the geometry factor can be calculated as in Eqn. 3:

$$Z_I = \frac{\cos\phi_t \sin\phi_t}{2m_N} \frac{m_r}{m_r + 1} \tag{3}$$

Where ϕ_t is the pressure angle. m_N is the load sharing ratio (for spur gears m_N=1). m_r is defined as the gear ratio in Eqn. 4.

$$m_r = \frac{Z_G}{Z_P} \tag{4}$$

Where Z_P and Z_G are the teeth number of pinion and gear, respectively.

3. Results and Discussion

In this study, spur gears with different number of teeth, modules and face width without profile modification are investigated. In order to calculate theoretical contact stress, it is necessary to calculate AGMA parameters in Eq. (1). Also, this study contains comparisons of the contact stress results of theoretical calculations and numerical works obtained from the AGMA equations and MARC Software respectively. The maximum contact stress values of FEM are taken from the second tooth pair contact of pinion.

In order to achieve the required objectives in this study, the three parameters of spur gear geometry are changed, which are number of teeth, module and face width. Two of these parameters are kept constant while third one is changed for case studies to obtain effect of each parameter on contact stress separately.

3.1 Number of Teeth

The results of the contact stress analysis show that increase of number of teeth causes to decrease in contact stress. Figure 2 and Table 5 presents the effect of number of teeth on the contact stress. It is clear that the change of the number of teeth from 20 to 120 reduces the contact stress about 83 %.

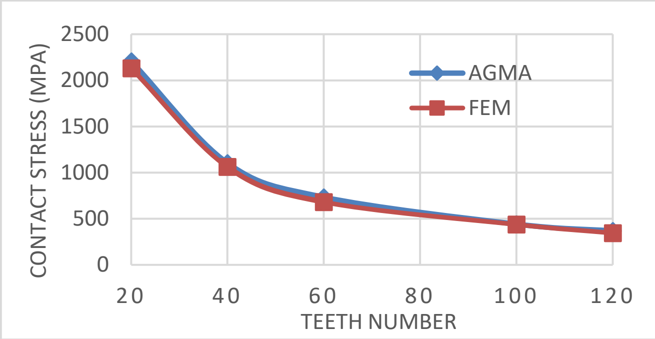


Figure 2. Effect of change of the number of teeth on contact stress

Table 5 Contact Stresses for different number of teeth

Number of Teeth	Contact Stresses by AGMA (MPa)	Contact Stresses by FEM (MPa)	Differences (%)
20	2206.42	2131.14	3.41
40	1103.21	1061.13	3.81
60	732.91	681.24	7.05
100	441.28	437.13	0.94
120	367.74	344.69	6.27

3.2 Module

The contact stresses based on AGMA stress equation and finite element analysis for modules ranging from 2 to 10 are compared in Figure 3 and Table 6. The contact stresses are inversely proportional to the module of the gear; as the module increases lead to the decrease in contact stresses.

Both the AGMA theories and FEA show the same pattern of results, although the nature of AGMA contact stresses is linear, but the FEA result of the drop of stresses shows non-linear nature which is possibly due to the size and type of the meshing.

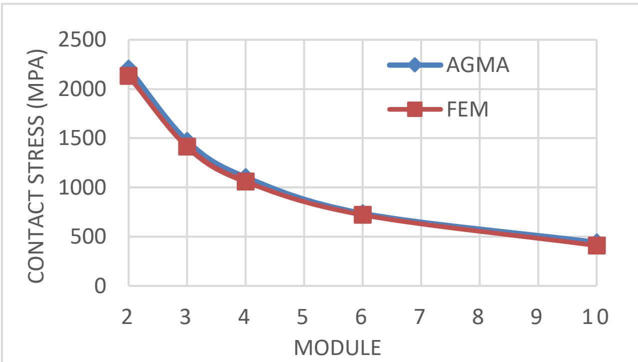


Figure 3. Effect of change of the module on contact stress

Table 6. Contact Stresses for different modules

Module (mm)	Contact Stresses by AGMA (MPa)	Contact Stresses by FEM (MPa)	Differences (%)
2	2208.48	2131.14	3.50
3	1472.32	1412.77	4.04
4	1104.24	1059.15	4.08
6	736.16	720.65	2.11
10	441.70	409.90	7.20

3.3 Face Width

The change of the maximum contact stress in gears with different values of face width is shown in Figure 4. It is observed that the maximum contact stress decreases with the increase of face width. Table 7 shows the comparison the contact stresses obtained from finite element works and theoretical calculations.

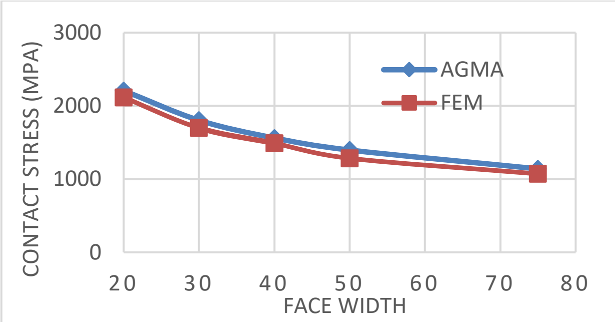


Figure 4. Effect of change of the face width on contact stress

Face width (mm)	Contact Stresses by AGMA (MPa)	Contact Stresses by FEM (MPa)	Differences (%)
20	2208.48	2115.39	4.22
30	1803.22	1698.30	5.82
40	1561.63	1489.85	4.60
50	1396.77	1283.90	8.08
75	1140.45	1071.84	6.02

Table 7 Contact Stresses for different face width

The comparison between numerical and theoretical calculations shows that there is a good compatibility between them; therefore the validity of the FEA is satisfied. The Marc FEA results show a similar trend with the theoretical results. This indicates that the FEA model is suitable for contact stress calculation. Results show the great compatibility between numerical and theoretical results of three cases. Since the results of AGMA calculations and FEA results of the contact stress are close to each other (maximum difference is about 8 %).

4. Conclusion

Based on this study of five different gear pairs, the following results can be concluded:
1. The FEM gear model is verified with AGMA equations. The models of gear pairs are suitable for analysis in terms of the contact stresses and the results obtained are in good compatibility with the analytical calculations.
2. AGMA based design calculations of gear are validated using FEM.
3. In all cases, contact stresses calculated by MSC Software MARC and AGMA are found in agreement and noticed the maximum percentage error between results is about 8 %.
4. The contact stresses decrease with increase of number of teeth, module, face width with zero coefficient of friction and the percent reduction in the contact stresses are given below for AGMA calculations:

a) 50 % when the number of teeth is changed from 20 to 40;
83 % when the number of teeth is changed from 20 to 120.

b) 33 % while module is increased from 2 mm to 3 mm; 80 % while module is increased from 2 mm to 10 mm.

c) 18 % when face width is increased from 20 mm to 30 mm; 48 % when face width is increased from 20 mm to 75 mm.

Similar reductions can be seen in FEM values and these results validate the FEM results.

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