Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

About Rock Strength Certificate

B.A. RYCHKOVA*, N.M. KOMARTSOVA, M.A. KULAGİNA

Kyrgyz-Russian Slavic University, Department of Mechanics, Bishkek, Kyrgyzstan

Abstract:

The stress state of rocks in the massif is modeled by testing standard cylindrical samples on equipment according to the Karman's scheme, when some ratio between the axial compressive stress and the principal stresses from uniform lateral pressure is given. In this case, their ratio at the time of destruction indicates the strength of rocks. These stresses can take many values, and it is impossible to carry out the whole complex of experiments with different ratios of these components. Therefore, various methods of calculation are being developed; they can be used to estimate the degree of danger of the stress state by the postulated dependencies between the main stresses, i.e. to predict the strength properties of materials at the moment of destruction. The envelope of limit stress circles on the Mohr diagram in coordinates "normal stress - shear stress" is considered as the rock strength certificate. To construct the envelope there was used the relationship between the maximum and minimum principal stresses, presented in the form of two different strength criteria. One of them is the well-known Hoek-Brown criterion, the other is proposed by T.B. Duyshenaliev and K. T. Koichumanov recently. The applicability of these criteria was proved on A.N. Stavrogin's and K. Mogi's experimental data, which were received by testing cylindrical samples of various rocks under triaxial compression, as well as in the case of uniaxial tension.

Keywords: Rock Strength; Envelope Of Limit Stress Circles; Strength Criterion; Principal Stresses; Mohr's Diagram

DOI:

1. INTRODUCTION

The rock strength certificate construction, according to GOST 211153.8-88 [1], consists in constructing the envelope of the Mohr limit stress circles which include experimental data on the tensile strength under uniaxial tension and compression as well as under at least three types of triaxial compression. Thus, these five initial tensile strengths serve as "reference points", which are used to construct the empirical envelope of the limit circles. The usage of major and minor principal stress dependency as a strength criterion allows to reduce the number of "reference points" necessary to construct such an envelope. This paper considers two such criteria: the T.B. Duishenaliev – K.T. Koichumanov criterion and the Hoek–Brown criterion.

According to Mohr, stress states, in which the material is destroyed, can be represented as limit circles in coordinates (normal and shear stresses):

$$\left(\frac{\sigma_1+\sigma_3}{2}-\sigma\right)^2+\tau^2=\left(\frac{\sigma_1-\sigma_3}{2}\right)^2$$

where, principal stresses correspond to the beginning moment of destruction.

This equation (for the convenience of its study) is written as [2]:

$$\varphi(\sigma,\tau,\sigma_1)=\sigma^2+\tau^2-(\sigma_1+\sigma_3)\sigma+\sigma_1\sigma_3=0 \qquad (1.1)$$

where the parameter of the circles family is stress σ_1.

Corresponding Author E-mail: rychkovba@mail.ru

Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

The expression (1.1) can be considered as an algebraic second-degree equation regarding principal stresses σ_1 and σ_3.

The general second-degree equation regarding x and y has the following formula [3]:

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0 \quad (1.2)$$

where $a_{ik} = a_{ki}$ (i, k=1, 2, 3).

In the given case (1.1), taking $\sigma_1 = x$, $\sigma_3 = y$ the coefficients a_{ik} can be expressed as:

$$a_{11} = a_{22} = 0\,;\, a_{12} = a_{21} = 1/2\,;\; a_{13} = a_{31} = -\sigma/2\,;\; a_{23} = a_{32} = -\sigma/2 \quad (1.3)$$

The invariants of equation (1.2) are used to estimate the classification of the curve corresponding to it as follows:

$$D_{инв.} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix},\; A_{инв.} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

According to (1.3):
$D_{инв.} = -1/4 < 0$, $A_{инв.} = -\tau^2/4 \neq 0$

In accordance with these values of the invariants Ainv.and Dinv., expression (1.1) is the hyperbola equation [3]. This property was used [2] to formulate the dependency between the principal stresses which reflects the established property of the Mohr limit circles, i.e. to represent the strength criterion.

2. THE STRENGTH CRITERIA

By definition, the hyperbola equation in the space of principal stresses is represented as follows [3]:

$$\sqrt{\sigma_1^2 + (\sigma_3 - b)^2} - \sqrt{\sigma_1^2 + (\sigma_3 - a)^2} = d \; (a,\, b,\, d - const).$$

Solving this equation regarding σ_3, we find [2]:

$$\sigma_3 = \frac{a+b}{2} + \sqrt{\frac{d^2\sigma_1^2}{(a-b)^2 - d^2} + \frac{d^2}{4}} \quad (2.1)$$

According to the well-known theorem [4], the circles family envelope of the type (1.1) must also satisfy the equation:

$$\phi_{\sigma_1}(\sigma, \tau, \sigma_1) = 0 \quad \phi_{\sigma_1} = \partial\phi / \partial\sigma_1 \quad (2.2)$$

Solving equations (1.1) and (2.2) together, we find the coordinates of the limit stress circles' envelope:

$$\sigma = \frac{\sigma_3 + \sigma_1\sigma_3'}{1+\sigma_3'} \quad \tau = \pm\frac{\sigma_1 - \sigma_3}{1+\sigma_3'}\sqrt{\sigma_3'} \left(\sigma_3' = \frac{\partial\sigma_3}{\partial\sigma_1}\right) \quad (2.3)$$

Hyperbola parameters replacement introduction (2.1):

$$\frac{a+b}{2} = A_k \quad \frac{d^2}{4} = B^2{}_k \quad \frac{d^2}{(a-b)^2 - d^2} = Q \quad (2.4)$$

In view of (2.4), expression (2.1) and the derivative have the following form:

$$\sigma_3 = A_k + \sqrt{Q\sigma_1^2 + B_k{}^2},\; \sigma_3' = \frac{Q\sigma_1}{\sqrt{Q\sigma_1^2 + B_k^2}} \quad (2.5)$$

Material Dependent (2.5) Parameters can be determined based on experimental data in any three triaxial compression stress states (selected as "reference points").

