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# A Study on Modeling of Lifetime with Right-Truncated Composite Lognormal-Pareto Distribution: Actuarial Premium Calculations

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## Highlights

- This paper focused on premium calculations of the right truncated marginals $(L_{tr}, P_{tr})$ and composite $(LP_{tr})$ lifetime models.
- Premiums were calculated for whole life and term life insurances in the single and joint life statuses.
- Models giving the highest and lowest premium coefficients were determined.
- It was seen from the results that there are significantly differences in the premiums.

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**Abstract**

The modeling of lifetime is important to compute actuarial quantities such as the premium on insurance and annuity products. De Moivre, Gompertz, and Makeham are laws of mortality frequently used in lifetime modeling. Composite distributions have also been used to model lifetime, recently. However, there are not many actuarial applications of these models in the literature. Therefore, the main aim of the study is to perform a case study that gives a comparison of marginal and composite models on premiums. For this purpose, firstly, it is aimed to achieve a new mortality function for a lifetime using composite distribution. The second aim is to analytically compute premiums for whole life and term life insurance products. Here, it is assumed that lifetime distribution is modeled with lognormal, Type 2 Pareto (Pareto) and composite lognormal-Pareto. Firstly, the right truncated distributions of the models were obtained under the consideration that the last age of death was 100. Afterwards, the survival and mortality functions were inferenced using Mathematica 10.2 for the right truncated models. Finally, premium coefficients were analytically presented for whole life and term life insurances in single and joint life statuses. The results show that there are significantly differences in these premium coefficients. It has been observed that the premium coefficients for the term life insurance were higher than the premium coefficients for whole life insurance. In addition, the premium coefficients of the insurances issued for the joint life were smaller than the premium coefficients for the single life.

## 1. INTRODUCTION

It is important to model future lifetimes in order to perform actuarial calculations on annuity and insurance products including single life and joint life states. There are well-known laws of mortality, such as De Moivre, Gompertz, and Makeham, in the literature on lifetime. However, with these laws, an entire age range can be modeled. In reality, however life data can be better modeled with different probability functions for certain age ranges that can be classified as young, middle and old. On the other hand, Pareto distribution is used in the modeling of major damages. However, while lognormal or Weibull distributions are preferred in modeling small high-frequency damages (right tail) and large low-frequency damages (left tail), these distributions may not be able to model the right and left tail data well at the same time [1]. Composite distribution models created with different probability distributions have been developed to overcome such situations [1-4]. There are also other studies that show that composite distributions fit better in lifetime data [5,6]. Klugman et al. [1] and Cooray and Ananda [2] used Danish fire insurance data to show that composite models fit better than standard univariate distributions. In their work, they used the lognormal and Pareto probability functions. Scollnik [3] developed the composite lognormal-Pareto model by weighting the coefficients of each piecewise function and used the generalized Pareto distribution in his work. Teodorescu and Vernic [7] studied the exponential-Pareto composite model to predict the future

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claims that will be helpful in evaluating premium calculations. By combining the Type 2 Pareto (Pareto) and Burr distributions with lognormal, Nadarajah and Bakar [8] developed lognormal-Pareto and lognormal-Burr composite models. They also showed composite lognormal - Pareto model fit better to the same Danish fire insurance data.

Dominicy and Sinner [4] included descriptive examples showing that various composite models are better suited to data in their study. They carried out the goodness of fit with internet and Danish fire insurance data for composite models developed by weighting Pareto, Lomax, Generalized Lomax, Generalized Extreme Distributions with Lognormal and Weibull models. They have shown that the lognormal-Pareto fits better.

The data sets used in the studies [5, 6] are “multivariate survival data” and “survival models in which the dependence between the two survival times is through stochastically related unobserved components. However, these data sets are also different from “lifetime” data which are used in the actuarial premium calculations. Although it is stated here that composite models fit better in the data sets used in the studies, the aim of this study is not to show that composite models fit the lifetime data better. Researchers can try to demonstrate that these existing composite models or new models better fit insurance data.

The aims of this study are to model a new mortality function for “lifetime” data with lognormal-Pareto 2 composite distribution and to make a case study evaluating premium calculations for whole life and term life insurance products using this new composite mortality function. Also, at the end of the study, it is aimed to compare the composite model with other models in terms of premium coefficients for whole life and the term life insurances. Thus, the models giving the highest and lowest premium coefficients according to the insurance products can be determined for the selected ages.

Dickson et al. [9] and Menge and Glover [10] also provided methods for calculating the quantities that are important in actuary, such as the probability of life and death, life expectancy, premium for whole life insurance and term life insurance under the single and the joint life states in detail. There are studies that is made theoretical premium calculations using known mortality laws such as De Moivre, Gompertz and Makeham [11-17]. Recently, lognormal and Pareto distributions have been used by Büyükyazıcı and Karagül [18] to calculate the reinsurance premium in the insurance area. However, no previous actuarial applications including premium calculations made using composite models are found in the literature review.

Therefore, in this study, only the lognormal-Pareto (LP) composite model created with lognormal (L) and Type 2 Pareto (P) marginal probability distributions are used in order to show that there may be significant differences, especially for ages after a certain age. For this purpose, a new mortality law is obtained by modeling lifetimes, which are examined at conditions before and after a certain age by using a composite probability distribution. The net single and annual premiums are calculated according to this new mortality law.

Here, it is assumed that the last death age is 100 and (0,100) age range is considered as two age groups as (0, 60) and (60, 100). The correct determination of insurance and annuity premiums while it is important for the age range of (0, 60), it is also important for the age range of (60,100) because most deaths occur within this age range. It is known that COVID 19, which is a pandemic epidemic today, has a fatal effect especially on individuals over 65 years of age [19, 20]. Therefore, in the future, this may require updating the death benefits and premium calculations for these age ranges. On the other hand, there are studies that the elderly population is predicted to increase. [21-23]. In this case, even now there are insurance companies selling life insurance for the individuals at over the age of 60, this number can be expected to increase in the future. Therefore, it can be said that the age groups selected in this study are significant.

In this sense, the study tries to show changes in premiums for these age groups according to lognormal, Pareto and lognormal-Pareto models. Firstly, the right-truncated probability distributions of lognormal and type 2 Pareto distributions are obtained. Secondly, using these truncated distributions for the age range (0, 100), the lognormal - Pareto composite probability function are created. Then, survival and mortality functions based on these probability models, which are shortly shown with the notations $L_{tr}$, $P_{tr}$ and $LP_{tr}$ respectively, are obtained. These actuarial quantities are examined for the single life and the joint life states consisting of two individuals with independent lifetimes, and the results were presented with tables and graphs according to some parameter values. In addition, according to these states and models, the present values (net single premiums) for life insurances and life annuities are calculated and annual premium values are compared.

From the results of this study, it is seen that premiums increase as the age progresses. It can be said that the premium coefficients for the term life insurance are higher than the premium coefficients for whole life insurance. Another result is that the premium coefficients for the insurances issued for the joint life are smaller than the premium coefficients for the single life. On the other hand, it is also made the comparison of results in terms of the models. Computational results show that, there is a difference in the premiums for the single life and the joint life states. If two individuals in spouse status want to purchase an insurance product separately (the single life status), the sum of the premiums to be paid will be higher than the premium determined in the joint life status.

The paper is organized as follows: in section 2, marginal probability functions and the composite model are introduced. In the same section, the survival- mortality functions and premium formulas are also given for the single life and the joint life consisting of two individuals with the independent future lifetimes. In section 3, the right-truncated marginal functions and the composite distribution are created using lognormal, Pareto and lognormal-Pareto models. In addition, the survival and the mortality functions are obtained. For these models right-truncated, the net single and the net annual premium calculations are inferenced in section 4 and presented with tables and graphs for some selected parameter values. Finally, the results are given in same section and evaluated in the discussion and conclusion section of the study.

