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Exact Solutions for Generalized (3+1)-Dimensional Shallow Water-Like (SWL) Equation

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Abstract: In this paper, we demonstrate the exact solutions of the generalized (3+1)-dimensional shallow water-like (SWL) equation by using Bernoulli sub-equation function method. Some new solutions are successfully constructed. We carried out all the computations by Wolfram Mathematica.

Keywords: Generalized (3+1) shallow water-like (SWL) equation , Bernoulli sub-equation function method, Exact solution.

1 Introduction

In this paper, generalized (3 + 1)-dimensional shallow water-like (SWL) equation [1, 2] which is one of nonlinear evolution equations will be discussed and new solutions will be examined.

$$u_{xxxy} + 3u_{xx} * u_y + 3u_x * u_{xy} - u_{yt} - u_{xz} = 0 \tag{1}$$

There are several studies on this equation. Rational solutions and lump solutions are obtained for equation(1) by Zhang et.al.[1] and Grammian and Pfaffian solutions are obtained by Tang et.al.[2]. Also, this equation solved by Tian and Gao[3] via the tanh method,by Zayed[4] via the (G'/G)-expansion method. Lump-type solutions and their interaction solutions are generated by Sadat[5].

2 Material and method

In this part, we use the Bernoulli sub-equation function method[6, 7]. for solutions equation(1).
Step 1. Let's consider the following partial differential equation;

$$P(u, u_x, u_y, u_z, u_t, u_{xx}, u_{xy}, ...) = 0 \tag{2}$$

and take the wave transformation;

$$u(x, y, z, t) = U(\eta), \eta = x + ky + mz - wt \tag{3}$$

where k, m and w are nonzero constants.Substituting equation(3) into equation(2), gives the following nonlinear ordinary differential equation:

$$N = (U, U^{'}, U^{''}, U^{'''}, ...) = 0 \tag{4}$$

Step 2. Considering trial equation of solution in equation(4), it can be written as following;

$$U(\eta) = \sum_{i=0}^{n} a_i F^i(\eta) = a_0 + a_1 * F + a_2 * F^2 + ... + a_n * F^n \tag{5}$$

According to the Bernoulli theory, we can consider the general form of Bernoulli differential equation for as following;

$$F^{'} = \alpha F + \beta * F^M, \alpha \neq 0, \beta \neq 0, M \in R - 0, 1, 2 \tag{6}$$

where $F = F(\eta)$ is Bernoulli differential polynomial. Substituting equation(5,6) into equation(4), it converts an equations of polynomial (F) as following;

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$$\Omega(F) = \rho_s F^s + ... + \rho_1 F + \rho_0 = 0 \tag{7}$$

According to the balance principle, we can determine the relationship between n and M.
Step 3. The coefficients of (F) all be zero will yield us an algebraic system of equations;

$$\rho_i = 0, i = 0, ..., s \tag{8}$$

Solving this system , we will specify the values of $a_0, ..., a_n$. Step 4. When we solve nonlinear Bernoulli differential equation equation (6), we obtain the following two situations according to and ;

$$F(\eta) = [\frac{}{} + \frac{E}{e^{(M-1)\eta}}]^{\frac{1}{1-M}}, \neq \tag{9}$$

$$F(\eta) = [\frac{(E-1) + (E+1)tanh(\frac{(1-M)\eta}{2}}{1 - tanh(\frac{(1-M)\eta}{2}}]^{\frac{1}{1-M}}, = , E \in R \tag{10}$$

3 Implementation of proposed method

In this section, application of the Bernoulli sub-equation function method to SWL equation is presented. Using the wave transformation on equation(1)

$$(u(x, y, z, t) = U(\eta), \eta = x + ky + mz - wt, \tag{11}$$

we get the following nonlinear ordinary differential equation:

$$(kU^4 + 6kU^{'}U^{''} + (kw - m)U^{''} = 0, \tag{12}$$

Integrating the equation in equation (12), we get

$$(kU^{'''} + 3k(U^{'})^2 + (kw - m)U^{'} = 0, \tag{13}$$

Finally, If we write V instead of U', the equation (13) becomes a second order nonlinear ordinary differential equation:

$$(kV^{''} + 3kV^2 + (kw - m)V = 0, \tag{14}$$

Balancing equation(14) by considering the highest derivative $(V^{''})$ and the highest power (V^2) , we obtain $n + 2 = 2M$. When determining the value of M, we pay attention to the fact that it is greater than two and take the smallest M value for easier calculation.Choosing $M = 3$ gives $n = 4$. Thus, the trial solution to equation(1) takes the following form:
Choosing $M = 3$,$m = 1$, gives $n = 3$. Thus, the trial solution to Eq.(1) takes the following form:

$$U(\eta) = a_0 + a_1 * F + a_2 * F^2 + a_3 * F^3 + a_4 * F^4 \tag{15}$$

where $F^{'} = wF + dF^3, \neq 0, \neq 0$. Substituting Eq.(15), its second derivative and power along with $F^{'} = wF + dF^3, \neq 0, \neq 0$ into equation(14), yields a polynomial in F. Solving the system of the algebraic equations, yields the values of the parameter involved. Substituting the obtained values of the parameters into equation (15), yields the solutions to equation (1). We can find following coefficients: Case 1

$$a_0 = -(1+k)w/3k; a_1 = 0; a_2 = (4w^(32))/(k/(1+k)); a_3 = 0; a_4 = -8w^2; = -w/(2(k/(1+k))) \tag{16}$$