We obtain a system of three equations for three constants (A_k, B_k, Q) on the basis of the formula (2.5) expressing the dependence in three types of stress state. The solution results in the following:

$$A_k = \frac{\left(c_1^2\sigma_{1(1)}^2 - c_2^2\sigma_{1(2)}^2\right) - q(c_2^2\,\sigma_{1(1)}^2 - c_3^2\,\sigma_{1(3)}^2)}{2\left[\left(c_1\sigma_{1(1)} - c_2\sigma_{1(2)}\right) - \left(c_1\sigma_{1(1)} - c_3\sigma_{1(3)}\right)q\right]} \quad (2.6)$$

where

$$q = \frac{(\sigma_{1(1)})^2 - \sigma_c^2}{(\sigma_{1(2)})^2 - \sigma_c^2} \quad \sigma_{1(i)} = \sigma_1\big|_{c=c_i} \quad \sigma_c = \sigma_1\big|_{c=0} \quad (2.7)$$

$$Q = \frac{c_2^2\sigma_{1(2)}^2 - 2c_2\sigma_{1(2)}A_k}{\sigma_{1(2)}^2 - \sigma_{1(1)}^2} \quad (2.8)$$

Corresponding Author E-mail: rychkovba@mail.ru

Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

$$B_k = \sqrt{A_k^2 - Q\sigma_c^2} \tag{2.9}$$

It is shown [2, 5] that if the $\sigma_1 \to \infty$ the parameter $Q \to 1$. Therefore, along with dependence (2.5), the following dependence, alternative to it, should be considered:

$$\sigma_3 = A_k + \sqrt{\sigma_1^2 + B_k{}^2} \tag{2.10}$$

First, we indicate the following: the equation of Mohr circles can be represented in the formula different from (1.1):

$$\varphi(\sigma,\tau,c) = \sigma^2 + \tau^2 - (1+c)\sigma_1\sigma + c\sigma_1^2 = 0 \tag{2.11}$$

where the type of the stress state $c = \sigma_3 / \sigma_1$ is a parameter of the given circles family.

In this case, the limit stress circles' envelope coordinates have the form [5]:

$$\sigma = \frac{\sigma_1(\sigma_1 + 2c(\sigma_1)_c)}{\sigma_1 + (1+c)(\sigma_1)_c}, \quad \tau = \frac{(1-c)\sigma_1\sqrt{(\sigma_1 + c(\sigma_1)_c)(\sigma_1)_c}}{\sigma_1 + (1+c)(\sigma_1)_c}$$
$$(\sigma_1)_c = \frac{\partial\sigma_1}{\partial c} = \frac{c[2A_k^2 - (1-c^2)B_k^2] - A_k(1+c^2)\sqrt{A_k^2 - (1-c^2)B_k^2}}{(1-c^2)^2\sqrt{A_k^2 - (1-c^2)B_k^2}} \tag{2.12}$$

Based on what has been stated above that dependency $\sigma_3 = \sigma_3(\sigma_1)$ (expressed in the formula (2.5) or (2.10)) was found [2] strictly mathematically. In this case, the established property of the Mohr limit circles was used, namely the Mohr circle equation in the space of principal stresses is the hyperbola equation. Thus, this dependence is one of the possible criteria for rock strength.

A criterion similar in structure was proposed in the works [6,7]. For undisturbed rocks it (unlike the original) can be represented [8,9] in the form:

$$\sigma_1 = \sigma_3 + \sqrt{A_h\sigma_3 + B_h^2} \quad (A_h, B_h - const) \tag{2.13}$$

As emphasized by the authors of this criterion [7] and other researchers [10], the criterion (16) is empirical. It was established by a trial and error method in accordance with the calculated and experimental data of various rocks samples triaxial compression.

The formula (16) is represented by the second degree algebraic equation:

$$\sigma_1^2 - 2\sigma_1\sigma_3 + \sigma_3^2 - A_h\sigma_3 - B_h^2 = 0 \tag{2.14}$$

In accordance with the second degree equations classification [3], the equation (2.14) (according to its invariants) is the parabola equation.

3. THE METHODOLOGY FOR DETERMINING MATERIAL CONSTANTS INCLUDED IN THE TWO STRENGTH CRITERIA

We first consider the criteria represented by the formulas (2.10) and (2.13), respectively.

We can find the constants A_k and B_k (included in Duishenaliev-Koichumanov criterion) using either the experimental data of triaxial and uniaxial compression, or the uniaxial tensile and compression strengths as follows.

The first of these options is to be considered as follows. If $\sigma_3 = 0$ under uniaxial compression, the stress σ_1 is equal to the compressive strength (σ_c), i.e. $\sigma_1 = \sigma_c$. This stress will serve as the first "reference point" to determine the desired constants. In this case, it follows from the formula (2.10):

$$A_k^2 - B_k^2 = \sigma_c^2 \tag{3.1}$$

Considering that in an arbitrary form of the stress state $\sigma_3 = c\sigma_1$, we express constant A_k according to (13):

$$A_k = \frac{(c^2-1)\sigma_1^2 + \sigma_c^2}{2c\sigma_1} \tag{3.2}$$

In the formula (3.2), stress σ_1 can be selected for any particular type of the stress state carried out in the experiment. It is advisable to choose such a stress at the maximum lateral pressure carried out in the experiment (i.e., at $c = c_{\max}$). Then this stress will serve as the second "reference point" for determine the constant A_k according to (3.2). Then constant B_k is found from formula (3.1). As a result (with the known constants A_k and B_k), for an arbitrary form of the stress state, we represent the stress σ_1 in the form:

Corresponding Author E-mail: rychkovba@mail.ru

Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

$$\sigma_1(c)=\frac{-A_k c+\sqrt{A_k{}^2-(1-c^2)B_k{}^2}}{1-c^2} \tag{3.3}$$

Dependence (2.10) (for the constants A_k and B_k found in the aforementioned process) allows to determine the tensile strength:

$$\sigma_p=A_k+B_k \tag{3.4}$$

As in the criterion (2.10) consideration, using the case of uniaxial compression and any particular type of stress state as the "reference points", we express the constants A_h and B_h, entering into the criterion (2.13):

$$B_h=\sigma_c;\quad A_h=\frac{\sigma_1^2(1-c)^2-\sigma_c^2}{\sigma_1 c} \tag{3.5}$$

In order to construct the envelope in the case of the Mohr circles triaxial compression with the known constants A_h and B_h, and to determine the tensile strength, we use the ratio:

$$\sigma_1=\frac{A_h c+\sqrt{A_h^2c^2+4(1-c)^2B_h^2}}{2(1-c)^2} \tag{3.6}$$

$$(\sigma_1)_c=\frac{A_h(1+c)\sqrt{A_h^2c^2+4(1-c)^2B_h^2}+4(1-c)^2B_h^2+A_h{}^2c(1+c)}{2(1-c)^3\sqrt{A_h^2c^2+4(1-c)^2B_h^2}}$$

$$\left((\sigma_1)_c=\frac{\partial\sigma_1}{\partial c}\right) \tag{3.7}$$

$$\sigma_p=\frac{A_h-\sqrt{A_h^2+4\sigma_c^2}}{2} \tag{3.8}$$

According to the second variant of determining material constants, the following intervals of the main stresses change are considered: $\sigma_1\in[0,\sigma_c]$, $\sigma_3\in[\sigma_p,0]$. Then the dependence $\sigma_3=\sigma_3(\sigma_1)$ according to the Duishenaliev-Koichumanov criterion is expressed by the formula (2.10), and the envelope coordinates are determined by the formula (2.3). The parameter A_k is calculated as follows:

$$A_k=\frac{\sigma_c^2+\sigma_p^2}{2\sigma_p} \tag{3.9}$$

The parameter B_k in this case is still determined from the formula (3.1).

According to the Hoek-Brown criterion, based on the original formula (2.13) as well as the value of the parameter B_h according to formula (3.5), the dependence of $\sigma_3=\sigma_3(\sigma_1)$ is determined by the formula:

$$\sigma_3=\frac{2\sigma_1+A_h-\sqrt{4(\sigma_1A_h+\sigma_c^2)+A_h^2}}{2} \tag{3.10}$$

and the envelope coordinates are given by formulas (2.3), in which the σ_3' derivative has the form:

$$\sigma_3'=\frac{4(A_h\sigma_3+\sigma_c^2)-2A_h\sqrt{A_h\sigma_3+\sigma_c^2}}{4(A_h\sigma_3+\sigma_c^2)-A_h^2} \tag{3.11}$$

where the parameter A_h is calculated not according to the formula (3.5), but as follows:

$$A_h=\frac{\sigma_p^2-\sigma_c^2}{\sigma_p} \tag{3.12}$$

When processing the experimental data for various rocks, it was found that for some of them it is preferable to use the Duishenaliev-Koichumanov criterion in the form (2.5) rather than (2.10). In these cases, to determine the material constants (A_k,B_k,Q), according to the formulas (2.6) - (2.9), it is necessary to use three "reference points".

4.THE LIMIT STRESS CIRCLES ENVELOPE CONSTRUCTION

4.1. Initial experimental data

Table 4.1 shows the experimental data for talchochlorite cylindrical samples of triaxial compression , presented in the monograph [11]: the axial stress values

Corresponding Author E-mail: rychkovba@mail.ru

Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

for the stress state performed types (c) as well as (empirical) envelope coordinates (and) for the corresponding Mohr circles. Hereinafter, for all the stresses used, to distinguish between the experimental and theoretical values, the corresponding superscripts (experimental and theoretical values) are assigned. The dimension of these stresses and material parameters (having the stress dimension) is 9,81-1 MPa which is not indicated in the tables. Tables 4.2 - 4.5 show the initial data for the other rocks considered.

Table 1. Talchochlorite [11].

$\sigma_1^{\exp}$	c	$\sigma^{\exp}$	$\tau^{\exp}$
945	0	200	390
1320	0.069	400	540
1420	0.116	500	600
1730	0.178	120	700
2340	0.233	1200	880
2790	0.321	1700	980
3850	0.407	2540	1120
5480	0.51	4000	1320

Table 2. Marble – II [11].

$\sigma_1^{\exp}$	c	$\sigma^{\exp}$	$\tau^{\exp}$
765	0	200	340
1115	0.069	350	440
1452	0.116	440	500
2462	0.178	640	640
2850	0.232	1180	920
5150	0.321	3000	1600
7120	0.408	4960	1120
8520	0.508	6380	1320

Table 3. Sandstone P-03[11].

$\sigma_1^{\exp}$	c	$\sigma^{\exp}$	$\tau^{\exp}$
2810	0	300	930
4915	0.07	1480	1960
5760	0.116	1920	2320
7940	0.178	3000	3000
13200	0.227	5800	4600

Table 4. Carrara marble [12]

$\sigma_1^{\exp}$	$\sigma_3^{\exp}$	c
1360	0	0
2350	250	0.106
3150	500	0.159
3565	685	0.192
4055	845	0.208
5550	1650	0.297

Table 5. Manazuru andesite[13]

$\sigma_1^{\exp}$	$\sigma_3^{\exp}$	c
140	0	0
349	16	0.046
381	20	0.05
552	40	0.07
671	70	0.10
806	100	0.12
“875	110	0.13
881	130	0.15

Based on these initial data, the material constants taken to study the two strength criteria are determined first.