## 2. METHODOLOGY

### 2.1. Marginal Probability Functions and Composite Model

$X$ random variable has lognormal distribution with $logX = Y$ transformation with $Y$ being a random variable with a normal distribution. The probability density function and distribution function of the lognormal distribution are defined as follows, respectively:

$$f(x,\mu,\sigma^2) = \frac{1}{x\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{(logx-\mu)^2}{2\sigma^2}}, x > 0 \tag{1}$$

$$F(x,\mu,\sigma^2) = \phi\left(\frac{logx-\mu}{\sigma}\right), \qquad x > 0\,. \tag{2}$$

Here $\mu \in IR$, is the location parameter, $\sigma > 0$ is the scale parameter. $\phi(x)$ shows the cumulative distribution function of the standard normal distribution.

Probability density function and distribution function of Type 2 Pareto ($\mu = 0$), also known as Lomax distribution ( $\mathrm{k} > 0$, $\alpha > 0$):

$$f(x) = \frac{\alpha}{k}\left(1+\frac{x}{k}\right)^{-(\alpha+1)}, x > 0 \tag{3}$$

$$F(x) = 1-\left(1+\frac{x}{k}\right)^{-\alpha}, x > 0\,. \tag{4}$$

The probability function and distribution function of the composite model defined by two probability density functions $f_1(x)$ and $f_2(x)$, by Klugman et al. [1] are defined as follows:

$$f(x) = \begin{cases} cf_1^*(x), & 0 < x \le \theta \\ (1-c)f_2^*(x), & \theta < x < \infty \end{cases} \tag{5}$$

$$F(x) = \begin{cases} cf_1^*(x), & 0 < x \le \theta \\ (1-c)f_2^*(x), & \theta < x < \infty \end{cases} . \tag{6}$$

Here, $f_1^*(x) = \frac{f_1(x)}{\int_0^\theta f_1(x)dx}$ and $f_2^*(x) = \frac{f_2(x)}{\int_\theta^x f_2(x)dx}$, show probability functions of truncated limit ranges, $0 < c < 1$ shows weightings and $\theta$ shows the limits of the distribution. Here $f(x)$ function has to provide $f(\theta^-) = f(\theta^+)$, continuity and $f'(\theta^-) = f'(\theta^+)$, differentiation conditions. So here the following equations related to the $c$ constant could be written according to the continuity and differentiation conditions, respectively:

$$c = \frac{f_2(\theta)F_1(\theta)}{f_2(\theta)F_1(\theta) + f_1(\theta)\big(1 - F_2(\theta)\big)} \tag{7}$$

and

$$c = \frac{f_2'(\theta)F_1(\theta)}{f_2'(\theta)F_1(\theta) + f_1'(\theta)(1 - F_2(\theta))} . \tag{8}$$

Here, the requirement below is obtained by equalization of the Equations (7) and (8)

$$\frac{f_1(\theta)}{f_2(\theta)} = \frac{f_1'(\theta)}{f_2'(\theta)} . \tag{9}$$

**2.2. The Survival and Mortality Functions for the Single and the Joint Life Statuses**

***The single life status:***

The future lifetime of a newborn baby is denoted by the random variable $T$. The random variable $T$ is also called the survival time. $F(t) = P(T \le t)$ is the distribution function of $T$. That is, $F(t)$ is the probability that T is less than or equal to $t$. The survival function, $S(t)$ and the hazard function, $h(t)$ are defined by $S(t) = 1 - F(t)$ and $h(t) = f(t)/S(t)$, respectively.

The future lifetime beyond the age of $x$ given the individual has survived to age is denoted by $T_x$ and it is defined as $T_x = T - x|T \ge x$. The distribution function and survival function of $T_x$ are expressed as follows, respectively:

$$F_X(t) = P(T < x + t|T \ge x) \tag{10}$$

$$S_X(t) = P(T \ge x + t|T \ge x) \tag{11}$$

The probability density function and the mortality function of $T_x$ are given by, respectively, $f_X(t) = -\frac{d}{dt}S_X(t)$ and $\mu(x+t) = \frac{f_X(t)}{S_X(t)}$. Here, the actuarial notations ${}_tq_x$ and ${}_tp_x$ are used instead of $F_X(t)$ and $S_X(t)$, respectively. ${}_tq_x$ is the probability that a person aged $x$ will die before the age of $x + t$. ${}_tp_x$ is the probability that a person aged $x$ will survive to the age of $x + t$. There are the following relations between mortality and survival functions [24, 25].

$$f_X(t) = {}_tp_x\mu(x+t) \tag{12}$$

$${}_tp_x = \mathrm{e}^{-\int_0^t \mu(x+s)\,\mathrm{d}s}, 0 \le x < w, 0 \le t < w - x \tag{13}$$

***The joint life status:***

For the joint life status consisting of two lives aged $x$ and $y$ with independent future lifetimes, these notations and formulas related to the survival and death probabilities are defined below according to joint life (first death) and last survivor states. It is assumed here that the future life times of these individuals are independent [9,10].

*The first death state*: The probability that both lives aged $x$ and $y$ survive $t$ years is denoted by ${}_tp_{xy}$ and defined as

$$ {}_tp_{xy} = {}_tp_x \, {}_tp_y \, . \tag{14}$$

The mortality function for the joint lives is defined by

$$\mu_{x+t:y+t} = \mu_{x+t} + \mu_{y+t} \; . \tag{15}$$

*The last survivor state*: The probability of the last survivor living at least t years out of two people at $x$ and $y$ years of age is indicated by ${}_tp_{\overline{xy}}$ and defined as

$$ {}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_x \; {}_tp_y \, . \tag{16}$$

The mortality function for the last survivor state is defined by

$$ {}_t\mu_{\overline{xy}} = \frac{{}_tq_y \; {}_tp_x \; \mu_{x+t} + {}_tq_x \; {}_tp_y \; \mu_{y+t}}{{}_tq_y \; {}_tp_x + {}_tq_x \; {}_tp_y \; + {}_tp_x \; {}_tp_y} \; . \tag{17}$$

### 2.3. The Net Single and the Net Annual Premium Calculations

***The single life status:*** Formulas that give the present values (or the net single premium) of the whole (temporary) life annuity $a_x(a_{x:n]})$ and whole (term) life insurance $A_x(A_{x:n]})$ with a death benefit of 1 unit issued for an $x$-years-old person and the net annual premium $P_x(P_{x:n]})$ calculations are provided in Table 1 for the single life status [9, 10].

***Table 1.*** *Formulas that gives the net single and annual premiums for the single life status*

| | The single premiums (present values) | The net annual premiums |
|---|---|---|
| Whole life | $a_x = \int_0^\infty e^{-\delta t} \, {}_tp_x dt$<br>$A_x = 1 - \delta a_x$ | $P_x = \frac{A_x}{a_x}$ |
| Temporary (Term) life | $a_{x:n]} = \int_0^n e^{-\delta t} \, {}_tp_x dt$<br>$A_{x:n]} = 1 - \delta a_{x:n]}$ | $P_{x:n]} = \frac{A_{x:n]}}{a_{x:n]}}$ |

***The joint life status:*** The notations and formulas related to the present values (or the net single premium) of whole (temporary) life annuity $a_{xy}(a_{xy:n]})$ and whole (term) life insurance $A_{xy}$ $(A_{xy:n]})$ with a death benefit of 1 unit issued for the joint life status consisting of two lives aged x and y with independent future lifetimes are defined according to the joint life (first death) and the last survivor states in Table 2 [9,10]. Also, the annual premium calculations $P_{xy}$ $(P_{xy:n]})$ are given.