Case 2

$$a_0 = 0; a_1 = 0; a_2 = (4w(-(1+k)w))/k; a_3 = 0; a_4 = -8w^2; = -(-(1+k)w)/(2k) \tag{17}$$

Case 3

$$a_0 = -(4^2)/3; a_1 = 0; a_2 = -(32k^3)/(1+k); a_3 = 0; a_4 = -(128k^{24})/(1+k)^2; w = (4k^2)/(1+k). \tag{18}$$

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Substituting equation(16) into Eq.(15), gives

$$u_1\left(x,y,z,t\right)=\frac{4(1+k)w^{\frac{5}{2}}\left(2d\sqrt{\frac{k}{1+k}}-e^{\frac{\sqrt{w}(x+ky+mz-wt)}{\sqrt{\frac{k}{1+k}}}}\sqrt{w}\in\right)}{d\left(-4d^2k+e^{\frac{2\sqrt{w}(x+ky+mz-wt)}{\sqrt{\frac{k}{1+k}}}}w\in^2+e^{\frac{2\sqrt{w}(x+ky+mz-wt)}{\sqrt{\frac{k}{1+k}}}}kw\in^2\right)}$$
$$-\frac{(1+k)w(x+ky+mz-wt)}{3k}+\frac{2(1+k)w^2(x+ky+mz-wt)}{dk}$$
$$-\frac{2(1+k)w^3(x+ky+mz-wt)}{d^2k}-\frac{2w^{\frac{3}{2}}Arc\tanh[\frac{e^{\frac{2\sqrt{w}(x+ky+mz-wt)}{\sqrt{\frac{k}{1+k}}}}\sqrt{1+k}\sqrt{w\in}}{2d\sqrt{k}}]}{d\sqrt{\frac{k}{1+k}}} \tag{19}$$
$$+\frac{2\sqrt{1+k}w^{\frac{5}{2}}Arc\tanh[\frac{e^{\frac{2\sqrt{w}(x+ky+mz-wt)}{\sqrt{\frac{k}{1+k}}}}\sqrt{1+k}\sqrt{w\in}}{2d\sqrt{k}}]}{d^2\sqrt{k}}$$
$$-\frac{w^{\frac{3}{2}}Log[4d^2k-e^{\frac{2\sqrt{w}(x+ky+mz-wt)}{\sqrt{\frac{k}{1+k}}}}w\in^2-e^{\frac{2\sqrt{w}(x+ky+mz-wt)}{\sqrt{\frac{k}{1+k}}}}kw\in^2]}{d\sqrt{\frac{k}{1+k}}}$$
$$+\frac{w^{\frac{5}{2}}Log[4d^2k-e^{\frac{2\sqrt{w}(x+ky+mz-wt)}{\sqrt{\frac{k}{1+k}}}}w\in^2-e^{\frac{2\sqrt{w}(x+ky+mz-wt)}{\sqrt{\frac{k}{1+k}}}}kw\in^2]}{d^2\sqrt{\frac{k}{1+k}}}$$

Substituting Eq.(17) into Eq.(15), gives

$$u_2\left(x,y,z,t\right)=\frac{\left(1+k\right)w^2\left(\begin{array}{l}\frac{8d^2kw-4de^{\frac{\sqrt{-(1+k)w}(x+ky+mz-wt)}{\sqrt{k}}}\sqrt{k}w\sqrt{-(1+k)w}\in}{\sqrt{-(1+k)w}\left(4d^2k+e^{\frac{\sqrt{-(1+k)w}(x+ky+mz-wt)}{\sqrt{k}}}(1+k)w\in^2\right)}\\+\frac{2(-d+w)(x+ky+mz-wt)}{\sqrt{k}}\\+\frac{2(d-w)Arc\tan\left[e^{\frac{\sqrt{-(1+k)w}(x+ky+mz-wt)}{\sqrt{k}}}\sqrt{w}\in\right]}{\sqrt{1+k}\sqrt{w}}\\+\frac{(d-w)Log\left[4d^2k+e^{\frac{\sqrt{-(1+k)w}(x+ky+mz-wt)}{\sqrt{k}}}(1+k)w\in^2\right]}{\sqrt{-(1+k)w}}\end{array}\right)}{d^2\sqrt{k}} \tag{20}$$

Substituting Eq.(18) into Eq.(15), gives

$$u_3\left(x,y,z,t\right)=\frac{4}{3}\sigma^2\left(\begin{array}{l}-\left(x+ky+mz-wt\right)+\frac{48k^2\in\sigma^4}{d^2(1+k)^2\left(de^{2(x+ky+mz-wt)\sigma}-\in\sigma\right)}\\+\frac{12k\sigma\left(d+dk-4k\sigma^2\right)Log\left[de^{2(x+ky+mz-wt)\sigma}-\in\sigma\right]}{d^2(1+k)^2}\end{array}\right) \tag{21}$$

4 Result and discussion

New solutions are obtained for the SWL equation using the Bernoulli sub-equation function method. We have seen that the results we obtained are new solutions when we compare them with previous ones.

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