4.2. Material Constants Determining

We first consider the methods of the constants determination illustrated in the example of the initial data for talchochlorite.

4.2.1. The T.B. Duishenaliev - K.T. Koichumanov Criterion Constants

Method 1. According to the formulas (2.6), (2.8), (2.9), taking stresses $\sigma_{1(1)}^{\exp} = 945$, $\sigma_{1(2)}^{\exp} = 2340$, $\sigma_{1(3)}^{\exp} = 5480$ (if *c =0, c=0.233, c=0.51*) as “reference points”, we get Q=1.113. Since, theoretically Q≤1, we take Q=1. Then, having the stresses at $c = 0$ and $c = 0.51$ as the “reference points”, according to the formulas (3.1) and (3.2), we get

$$A_k = -3815.38\,,\ B_k = 3696.502 \qquad (4.1)$$

In this case, in accordance with formula (3.4), the tensile strength calculated value is $\sigma_p^T = -118.882$. The tensile strength experimental value is $\sigma_p^{\exp} = -130$.

Corresponding Author E-mail: rychkovba@mail.ru

Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

Method 2. If we take the experimental values of compressive strengths ($\sigma_c^{\exp} = 945$) and tensile strengths ($\sigma_p^{\exp} = -130$) as “reference points”, then according to (3.4) and (3.9) it results:

$$A_k = -3499.71\,,\; B_k = 3369.712 \tag{4.2}$$

4.2.2. The Hoek - Brown Criterion Constants

Method 1. Taking “reference points” at $c = 0$ and $c = 0.51$, according to (3.5) the result is:

$$A_h = 2260.367\,,\; B_h = 945 \tag{4.3}$$

In this case, in accordance with formula (3.8), the tensile strength value σ_p^T = -343,02, that outdoes more than 2.5 times this limit experimental value in absolute module.

Method 2. If the "reference points" are the tensile and compression strength limits, then according to (3.12)

$$A_h = 6739.423\,,\; B_h = 945 \tag{4.4}$$

The described method of material constants determination for talchochlorite is implemented for a large group of rocks, from which all calculated data for five rocks are summarized as an example in Table 4.6. Experimental data in this example was selected for the following rocks: 1 - talchochlorite [11], 2 - marble-II [11], 3 - sandstone P-03 [11], 4 - Carrara marble [12], 5 - Manazuru andesite [13].

Corresponding Author E-mail: rychkovba@mail.ru

Table 4.6. “Reference points” and material constants

Rock	*“Reference points”*	A_k	B_k	Q	$\sigma^T_{p(k)}$	A_h	B_h	$\sigma^T_{p(h)}$
1	c =0; c=0.51	-3815,4	3696,5	1	-118,9	2260,4	945	-343,0
	$\sigma_p^{\exp} = -130; \sigma_c^{\exp} = 945$	-3499.7	3369,7	1		6739,4	945	
2	c =0; c=0.508	-6154,1	6106,4	1	-47,73	3921,6	765	-143,85
	$\sigma_p^{\exp} = -50; \sigma_c^{\exp} = 765$	-5877,3	5827,3	1		11654,5	765	
3	c =0; c=0.116; c=0.227	-2157,7	1901,5	0,13	-256,21	–	–	–
	$\sigma_p^{\exp} = -250; \sigma_c^{\exp} = 2810$							
	c =0; c=0.227	–	–-	–	–	32110	2810	-244,05
4	c =0; c =0.297	-7948,6	7831,4	1	-117,2	8097,2	1360	-222,3
5	*c =0;c =0.15*	-2844,9	2841,4	1	-3,45	4187,7	140	-4,67

Note 1. The tensile strength $\sigma^T_{p(\kappa)}$ calculated by the criterion of T.B.Duishenaliev - K.T. Koichumanov criterion practically coincides with its experimental value (and according to the Hoek – Brown criterion $\sigma^T_{p(h)}$ significantly overrates $\sigma_p^{\exp}$).

4.3. Envelope construction

Tables 4.7–4.19 show the calculated stresses, and Figures 4.1–4.8 show Mohr diagrams.

Table 7. Talchochlorite. The T.B. Duishenaliev - K.T. Koichumanov criterion. Method 1.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{\exp}}{\sigma_1^{\exp}}$*100%	σ^T	$\frac{\sigma^T - \sigma^{\exp}}{\sigma^{\exp}}$*100%	τ^T	$\frac{\tau^T - \tau^{\exp}}{\tau^{\exp}}$*100%
0	945	0	187.5952	-6.20	376.9423	-3.35
0.069	1248.019	-5.45	367.7087	-8.07	497.8868	-7.80
0.116	1500.508	5.67	536.6028	7.32	591.1497	-1.48
0.178	1890.541	9.28	822.7351	14.27	720.5466	2.94
0.233	2292.023	-2.05	1140.737	-4.94	835.7513	-5.03
0.321	3056.594	9.56	1788.963	5.23	1011.923	3.26
0.407	3990.536	4.46	2625.536	3.37	1169.143	4.39
0.51	5480	0	4011.895	0.30	1336.721	1.27