***Table 2.*** *Formulas that gives the net single and net annual premiums for the joint life status*

| | *First death state* | *Last survivor state* |
|---|---|---|
| Whole life | $a_{xy} = \int_0^\infty e^{-\delta t}\,{}_tp_{xy}dt$ <br> $A_{xy} = 1 - \delta a_{xy}$ <br> $P_{xy} = \frac{A_{xy}}{a_{xy}}$ | $a_{\overline{xy}} = \int_0^\infty e^{-\delta t}\,{}_tp_{\overline{xy}}dt$ <br> $A_{\overline{xy}} = 1 - \delta a_{\overline{xy}} = A_x + A_y - A_{xy}$ <br> $P_{\overline{xy}} = \frac{A_{\overline{xy}}}{a_{\overline{xy}}}$ |
| Temporary (Term) life | $a_{xy:n]} = \int_0^n e^{-\delta t}\,{}_tp_{xy}dt$ <br> $A_{xy:n]} = 1 - \delta a_{xy:n]}$ <br> $P_{xy:n]} = \frac{A_{xy:n]}}{a_{xy:n]}}$ | $a_{\overline{xy}:n]} = \int_0^n e^{-\delta t}\,{}_tp_{\overline{xy}}dt$ <br> $A_{\overline{xy}:n]} = 1 - \delta a_{\overline{xy}:n]} = A_{x:n]} + A_{y:n]} - A_{xy:n]}$ <br> $P_{\overline{xy}:n]} = \frac{A_{\overline{xy}:n]}}{a_{\overline{xy}:n]}}$ |

The constant, $\delta$ in Tables 1 and 2 shows continuously payable interest rate, i.e. the force of interest rate.

## 3. MODELING LIFETIME WITH A RIGHT-TRUNCATED COMPOSITE MODEL

### 3.1. A Right-Truncated Composite Model

In this study, right-truncated lognormal ($fL_{tr}$) and Pareto ($fP_{tr}$) probability distributions were used to create the composite probability model ($fLP_{tr}$) given with the Equation (5) in the range $0 < x < 100$ instead of $f_1(x)$ and $f_2(x)$ respectively. The composite lognormal-Pareto probability function and distribution function, which is formed by right-truncated marginal functions obtained in this way, were obtained as in Equations (22) and (23), respectively.

The pdf and cdf of $L_{tr}$ model:

$$fL_{tr}(x,\mu,\sigma^2) = \frac{1}{x\sqrt{2\pi}\sigma\phi\left(\frac{log\,[100]-\mu}{\sigma}\right)} e^{-\frac{(log[x]-\mu)^2}{2\sigma^2}}, \quad 0 < x < 100 \tag{18}$$

$$FL_{tr}(x,\mu,\sigma^2) = \frac{\phi\left(\frac{log\,[x]-\mu}{\sigma}\right)}{\phi\left(\frac{log\,[100]-\mu}{\sigma}\right)}, \qquad 0 < x < 100\ . \tag{19}$$

The pdf and cdf of $P_{tr}$ model:

$$fP_{tr}(x,k,\alpha) = \frac{\left(\frac{k+x}{k}\right)^{-1-\alpha}\alpha}{\left(1-\left(1+\frac{100}{k}\right)^{-\alpha}\right)k}, \qquad 0 < x < 100 \tag{20}$$

$$FP_{tr}(x,k,\alpha) = \left(1+\frac{1}{-1+\left(\frac{100+k}{k}\right)^{\alpha}}\right)\left(1-\left(\frac{k+x}{k}\right)^{-\alpha}\right), 0 < x < 100. \tag{21}$$

The pdf and cdf of $LP_{tr}$ model:

$$fLP_{tr}(x) = \begin{cases} c\dfrac{e^{-\frac{(\mu-log[x])^2}{2\sigma^2}}}{\sqrt{2\pi}x\sigma\phi\left(\dfrac{log\,[\theta]-\mu}{\sigma}\right)}, & 0 < x \leq \theta \\ \dfrac{(1-c)(\frac{k+x}{k})^{-\alpha}\alpha}{(k+x)(-(\frac{100+k}{k})^{-\alpha}+(\frac{k+\theta}{k})^{-\alpha})}, & \theta < x < 100 \end{cases} \tag{22}$$

$$FLP_{tr}(x) = \begin{cases} c\left(\dfrac{\phi\left(\frac{log[x]-\mu}{\sigma}\right)}{\phi\left(\frac{log[\theta]-\mu}{\sigma}\right)}\right), & 0 < x \leq \theta \\ \dfrac{c\left(\frac{k+\theta}{k}\right)^{\alpha}\left[\left(\frac{100+k}{k+x}\right)^{\alpha}-1\right]+\left(\frac{100+k}{k}\right)^{\alpha}\left[1-\left(\frac{k+\theta}{k+x}\right)^{\alpha}\right]}{\left[\left(\frac{100+k}{k}\right)^{\alpha}-\left(\frac{k+\theta}{k}\right)^{\alpha}\right]}, & \theta < x < 100. \end{cases} \tag{23}$$

Here c is as follows:

$$c = \frac{\alpha\theta\sigma\phi\left(\frac{log\,[\theta]-\mu}{\sigma}\right)}{\frac{(k+\theta)}{\sqrt{2\pi}}(1-\left(\frac{k+\theta}{100+k}\right)^{\alpha})e^{-\frac{(\mu-\mathrm{Log}[\theta])^2}{2\sigma^2}}+\alpha\theta\sigma\phi\left(\frac{log\,[\theta]-\mu}{\sigma}\right)},\quad 0 < c < 1\ . \tag{24}$$

Solving the equations under the continuity and the differentiability conditions given by Equations (7) and (8) respectively, between in the parameters of the composite lognormal-Pareto distribution

$$\mu = log(\theta) - \left(\frac{\alpha\theta - k}{\theta + k}\right)\sigma^2 \tag{25}$$

relation is achieved. In the fourth section, this relation has been used in parameter selection. The derivatives required solving of 'c' in the Equation (24) are given in Appendix A1. Also, it is shown in Appendix A2 that $c$ is in the range (0,1).

### 3.2. The Survival and Mortality Functions for Truncated Models

In a single-life status, survival and mortality functions for the $L_{tr}$, $P_{tr}$ and $LP_{tr}$ distributions were obtained using Equations (12) and (13), respectively as follows. Mathematical expressions of survival and mortality functions, which are also found for the joint life status, are not given here because they are very long. However, the numerical results of these actuarial quantities calculated at the selected parameter values with the help of Mathematica 10.2 program are given in Table 2. Graphs showing changes according to age are also presented in Figure 3.

$L_{tr}$ model:

$$
{}_tp_x=\frac{\phi\left(\frac{log\,[100]-\mu}{\sigma}\right)-\phi\left(\frac{log\,[x+t]-\mu}{\sigma}\right)}{\phi\left(\frac{log\,[100]-\mu}{\sigma}\right)-\phi\left(\frac{log\,[x]-\mu}{\sigma}\right)} \tag{26}
$$

$$
\mu(x+t)=\frac{e^{-\frac{(\mu-log[t+x])^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma(t+x)\left(\phi\left(\frac{log\,[100]-\mu}{\sigma}\right)-\phi\left(\frac{log\,[x+t]-\mu}{\sigma}\right)\right)}\,. \tag{27}
$$

$P_{tr}$ model:

$$
{}_tp_x=\frac{(\frac{k+x}{k})^{\alpha}(\frac{k+t+x}{k})^{-\alpha}(-(\frac{100+k}{k})^{\alpha}+(\frac{k+t+x}{k})^{\alpha})}{-(\frac{100+k}{k})^{\alpha}+(\frac{k+x}{k})^{\alpha}} \tag{28}
$$

$$
\mu(x+t)=\frac{(\frac{100+k}{k})^{\alpha}\alpha}{(k+t+x)((\frac{100+k}{k})^{\alpha}-(\frac{k+t+x}{k})^{\alpha})} \tag{29}
$$

$LP_{tr}$ model:

$$
{}_tp_x=\begin{cases}\dfrac{1-\frac{1}{m}\phi\left(\frac{log[x+t]-\mu}{\sigma}\right)}{1-\frac{1}{m}\phi\left(\frac{log[x]-\mu}{\sigma}\right)},0<x\le\theta\\[2ex] \dfrac{(\frac{k+x}{k})^{\alpha}(\frac{k+t+x}{k})^{-\alpha}(-(\frac{100+k}{k})^{\alpha}+(\frac{k+t+x}{k})^{\alpha})}{-(\frac{100+k}{k})^{\alpha}+(\frac{k+x}{k})^{\alpha}},\ \ \theta<x<100\end{cases} \tag{30}
$$

$$
\mu(x+t)=\begin{cases}\dfrac{ce^{-\frac{(\mu-log[t+x])^2}{2\sigma^2}}}{\sqrt{2\pi}(t+x)\sigma\left(\phi\left(\frac{log[\theta]-\mu}{\sigma}\right)-c\phi\left(\frac{log[x+t]-\mu}{\sigma}\right)\right)},0<x\le\theta\\[2ex] \left(\dfrac{\alpha}{k+t+x}\right)(\dfrac{1}{1-\left(\frac{k+t+x}{100+k}\right)^{\alpha}}),\ \ \theta<x<100\end{cases} \tag{31}
$$

where $m=\frac{1}{c}\phi\left(\frac{log\,[\theta]-\mu}{\sigma}\right)$.

## 4. ACTUARIAL CALCULATIONS

In this section, the actuarial quantities given in Section 3 are calculated for the selected parameters of $\theta=60,\ \mu=1,\ \sigma=0.5,\ k=5,\ \alpha=0.48$ and $\delta=0.06$ under the models ($L_{tr}, P_{tr}$ and $LP_{tr}$). Here, parameters that provide Equations (24) and (25) are selected to perform the case study. For the parameters, the net single and annual premiums for life annuities (whole/temporary) and life insurance (whole/term) under the single life and the joint life statuses are computed using the Mathematica 10.2 program. For selected ages, the values of the survival, mortality and premium functions are given in Table 3 for the single life status and in Tables 4-5 for the joint life status. For the joint life status, the actuarial quantities are evaluated in the first death state for whole life insurance. Here, it is assumed that the net annual premium is payable throughout the entire lifetime of the insured for a whole life insurance (is called 'ordinary life' [10]) and is be n payment for n- year term insurance.

***Table 3.*** *Values at selected ages of the survival* $({}_tp_x)$*, mortality* $({}_t\mu_x)$ *and premium* $(P_x, P_{x:n\rceil})$ *functions under the single life status*

| | | Lognormal | | | | Pareto | | | | Lognormal-Pareto | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | x | 10 | 30 | 50 | 70 | 10 | 30 | 50 | 70 | 10 | 30 | 50 | 70 |
| t=5 | ${}_tp_x$ | 0.9948 | 0.9074 | 0.8342 | 0.7614 | 0.7875 | 0.8485 | 0.8467 | 0.7954 | 0.9951 | 0.9132 | 0.8511 | 0.7954 |
| | $\mu_{x+t}$ | 0.0021 | 0.0219 | 0.0380 | 0.0581 | 0.0437 | 0.0323 | 0.0339 | 0.0480 | 0.0020 | 0.0204 | 0.0335 | 0.0490 |
| t=10 | ${}_tp_x$ | 0.9752 | 0.8037 | 0.6832 | 0.5563 | 0.6417 | 0.7227 | 0.7112 | 0.6090 | 0.9766 | 0.8161 | 0.7154 | 0.6090 |
| | $\mu_{x+t}$ | 0.0061 | 0.0265 | 0.0418 | 0.0681 | 0.0385 | 0.0319 | 0.0359 | 0.0585 | 0.0057 | 0.0244 | 0.0359 | 0.0585 |
| Whole life | $a_x$ | 14.3563 | 11.5444 | 9.7576 | 7.8668 | 9.8467 | 10.7204 | 10.2043 | 8.3773 | 14.2900 | 11.1335 | 6.5236 | 8.3773 |
| | $A_x$ | 0.1386 | 0.3073 | 0.4145 | 0.5279 | 0.4092 | 0.3567 | 0.3877 | 0.4973 | 0.1425 | 0.3319 | 0.6085 | 0.4973 |
| | $P_x$ | 0.0096 | 0.0266 | 0.0424 | 0.0671 | 0.0415 | 0.0333 | 0.0379 | 0.0593 | 0.0099 | 0.0298 | 0.0932 | 0.0593 |
| Term life (n=5) | $a_{x:n\rceil}$ | 4.3125 | 4.1353 | 3.9749 | 3.8176 | 3.8511 | 3.9981 | 3.9980 | 3.8930 | 4.3129 | 4.1469 | 4.0099 | 3.8929 |
| | $A_{x:n\rceil}$ | 0.7412 | 0.7518 | 0.7615 | 0.7709 | 0.7689 | 0.7601 | 0.7601 | 0.7664 | 0.7412 | 0.7511 | 0.7594 | 0.7664 |
| | $P_{x:n\rceil}$ | 0.1718 | 0.1818 | 0.1915 | 0.2019 | 0.1996 | 0.1901 | 0.1901 | 0.1968 | 0.1718 | 0.1811 | 0.1893 | 0.1968 |
| Term life (n=10) | $a_{x:n\rceil}$ | 7.4715 | 6.8836 | 6.4109 | 5.9342 | 6.1369 | 6.5163 | 6.4973 | 6.1507 | 7.4742 | 6.9235 | 6.5236 | 6.1507 |
| | $A_{x:n\rceil}$ | 0.5517 | 0.5869 | 0.6153 | 0.6439 | 0.6318 | 0.6090 | 0.6102 | 0.6309 | 0.5515 | 0.5845 | 0.6085 | 0.6309 |
| | $P_{x:n\rceil}$ | 0.0738 | 0.0852 | 0.0959 | 0.1085 | 0.1029 | 0.0934 | 0.0939 | 0.1026 | 0.0737 | 0.0844 | 0.0933 | 0.1025 |

***Table 4.*** *Values at selected ages of survival function* $({}_tp_{xy})$ *and mortality rates* $({}_t\mu_{xy})$ *under the joint life status*

| | | Lognormal | | | | Pareto | | | | Lognormal-Pareto | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x \ y | | 10 | 30 | 50 | 70 | 10 | 30 | 50 | 70 | 10 | 30 | 50 | 70 |
| 10 | ${}_tp_{xy}$ | 0.9897 | 0.9028 | 0.8300 | 0.7575 | 0.6202 | 0.6682 | 0.6668 | 0.6264 | 0.9903 | 0.9088 | 0.8470 | 0.9648 |
| | ${}_t\mu_{xy}$ | 0.0042 | 0.0240 | 0.0402 | 0.0602 | 0.0874 | 0.0760 | 0.0776 | 0.0927 | 0.0040 | 0.0224 | 0.0355 | 0.0510 |
| 30 | ${}_tp_{xy}$ | 0.9028 | 0.8234 | 0.7570 | 0.6910 | 0.6682 | 0.7199 | 0.7184 | 0.6749 | 0.9088 | 0.8340 | 0.7773 | 0.8854 |
| | ${}_t\mu_{xy}$ | 0.0240 | 0.0438 | 0.0599 | 0.0800 | 0.0760 | 0.0647 | 0.0663 | 0.0813 | 0.0224 | 0.0408 | 0.0539 | 0.0694 |
| 50 | ${}_tp_{xy}$ | 0.8300 | 0.7570 | 0.6960 | 0.6352 | 0.6668 | 0.7184 | 0.7169 | 0.6735 | 0.8470 | 0.7773 | 0.7244 | 0.8251 |
| | ${}_t\mu_{xy}$ | 0.0402 | 0.0599 | 0.0761 | 0.0961 | 0.0776 | 0.0663 | 0.0679 | 0.0829 | 0.0355 | 0.0539 | 0.0670 | 0.0825 |
| 70 | ${}_tp_{xy}$ | 0.7575 | 0.6910 | 0.6352 | 0.5798 | 0.6264 | 0.6749 | 0.6735 | 0.6327 | 0.9648 | 0.8854 | 0.8251 | 0.9399 |
| | ${}_t\mu_{xy}$ | 0.0602 | 0.0800 | 0.0961 | 0.1162 | 0.0927 | 0.0813 | 0.0829 | 0.0980 | 0.0510 | 0.0694 | 0.0825 | 0.0980 |