Corresponding Author E-mail: rychkovba@mail.ru

Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

Table 8. Talchochlorite. The T.B. Duishenaliev - K.T. Koichumanov criterion. Method 2.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{\exp}}{\sigma_1^{\exp}}*100\%$	σ^T	$\frac{\sigma^T - \sigma^{\exp}}{\sigma^{\exp}}*100\%$	τ^T	$\frac{\tau^T - \tau^{\exp}}{\tau^{\exp}}*100\%$
0	945	0	200.9186	0.46	386.652	-0.87
0.069	1220.474	-7.54	274.4054	-31.40	424.1879	-21.45
0.116	1448.104	1.80	710.3678	42.07	632.5656	5.43
0.178	1799.239	4.00	1089.565	51.33	738.8861	5.56
0.233	2161.378	-7.63	1471.297	22.61	817.1835	-7.14
0.321	2853.798	2.29	2154.347	26.73	930.6528	-5.04
0.407	3703.362	-3.05	2937.706	15.66	1046.529	-6.56
0.51	5062.967	-7.61	4162.666	4.07	1192.885	-9.63

Note 2. The tables 4.7 and 4.8 comparison shows when using the T.B. Duishenaliev - K.T. Koichumanov criterion, selection of “reference points” has little effect on all the calculated ultimate strengths under the considered stress conditions, as well as at the envelope display to the limit circles. This conclusion is confirmed by the earlier study [14] regarding the given criterion properties.

Table 9. Talchochlorite. The Hoek-Brown criterion. Method 1.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{\exp}}{\sigma_1^{\exp}}*100\%$	σ^T	$\frac{\sigma^T - \sigma^{\exp}}{\sigma^{\exp}}*100\%$	τ^T	$\frac{\tau^T - \tau^{\exp}}{\tau^{\exp}}*100\%$
0	945	0	28.69	-85.65	162.14	-58.43
0.069	1108.99	-15.98	207.08	-48.23	343.15	-36.45
0.116	1249.85	-11.98	347.30	-30.54	427.32	-28.78
0.178	1485.30	-14.14	556.06	-22.77	520.61	-25.63
0.233	1758.49	-24.85	781.61	-34.87	602.73	-31.51
0.321	2385.69	-14.49	1283.74	-24.49	755.47	-22.91
0.407	3369.78	-11.79	2076.30	-18.26	954.80	-14.75
0.51	5480	0	3829.04	-4.27	1306.70	-1.01

Table 10. Talchochlorite. The Hoek-Brown criterion. Method 2.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{\exp}}{\sigma_1^{\exp}}*100\%$	σ^T	$\frac{\sigma^T - \sigma^{\exp}}{\sigma^{\exp}}*100\%$	τ^T	$\frac{\tau^T - \tau^{\exp}}{\tau^{\exp}}*100\%$
0	945	0	26.77	-86.62	156.77	-59.80
0.069	1318.14	-0.14	265.71	-33.57	428.86	-20.58
0.116	1680.45	18.34	473.36	-5.33	579.73	-3.38
0.178	2340.18	35.27	865.65	20.23	813.76	16.25
0.233	3151.0	34.66	1380.22	15.02	1069.57	21.54
0.321	5074.07	81.87	2704.29	59.08	1596.48	62.91
0.407	8113.25	112.39	4975.48	95.89	2291.44	104.59
0.51	14570.58	165.89	10164.53	154.11	3470.46	162.91

Corresponding Author E-mail: rychkovba@mail.ru

Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

Note 3. If the material constants are determined from the "reference points" of uniaxial tension and compression, then according to the Hoek – Brown criterion, the calculated stresses under triaxial compression starting from the case $c = 0.178$, are significantly overestimated (see Table 4.10).

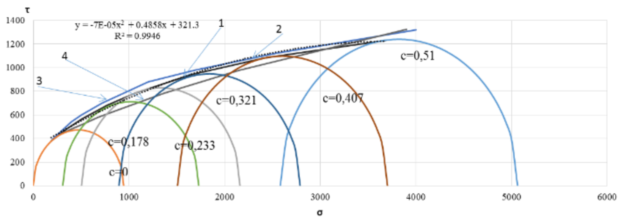

Figure. 1. Talchochlorite. 1 - calculated envelope using the T.B. Duishenaliev - K.T. Koichumanov strength criterion; 2 - approximation by the envelope 1 trend line (reliability of the approximation R^2=0.9946); 3 - empirical envelope; 4 - calculated envelope using the Hoek – Brown strength criterion.

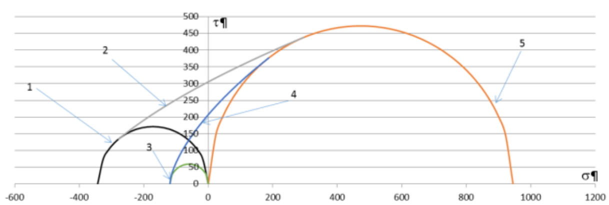

Figure. 2. Talchochlorite.1 - calculated tension circle constructed according to the Hoek – Brown criterion (constants were determined from the triaxial compression "reference points"); 5 - experimental compression circle used as initial data for both strength criteria; 3 - calculated tension circle constructed according to the T.B. Duishenaliev - K.T. Koichumanov criterion; 2 - envelope of the indicated tension and compression circles according to the Hoek-Brown criterion; 4 - envelope of the indicated tension and compression circles according to the T.B. Duishenaliev - K.T. Koichumanov criterion.

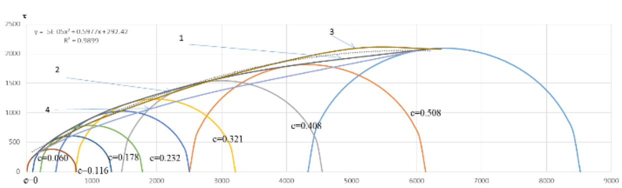

Figure. 3. Marble -II. 1 - Calculated envelope using the T.B. Duishenaliev - K.T. Koichumanov strength criterion; 2 - approximation by the envelope 1 trend line (reliability of the approximation (approximation reliability by the trend line R^2=0.9899); 3 - empirical envelope; 4 - calculated envelope using the Hoek – Brown strength criterion.