***Table 5.*** *The net single and the net annual premium values for the joint life status*

| | | Lognormal | | | | Pareto | | | | Lognormal-Pareto | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x \ y | | 10 | 30 | 50 | 70 | 10 | 30 | 50 | 70 | 10 | 30 | 50 | 70 |
| 10 | $a_{xy}$ | 13.1265 | 10.8298 | 9.3294 | 7.6901 | 6.8213 | 7.3630 | 7.1863 | 6.2987 | 13.2111 | 10.6889 | 6.4876 | 9.8468 |
| | $A_{xy}$ | 0.2124 | 0.3502 | 0.4402 | 0.5385 | 0.5907 | 0.5582 | 0.5688 | 0.6220 | 0.2073 | 0.3586 | 0.6107 | 0.4091 |
| | $P_{xy}$ | 0.0161 | 0.0323 | 0.0471 | 0.0700 | 0.0865 | 0.0758 | 0.0791 | 0.0987 | 0.0156 | 0.0335 | 0.0941 | 0.0415 |
| 30 | $a_{xy}$ | 10.8298 | 9.1644 | 8.0488 | 6.8295 | 7.3630 | 7.9673 | 7.7700 | 6.7663 | 10.6889 | 9.2404 | 6.0387 | 10.7204 |
| | $A_{xy}$ | 0.3502 | 0.4501 | 0.5170 | 0.5902 | 0.5582 | 0.5219 | 0.5337 | 0.5940 | 0.3586 | 0.4455 | 0.6376 | 0.3567 |
| | $P_{xy}$ | 0.0323 | 0.0491 | 0.0642 | 0.0864 | 0.0758 | 0.0655 | 0.0686 | 0.0877 | 0.0335 | 0.0482 | 0.1055 | 0.0332 |
| 50 | $a_{xy}$ | 9.3294 | 8.0488 | 7.1716 | 6.2111 | 7.1863 | 7.7700 | 7.6134 | 6.6924 | 6.4876 | 6.0387 | 5.7097 | 10.2043 |
| | $A_{xy}$ | 0.4402 | 0.5170 | 0.5696 | 0.6273 | 0.5688 | 0.5337 | 0.5431 | 0.5984 | 0.6107 | 0.6376 | 0.6574 | 0.3877 |
| | $P_{xy}$ | 0.0471 | 0.0642 | 0.0794 | 0.1010 | 0.0791 | 0.0686 | 0.0713 | 0.0894 | 0.0941 | 0.1055 | 0.1151 | 0.0379 |
| 70 | $a_{xy}$ | 7.6901 | 6.8295 | 6.2111 | 5.5509 | 6.2987 | 6.7663 | 6.6924 | 6.0892 | 12.4328 | 10.5604 | 6.3567 | 12.3331 |
| | $A_{xy}$ | 0.5385 | 0.5902 | 0.6273 | 0.6669 | 0.6220 | 0.5940 | 0.5984 | 0.6346 | 0.2540 | 0.3663 | 0.6185 | 0.2600 |
| | $P_{xy}$ | 0.0700 | 0.0864 | 0.1010 | 0.1201 | 0.0987 | 0.0877 | 0.0894 | 0.1042 | 0.0204 | 0.0346 | 0.0973 | 0.0210 |

The graphs showing changes according to the ages are presented in Figures 1-4. From these graphs, it is seen that the survival probability for each model is decreased by age (Figure 1b for the single life, Figure

3-upper for the joint life). In addition, changes in the mortality rates for the models handled by age can be monitored (Figure 1c for the single life, Figure 3-lower for the joint life). It is seen that the present value (net single premium) of whole life annuity payments decreases according to age (Figure 2a), and the present value (net single premium) of whole life insurance increase according to age (Figure 2b). Net annual premiums also increase according to age (Figure 2c for the single life, Figure 4 for the joint life). On the other hand, it can be understood from these graphs that the premiums are lower or higher for which model according to the ages given.

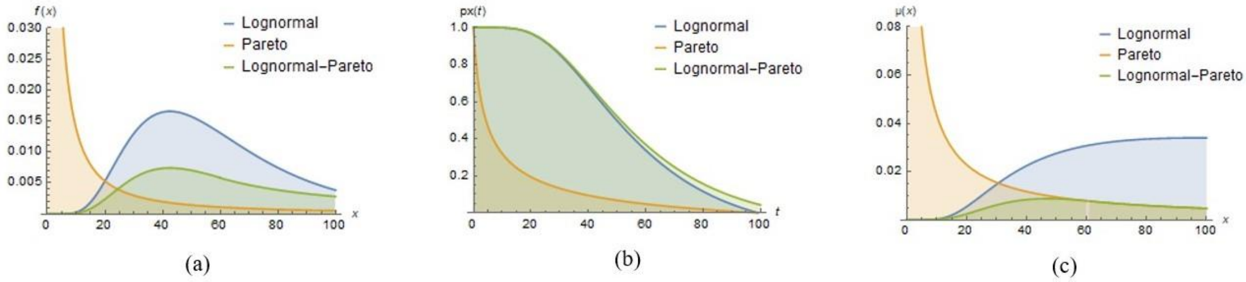


***Figure 1.*** *The probability functions (a), The survival functions (b), The mortality functions (c)*

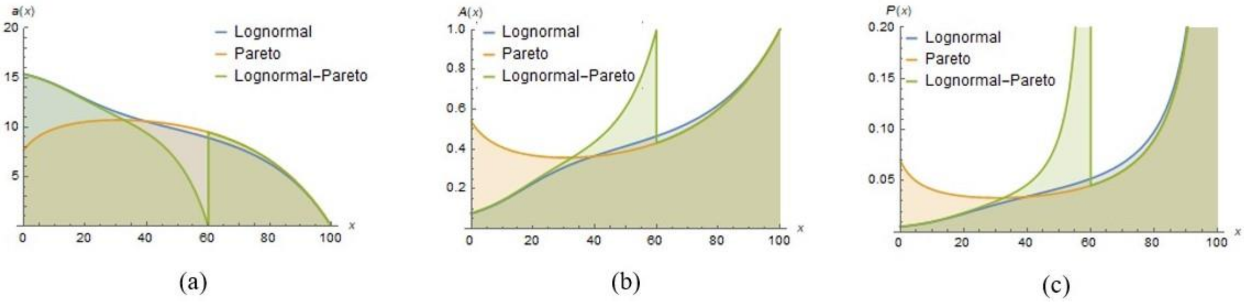


***Figure 2**. The net single premiums of annuity (a) and insurance (b), the net annual premiums (c) for the single life status*

Here, Tables 3 and 5 can be evaluated together. In general, according to the models discussed, there is a difference in premiums for single life and joint life statuses. If two individuals in spouse status want to purchase an insurance product separately (single life status), the total premiums are higher than the premium computed in the joint life status. Sample calculations showing this situation are given below for net single and annual premiums.

***The net single premiums*** for *a whole life insurance* issued for two individuals 30 ages which provides a death benefit of 1000 units at the end of the year of death, according to the $L_{tr}$ $P_{tr}$ and $LP_{tr}$ models, while for *the single life status* (see at the line $A_x$ in Table 3), these are 614.6 (=2×(0.3073×1000)), 713.4 (=2×(0.3567×1000)) and 663.8 (=2×(0.3319×1000)) units respectively, for *the joint life status* in the first death (see at the line $A_{xy}$ in Table 5), these are 450.1(0.4501 ×1000), 521.9 (=0.5219 ×1000) and 445.5 (=0.4455×1000) units.

***The net annual premiums*** for *an ordinary life policy* issued for two individuals 30 ages which provides a death benefit 1000 units paid at the end of the year of death, according to the $L_{tr}$ $P_{tr}$ and $LP_{tr}$ models, while for *the single life status* (see at the line $P_x$ in Table 3), these are 53.2 (=2×(0.0266×1000)), 66.6 (=2×(0.0333×1000)) and 59.6 (=2×(0.0298×1000)) units respectively, for *the joint life status in the first death* (see at the line $P_{xy}$ in Table 5), these are 49.1 (0.0491×1000), 65.5 (=0.0655×1000) and 48.2 (=0.0482×1000) units.

Further examples of Tables 3 and 5 are given in Chapter 4. Here, model comparisons are summarized in Tables 6 and 7 for *the net annual premium* calculations and the results are interpreted in detail.

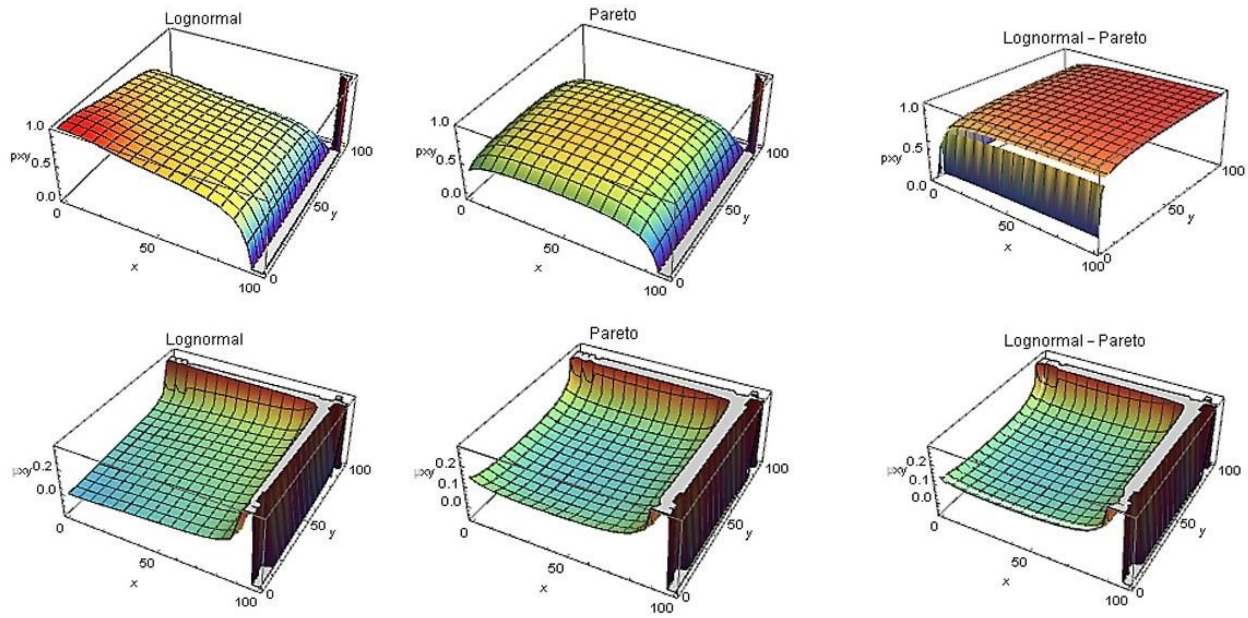


***Figure 3.*** *Graphics of* ${}_tp_{xy}$ *(upper) and* ${}_t\mu_{xy}$ *(lower) functions for the joint life status*

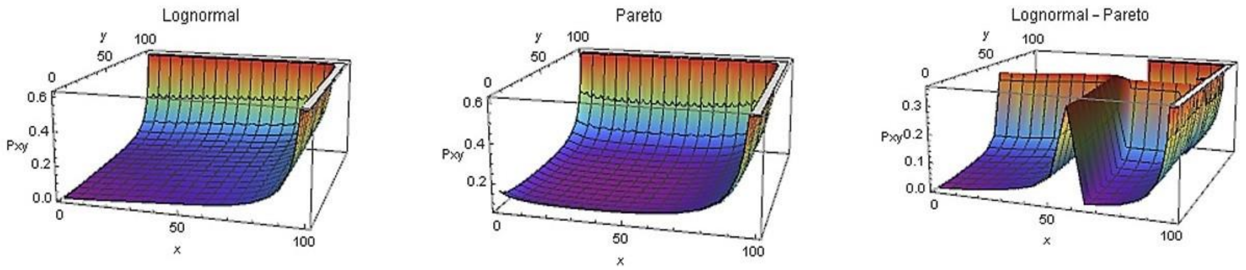


***Figure 4.*** *The net annual premium values for the joint life status*

### 4.1. Illustrative examples

In the previous section, using the information provided in Section 2, life probabilities and mortality rates are calculated for some ages and these results are given in Tables 2 and 4, respectively, for the single life and the joint life. By using these values, the present values of annuity and insurance and therefore net single premiums were calculated. In addition, graphics were drawn to see the changes of these quantities according to selected ages and models. The results obtained for the single life and the joint life are given in Tables 3 and 5 respectively. In order to interpret the results easily, Tables 6 and 7, which are the summary of these tables, were prepared. In addition, taking into account the term life insurance and the last survivor states in Table 7, all the interpretations of the study outputs are exemplified on these tables and the results are listed in this section.

Now, the findings of the study related to net annual premiums for the single life and the joint life statuses will be interpreted on a few illustrative examples. The net annual premium values of life insurance (whole /ten-year term) calculated for some ages according to $L_{tr}$, $P_{tr}$ and $LP_{tr}$ models for the single life and the joint life statuses with independent lifetimes are presented in Tables 6 and 7, respectively. Comparative results of the models according to the net annual premiums are given in detail at the end of the section after sample premium calculations for some ages. In addition, the summary of these explanations is given under each table.

***Table 6***. *The net annual premium values for the single life status*

| | Whole life | | | Term life | | |
|---|---|---|---|---|---|---|
| x | $L_{tr}\_P_x$ | $P_{tr}\_P_x$ | $LP_{tr}\_P_x$ | $L_{tr}\_P_{x:10]}$ | $P_{tr}\_P_{x:10]}$ | $LP_{tr}\_P_{x:10]}$ |
| 40 | 0.0346 | 0.0343 | 0.0463 | 0.0910 | 0.0929 | 0.0894 |
| 45 | 0.0385 | 0.0358 | 0.0618 | 0.0935 | 0.0932 | 0.0915 |
| 65 | 0.0585 | 0.0511 | 0.0511 | 0.1043 | 0.0990 | 0.0990 |
| 70 | 0.0671 | 0.0593 | 0.0593 | 0.1085 | 0.1025 | 0.1025 |
| | If $x < 60$ then $LP_{tr} > L_{tr} > P_{tr}$<br>If $x > 60$ then $L_{tr} > P_{tr} = LP_{tr}$ | | | If $x < 60$ then $LP_{tr} < L_{tr} < P_{tr}$<br>If $x > 60$ then $L_{tr} > P_{tr} = LP_{tr}$ | | |

Table 6 shows some the net annual premium calculations for the single lives at ages 40, 45, 65 and 70. While the net annual premiums, for a whole life insurance (an ordinary life policy) issued for individuals aged 40 , 45, 65 and 70 with a death benefit of 1000 units which will be paid at the end of the year of death, according to the $LP_{tr}$ model, are 46.3 (=0.0463×1000), 61.8 (=0.0618×1000), 51.1 (=0.0511×1000) and 59.3 (=0.0593×1000) units respectively, for ten-year term insurance (with ten-payment) it is 89.4 (0.0894×1000), 91.5 (=0.0915×1000), 99 (0.099×1000) and 102.5 (=0.1025×1000) units.

Table 7 shows some the net annual premium calculations for joint life at ages (40, 45), (45, 65) and (65, 70). The graphs showing the change of premiums by age for each model are presented in Figure 4. From here, it is observed that premiums increase with increasing ages.