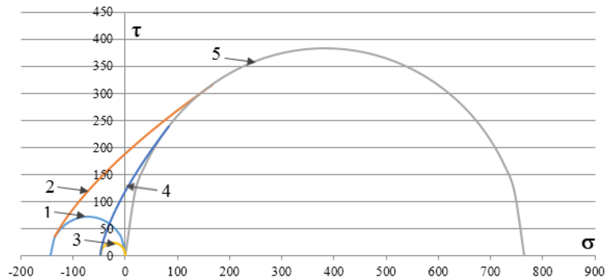

Figure 4. Marble-II. 1 - calculated tension circle constructed according to the Hoek – Brown criterion (constants were determined from the triaxial compression "reference" points); 5 - experimental compression circle; 3 - calculated tension circle constructed according to the T.B. Duishenaliev - K.T. Koichumanov criterion; 2 - envelope of the indicated tension and compression circles according to the Hoek-Brown criterion; 4 - envelope of the indicated tension and compression circles according to the T.B. Duishenaliev - K.T. Koichumanov criterion.

Note 4. The discrepancy between the calculation results for the two studied strength criteria with respect to Marble II is the same as for talchlorite.

Unlike the two rocks considered above, sandstone P - 03 was tested for triaxial compression at relatively low values of lateral pressure (up to the maximum value of the stress state type $c = 0.227$). The experimental data obtained in this case, as well as the value of the tensile strength, are reasonably well displayed by both considered strength criteria. Moreover, when using the criterion of T.B.Duishenaliev - K.T. Koichumanov it turned out to be necessary to apply it in the form of (8); the corresponding values of constants which includes three material constants A_k, B_k, Q given in table 4.6.

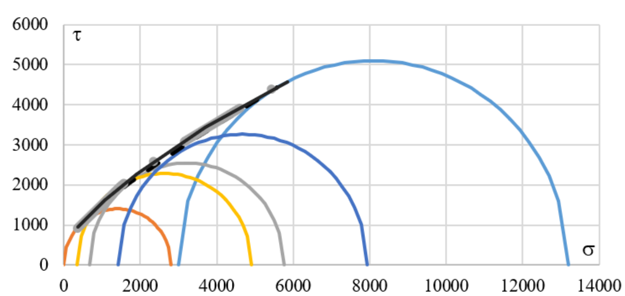

Figure 5. Mohr's circles for Sandstone P-03 under uniaxial (c = 0) and triaxial compression (c> 0) and envelopes to them (empirical - dash-dotted line, constructed according to two criteria: solid line - according to the Hoek-Brown criterion, dashed - according to the Duishenaliev-Koichumanov criterion)

Corresponding Author E-mail: rychkovba@mail.ru

Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

Table 11. Marble -II. The T.B. Duishenaliev - K.T. Koichumanov criterion. Method 1.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{\exp}}{\sigma_1^{\exp}}*100\%$	σ^T	$\frac{\sigma^T - \sigma^{\exp}}{\sigma^{\exp}}*100\%$	τ^T	$\frac{\tau^T - \tau^{\exp}}{\tau^{\exp}}*100\%$
0	765	0	84.581	-57.710	239.896	-29.442
0.069	1304.203	16.969	299.782	-14.348	459.042	4.328
0.116	1780.419	22.618	550.731	25.166	650.586	30.117
0.178	2503.931	1.703	1011.802	58.094	919.075	43.605
0.232	3210.607	12.653	1527.932	29.486	1147.891	24.771
0.321	4548.287	-11.684	2614.903	-12.837	1494.279	-6.608
0.408	6139.035	-13.778	4012.450	-19.104	1790.615	-13.913
0.508	8520	0	6207.642	-2.702	2084.715	0.227

Table 12. Marble -II. The T.B. Duishenaliev - K.T. Koichumanov criterion. Method 2.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{\exp}}{\sigma_1^{\exp}}*100\%$	σ^T	$\frac{\sigma^T - \sigma^{\exp}}{\sigma^{\exp}}*100\%$	τ^T	$\frac{\tau^T - \tau^{\exp}}{\tau^{\exp}}*100\%$
0	765	0	88.106	-55.947	244.210	-28.174
0.069	1275.835	14.425	297.315	-15.053	452.534	2.849
0.116	1725.841	18.860	537.621	22.187	633.193	26.639
0.178	2411.395	-2.0554	977.503	52.735	886.660	38.541
0.232	3082.808	8.169	1469.593	24.542	1103.168	19.910
0.321	4356.421	-15.410	2506.238	-16.459	1431.671	-10.521
0.408	5873.148	-17.512	3839.722	-22.586	1713.244	-17.633
0.508	8145.091	-4.400	5935.086	-6.974	1993.042	-4.181

Table 13. Marble-II. The Hoek – Brown criterion. Method 1.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{\exp}}{\sigma_1^{\exp}}*100\%$	σ^T	$\frac{\sigma^T - \sigma^{\exp}}{\sigma^{\exp}}*100\%$	τ^T	$\frac{\tau^T - \tau^{\exp}}{\tau^{\exp}}*100\%$
0	765	0	167.576	-16.212	316.408	-6.939
0.069	992.626	-10.975	292.611	-16.397	396.090	-9.98
0.116	1204.378	-17.054	416.742	-5.286	467.121	-6.576
0.178	1581.532	-35.762	651.948	1.867	586.814	-8.310
0.232	2031.985	-28.702	950.499	-19.449	719.803	-21.761
0.321	3137.128	-39.085	1736.203	-42.127	1010.709	-36.831
0.408	4909.064	-31.053	3089.226	-37.717	1406.037	-32.402
0.508	8520	0	6026.551	-5.540	2057.876	-1.064

Corresponding Author E-mail: rychkovba@mail.ru

Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

Table 14. Marble -II. The Hoek – Brown criterion. Method 2.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{\exp}}{\sigma_1^{\exp}}$*100%	σ^T	$\frac{\sigma^T - \sigma^{\exp}}{\sigma^{\exp}}$*100%	τ^T	$\frac{\tau^T - \tau^{\exp}}{\tau^{\exp}}$*100%
0	765	0	79.544	-60.228	233.504	-31.323
0.069	1407.487	26.232	300.369	-14.180	474.367	7.811
0.116	2088.57	43.841	600.348	36.443	729.996	45.999
0.178	3330.297	35.268	1255.84	96.225	1172.801	83.250
0.232	4791.241	68.114	2138.368	81.218	1650.445	79.396
0.321	8267.978	60.543	4501.937	50.065	2638.054	64.878
0.408	13689.81	92.273	8566.056	72.703	3907.931	87.881
0.508	24556.8	188.225	17342.07	171.820	5925.846	184.897

Table 15. Sandstone P-03. The T.B. Duishenaliev - K.T. Koichumanov criterion. Method 1.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{\exp}}{\sigma_1^{\exp}}$*100%	σ^T	$\frac{\sigma^T - \sigma^{\exp}}{\sigma^{\exp}}$*100%	τ^T	$\frac{\tau^T - \tau^{\exp}}{\tau^{\exp}}$*100%
0	2810	0	411.421	17.549	993.3912	6.816
0.07	4292.657	-12.662	1046.991	-29.257	1556.569	-20.583
0.116	5760	0	1745.828	-9.071	2079.891	-10.350
0.178	8853.78	11.509	3308.0535	10.268	3099.298	3.310
0.227	13200	0	5570.096	-3.964	4431.372	-3.666

Table 16. Sandstone P-03. The T.B. Duishenaliev - K.T. Koichumanov criterion. Method 2.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{\exp}}{\sigma_1^{\exp}}$*100%	σ^T	$\frac{\sigma^T - \sigma^{\exp}}{\sigma^{\exp}}$*100%	τ^T	$\frac{\tau^T - \tau^{\exp}}{\tau^{\exp}}$*100%
0	2810	0	364,29	4,08	943,90	1,49
0.07	4588,52	-6,64	1061,73	-28,26	1616,08	-17,55
0.116	6356,25	10,35	1894,10	-1,35	2271,94	-2,07
0.178	9667,96	21,76	3697,63	23,25	3435,37	14,51
0.227	13200	0	5851,74	0,89	4580,59	-0,42

Table 17. Carrara marble. The T.B. Duishenaliev - K.T. Koichumanov criterion. Method 1.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{\exp}}{\sigma_1^{\exp}}$*100%	σ^T	τ^T
0	1360	0	198.70	480.36
0.106	2468.43	5.04	707.87	885.40
0.159	3184.43	1.09	1151.38	1145.94
0.192	3691.88	3.56	1501.39	1317.15
0.208	3952.34	-2.53	1690.38	1400.22
0.297	5550	0	2952.66	1839.42

Corresponding Author E-mail: rychkovba@mail.ru

Table 18. Carrara marble. The Hoek – Brown criterion. Method 1.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{exp}}{\sigma_1^{exp}}*100\%$	σ^T	τ^T
0	1360	0	273.26	544.94
0.106	2154.0	-8.34	698.25	826.37
0.159	2762.18	-12.31	1059.38	1028.27
0.192	3254.73	-8.70	1368.18	1183.78
0.208	3528.98	-12.97	1545.3	1267.52
0.297	5550	0	2933.70	1832.63

Table 19. Manazuru andesite. The T.B. Duishenaliev - K.T. Koichumanov criterion. Method 1.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{exp}}{\sigma_1^{exp}}*100\%$	σ^T	τ^T
0	140	0	6.57	29.60
0.046	322.33	-7.64	45.93	92.80
0.05	354.9	-6.86	55.71	105.32
0.07	457.53	-17.11	91.37	146
0.10	631.47	-5.89	166.71	216.48
0.12	743.72	-7.73	223.90	261.58
0.13	753.20	-13.92	229.00	265.35
0.15	881	0	301.59	315.31

Table 20. Manazuru andesite. The Hoek – Brown criterion. Method 1.

c	σ_1^T	$\frac{\sigma_1^T - \sigma_1^{exp}}{\sigma_1^{exp}}*100\%$	σ^T	τ^T
0	140	0	8.26	32.98
0.046	286.12	-18.02	41.35	83.13
0.05	314.32	-17.50	49.48	93.46
0.07	408.49	-26	79.94	128.61
0.10	586.24	-12.63	148.85	195.85
0.12	713.00	-11.54	205.14	243.43
0.13	724.15	-17.24	210.33	247.58
0.15	881	0	286.85	305.27

Corresponding Author E-mail: rychkovba@mail.ru

Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

As can be seen from Figure 4.5, all envelopes (empirical and constructed according to two criteria) practically coincide.

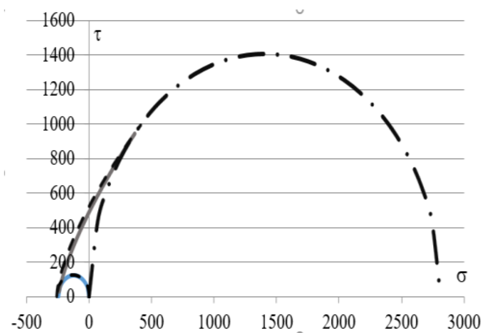

Figure 6. Mohr's circles under uniaxial tension and compression and envelopes for them for Sandstone P - 03. Dashed lines - according to Duishenaliev-Koichumanov criterion, solid lines - according to Hoek-Brown criterion, dash-dotted line - circle on uniaxial compression

As Fig. 4.6 shows, the calculated (by both considered criteria) and experimental values of the tensile strengths for Sandstone P-03 are close to each other, and the envelopes of the tensile and compression circles constructed according to these criteria also practically coincide. This means that the use for this rock of one or the other of the two "reference points" to determine the material constants is practically equivalent. This is directly confirmed when displaying the triaxial compression tensile strengths under all types of stress state carried out in the experiment. Therefore, the results of the calculation are presented only through the first method of determining material constants.