***Table 7.*** *The net annual premium values for the joint life status*

| | | First death | | | Last survivor | | |
|---|---|---|---|---|---|---|---|
| | (x,y) | $L_{tr}\_P_{xy}$ | $P_{tr}\_P_{xy}$ | $LP_{tr}\_P_{xy}$ | $L_{tr}\_P_{\overline{xy}}$ | $P_{tr}\_P_{\overline{xy}}$ | $LP_{tr}\_P_{\overline{xy}}$ |
| Whole life | (40,45) | 0.0685 | 0.0671 | 0.0819 | 0.0173 | 0.0159 | 0.0450 |
| | (45,65) | 0.0899 | 0.0808 | 0.0358 | 0.0249 | 0.0224 | 0.0657 |
| | (65,70) | 0.1134 | 0.0981 | 0.0213 | 0.0351 | 0.0308 | 0.1381 |
| | (x,y) | $L_{tr}\_P_{xy:10}$ | $P_{tr}\_P_{xy:10}$ | $LP_{tr}\_P_{xy:10]}$ | $L_{tr}\_P_{\overline{xy}:10}$ | $P_{tr}\_P_{\overline{xy}:}$ | $LP_{tr}\_P_{\overline{xy}:10]}$ |
| Term life | (40,45) | 0.1129 | 0.1147 | 0.1091 | 0.0760 | 0.0761 | 0.0755 |
| | (45,65) | 0.1271 | 0.1212 | 0.0957 | 0.0778 | 0.0770 | 0.0945 |
| | (65,70) | 0.1435 | 0.1312 | 0.0807 | 0.0807 | 0.0786 | 0.1275 |
| Whole life | If $(x < 60, y < 60)$ then $LP_{tr} > L_{tr} > P_{tr}$<br>If $(x < 60, y > 60)$ then $L_{tr} > P_{tr} > LP_{tr}$<br>If $(x > 60, y > 60)$ then $L_{tr} > P_{tr} > LP_{tr}$ | | | | | | |
| Term life | If $(x < 60, y < 60)$ then $P_{tr} > L_{tr} > LP_{tr}$<br>If $(x < 60, y > 60)$ then $L_{tr} > P_{tr} > LP_{tr}$<br>If $(x > 60, y > 60)$ then $L_{tr} > P_{tr} > LP_{tr}$ | | | | If $(x < 60, y < 60)$ then $P_{tr} > L_{tr} > LP_{tr}$<br>If $(x < 60, y > 60)$ then $LP_{tr} > L_{tr} > P_{tr}$<br>If $(x > 60, y > 60)$ then $LP_{tr} > L_{tr} > P_{tr}$ | | |

*For joint life at ages (40, 45)*; while for a whole life insurance (an ordinary life policy) jointly issued for two individuals at ages 40 and 45 with a death benefit of 1000 units to be payable at the end of the year of death if at least one of them dies, the net annual premium is 21.3 (=0.0213×1000) units according to $LP_{tr}$ model, it is 80.7 (=0.0807×1000) units for ten-year term insurance. These values were low compared to the $L_{tr}$ and $P_{tr}$ models. While the premium for 1000 units of whole life insurance to be paid in the event of the death of the last survivor is 138.1 (=0.1381×1000), it was found to be 127.5 (=0.1275×1000) units for ten-year term insurance.

*For joint life at ages (45, 65)*; while for a whole life insurance (an ordinary life policy) jointly issued for two individuals at ages 45 and 65 with a death benefit of 1000 units be payable at the end of the year of death if at least one of them dies, the net annual premium is 35.8 (=0.0358×1000) units according to $LP_{tr}$ model, it is 95.7 (=0.0957×1000) units for ten-year term insurance. These values are low compared to the $L_{tr}$ and $P_{tr}$ models. While the net annual premium for a whole life insurance with a death benefit of 1000

units to be paid in the event of the death of the last survivor is 65.7 (=0.0657×1000), it was found to be 94.5 (=0.0945×1000) units for ten-year term insurance.
*For the joint life at ages (65, 70)*; while for a whole life insurance (an ordinary life policy) jointly issued for two individuals at ages 65 and 70 with a death benefit of 1000 units to be payable at the end of the year of death if at least one of them dies, the net annual premium is 21.3 (=0.0213×1000) units according to $LP_{tr}$ model, it is 80.7 (=0.0807×1000) units for ten-year term insurance. These values are low according to the $L_{tr}$ and $P_{tr}$ models. While the net annual premium for 1000 units of whole life insurance to be paid in the event of the death of the last survivor is 138.1 (=0.1381×1000), it was found to be 127.5 (=0.1275×1000) units for ten-year term insurance.

### 4.2. Results

In fact, it is a known fact that premiums are determined high according to this situation since the risk of death increases with age. In general, according to these models ($L_{tr}$, $P_{tr}$ and $LP_{tr}$), it can be seen from the results in Tables 6 and 7 that the premium coefficients of higher ages are higher for both the single life and the joint life statuses. However, the aim of the study is to compare the premium coefficients for whole life and term life insurances based on these models. In this sense, the conclusions reached for the single life and the joint life are listed below according to the information in Tables 6 and 7, respectively.

*For the single life:*

- In insurance policies issued for ages below 60, for whole life insurance, while the highest and lowest premium coefficients are respectively determined by the $LP_{tr}$ model and the $P_{tr}$ model, for the temporary life insurance, these coefficients are determined by the $P_{tr}$ and the $LP_{tr}$ models, respectively.

-In insurance policies issued for ages older than 60, the highest premium coefficient for both whole life insurance and term life insurances is determined by the $L_{tr}$ model. The premium coefficients of $P_{tr}$ and $LP_{tr}$ models, which give the lowest premium coefficients for these ages, are almost the same. (Although there are differences in decimals, the values appear the same because only 4 digits are written)

*For the joint life:*

-In insurance policies issued for the joint life of both individuals under the age of 60, for whole life insurance, while the highest premium coefficient is determined by the $LP_{tr}$ model and the lowest premium coefficient is determined by the $P_{tr}$ model, for the temporary life insurance, respectively, by the $P_{tr}$ and $LP_{tr}$ model. This situation does not change according to the first death and last survivor conditions.

-In insurance policies issued for the joint life where at least one of the two individuals are over 60 years old, for the whole life insurance, while the highest and lowest premium coefficients are respectively determined with the $L_{tr}$ and the $LP_{tr}$, for the temporary life insurance, these coefficients change according to the first death and last living states: the highest and lowest premium coefficients are respectively determined with the $L_{tr}$ and the $LP_{tr}$ in the first death state, the $LP_{tr}$ and the $P_{tr}$ models in the last survivor state.

According to these results, in general, when the composite model ($LP_{tr}$) is compared to other models, the premium coefficients are determined as the higher or lower. In a whole life insurance issued for the single life, with the $LP_{tr}$, a high net annual premium for individuals aged under 60, and a low net annual premium for individuals aged over 60 are determined (see Table 6). In the case of the joint life, the premium values change according to the policy types issued for the first death and last survivor states. However, for a whole life insurance at the both cases, with the $LP_{tr}$, a high premium is determined for the joint life consisted individuals ages younger than 60. For a term life insurance, with the $LP_{tr}$, in the case of first death, while low premium is determined for the joint life consisting of individuals younger or older the age of 60 , in the last life state, high premium is determined for the joint life formed by individuals where at least one is older than 60 years (see Table 7).

## 5. DISCUSSION AND CONCLUSION

In this study, firstly with the assumption that the last age of death is 100, lognormal and right-truncated probability distributions of Type 2 Pareto distributions are obtained. Then the composite lognormal-Pareto probability function is created for the age range (0,100) using these truncated distributions. And finally, survival and mortality functions based on these probability models are obtained.

One of the importance of the study is that the premium coefficients will be decisive in determining the premiums for the risk groups formed for different age groups by insurance companies. In other words, insurance companies will be able to see high or low premium coefficients for the low or high - risk groups according to the selected model and parameters by the composite distribution method discussed in this study, and this will enable them to calculate actuarial premiums for certain ages.

The importance of the study is that it can be provide that more accurate actuarial decisions are made with the new composite mortality laws for the selected age groups. The other composite models can be developed use with the other probability distributions for lifetime. Consequently, it can be tried to demonstrate that these existing composite models or new models better fit insurance data in the insurance area. It is thought that the study will provide an important contribution to the literature with this perspective and the outputs obtained from the study.

## CONFLICTS OF INTEREST

No conflict of interest was declared by the author.

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**Appendix A1. The derivatives for solution of ‘c’**

In this appendix section, the derivative calculations required for the solution of $c$ using the Equations (7) and (8) are given.

The derivatives for the right-truncated lognormal:

$$\text{dfL}\ (\text{x}, \mu, \sigma) = -\frac{e^{-\frac{(-\mu+log[x])^2}{2\sigma^2}}}{\sqrt{2\pi}x^2\sigma\phi[\frac{-\mu+log\ [\![100]\!]}{\sigma}]} - \frac{e^{-\frac{(-\mu+log[x])^2}{2\sigma^2}}(-\mu+log[x])}{\sqrt{2\pi}x^2\sigma^3\phi[\frac{-\mu+log\ [\![100]\!]}{\sigma}]}$$

$$\text{dfL}(\theta, \mu, \sigma) = -\frac{e^{-\frac{(-\mu+\log[\theta])^2}{2\sigma^2}}}{\sqrt{2\pi}\theta^2\sigma\phi\left[\frac{-\mu+log\ [\![100]\!]}{\sigma}\right]} - \frac{e^{-\frac{(-\mu+\text{Log}[\theta])^2}{2\sigma^2}}(-\mu+\log[\theta])}{\sqrt{2\pi}\theta^2\sigma^3\phi\left[\frac{-\mu+log\ [\![100]\!]}{\sigma}\right]}.$$

The derivatives for the right-truncated Pareto 2:

$$\text{dfP}\ (\text{x}, \text{k}, \alpha) = \frac{(\frac{k+x}{k})^{-2-\alpha}(-1-\alpha)\alpha}{(1-(1+\frac{100}{k})^{-\alpha})k^2}$$

$$\text{dfP}(\theta, k, \alpha) = \frac{(-1-\alpha)\alpha\left(\frac{k+\theta}{k}\right)^{-2-\alpha}}{\left(1-\left(1+\frac{100}{k}\right)^{-\alpha}\right)k^2}.$$

By solving one of the Equations (7) or (8), the following solution is obtained for c in Mathematica 10.2

$$c = \frac{e^{\frac{(\mu-log[\theta])^2}{2\sigma^2}}(\frac{100+k}{k})^\alpha\sqrt{\pi}\alpha\theta\sigma\text{Erfc}[\frac{\mu-log[\theta]}{\sqrt{2}\sigma}]}{\sqrt{2}(k+\theta)((\frac{100+k}{k})^\alpha-(\frac{k+\theta}{k})^\alpha)+e^{\frac{(\mu-log[\theta])^2}{2\sigma^2}}(\frac{100+k}{k})^\alpha\sqrt{\pi}\alpha\theta\sigma\text{Erfc}[\frac{\mu-log[\theta]}{\sqrt{2}\sigma}]}.$$

or it can be written as in Equation (24). Here Erfc shows the error function and $\phi$ shows the standard normal distribution function and the relation between them can be written as $\frac{1}{2}\text{Erfc}\left[\frac{\mu-log[x]}{\sqrt{2}\sigma}\right] = \phi\left(\frac{logx-\mu}{\sigma}\right)$.

**Appendix A2. The showing of $c \in (0, 1)$**

By equalization the continuity and differentiability conditions determined by Equations (7) and (8), the following Equation is obtained. Here it is clear that both equations are $0 < c < 1$. For example, let’s show that this is for Equation (7):

$$0 < \frac{f_2(\theta)F_1(\theta)}{f_2(\theta)F_1(\theta)+f_1(\theta)\big(1-F_2(\theta)\big)} < 1$$

where the denominator is positive since $0 < f_1(\theta), f_2(\theta) < 1$ and $0 < F_1(\theta)\,, \big(1-F_2(\theta)\big) < 1$. If both sides of inequality are multiplied by the denominator, $0 < F_2(\theta) < 1$. This requires that c be in the range (0,1) for each selected continuous distribution function.

On the other hand, for the composite distribution used in the study, it can be seen from the graphs that c changes in the range (0,1). To draw these graphics, the following codes were used written in Mathematica 10.2:

$$\text{Manipulate[Plot[}\frac{e^{\frac{(\mu-log[\theta])^2}{2\sigma^2}}\left(\frac{100+k}{k}\right)^{\alpha}\sqrt{\pi}\alpha\theta\sigma\text{Erfc}\left[\frac{\mu-log[\theta]}{\sqrt{2}\sigma}\right]}{\sqrt{2}(k+\theta)\left(\left(\frac{100+k}{k}\right)^{\alpha}-\left(\frac{k+\theta}{k}\right)^{\alpha}\right)+e^{\frac{(\mu-log[\theta])^2}{2\sigma^2}}\left(\frac{100+k}{k}\right)^{\alpha}\sqrt{\pi}\alpha\theta\sigma\text{Erfc}\left[\frac{\mu-log[\theta]}{\sqrt{2}\sigma}\right]}$$

$$,\{\theta,0{,}100\},\text{AxesLabel}\rightarrow\{\theta,c\}],\{k,0{,}100\},\{\alpha,0{,}100\},\{\mu,-20{,}20\},\{\sigma,0{,}10\}]\ .$$

The graph showing the change of c in the range of (0,1) with respect to θ for the parameters (μ = 1, σ = 0.5, k = 5, α = 0.48) used in the study is given in Figure A1. Here, c is 0.999999998214263 for θ = 60. Also, for other parameter values, the change of c in the range of (0,1) with respect to θ can be seen by running the above code similarly. For example; It is seen from Figure A2 for parameters (μ = 2, σ = 6, k = 10, α = 10) and here c is 0.9887846832858257 for θ = 60.

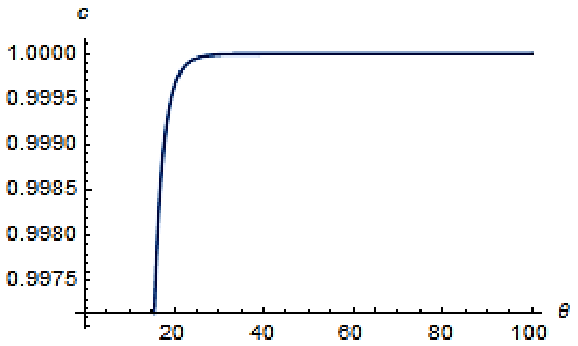


***Figure A1.*** *The change of c with respect to θ* $(\mu=1,\ \sigma=0.5,\ k=5,\ \alpha=0.48)$

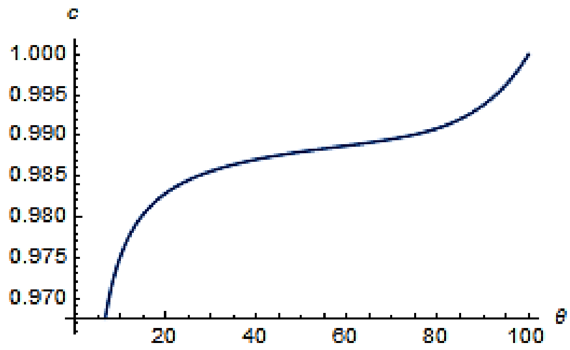


***Figure A2****. The change of c with respect to θ* $(\mu=2,\ \sigma=6,\ k=10,\ \alpha=10)$