Due to the absence of the tensile strength experimental value of Carrara marble [12], the calculation results presented below were obtained on the basis of the corresponding initial data of uniaxial and triaxial compression.

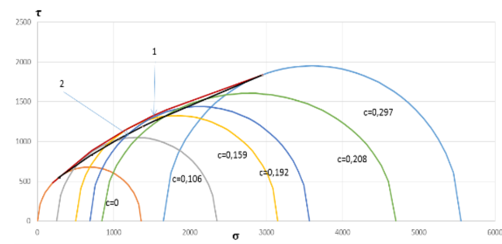

Figure 7. Carrara marble. 1- calculated envelope by the T.B. Duishenaliev - K.T. Koichumanov strength criterion; 2 - calculated envelope by the Hoek-Brown strength criterion.

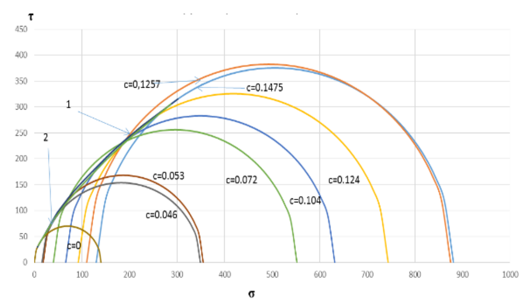

Figure 8.Manazuru andesite. 1- calculated envelope by the T.B. Duishenaliev - K.T. Koichumanov strength criterion; 2 - calculated envelope by the Hoek-Brown strength criterion.

5. CONCLUSION

The Hoek-Brown empirical criterion adequately reflects the tensile strengths under triaxial compression in cases when the stress state type parameter (c) varies in the interval c~0; 0.25. In these cases, as indicated in [10], the maximum stress values generated under lateral pressure are . When , this criterion does not adequately reflect the experimental data, as demonstrated above by the Talchochlorite and Marble- II examples. In these cases, preference should be given to the of T.B.Duishenaliev - K.T. Koichumanov criterion.

The T.B. Duishenaliev - K.T. Koichumanov criterion advantage (formulated strictly mathematically) consists in the fact that to determine the material parameters included in this criterion either triaxial compression experimental data (of two or three "reference points") can be used, or the uniaxial tensile strength value as one of the necessary "reference points" can be taken.

Corresponding Author E-mail: rychkovba@mail.ru

Acta Materialia Turcica

Volume 4, December 2020, Pages 18-31

In this case, the entire set of experimental limit stress circles under triaxial stress state, both in that and in the other case of determining material constants, is theoretically quite satisfactorily displayed. The Hoek – Brown criterion does not possess such a property.

REFERENCES

[1] State Standart 21153.8-88. Rocks. Method for determining the tensile strength in bulk compression. Moscow, Standartinform Publ., 1988. (In Russian).

[2] T.B. Duyshenaliev, K.T. Koichumanov, Uravnenie ogibauschei predelnyh krugov napryajenii. [The envelope equation of limit stress circles], Ilim Publ., Bishkek, 2006. (In Russian).

[3] Granino A. Korn, Theresa M. Korn, Mathematical handbook for scientists and engineers definitions, theorems and formulas. For reference and review. Second, enlargend and revised edition, M.S. McGraw-Hill Book Company. New York, San Francisco Toronto London Sydney, 1968 (Moscow: Nauka, 1974).

[4] A.V. Pogorelov, Differencialnaya geomtria [Differential geometry], Nauka Publ., Moscow, 1974. (In Russian).

[5] B.A. Rychkov, O prochnostnyh harakteristikah gornyh porod [About strength characteristics of rocks]. //Sovremennye problem mehaniki sploshnyh sred, Bishkek. 13 (2011) 310-317. (In Russian).

[6] E. Hoek, E.T. Brown,. Empirical strength criterion for rock masses, J.Geotech. Engng Div., ASCE 106(GT9) (1980) 1013-1035.

[7] E. Hoek, Strength of jointed rock masses, 23rd Rankine Lecture. Géotechnique. 33(3) (1983) 187–223.

[8] W.G. Pariseau, Fitting failure criteria to laboratory strength tests, International Journal of Rock Mechanics and Mining Sciences. 44 (2007) 637-646.

[9] W. G. Pariseau, Design Analysis in Rock Mechanics, Taylor and Francis, 2009.

[10] E. Eberhardt, Rock Mechanics and Rock Engineering, Springer Vienna, 45(6) (2012) 981–988.

[11] A.N. Stavrogin, A.G. Protosenya, Plastichnost' gornyh porod [Plasticity of Rocks], Nedra Publ., Moscow, 1979. (In Russian).

[12] T. von Karman: Festigkeitsversuche unter allseitigem Druck, Zeitschreift des Vereins Deutscher Ingenieure, 55 (1911) 1749–1757.

[13] K. Mogi, Experimental rock mechanics, CRC Press, London, 2007.

[14] B.A. Rychkov, Zh.Y. Mamatov, E.I. Kondrat'eva, Determination of the ultimate tensile strength of rocks by the uniaxial compression test data. J. of Mining Science, V.45. №3 (2009) pp.235-239. Translated from Fiziko-Tekhnicheskie Problemy Razrabotki Poleznyh Iskopaemyh. №3, pp.40-45, May-June, 2009. Original article submitted February 12, 2009.

Corresponding Author E-mail: rychkovba@mail.ru