GENERATING FUNCTIONS FOR THE BERNSTEIN TYPE POLYNOMIALS: A NEW APPROACH TO DERIVING IDENTITIES AND APPLICATIONS FOR THE POLYNOMIALS

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Abstract

The main aim of this paper is to construct generating functions for the Bernstein type polynomials. Using these generating functions, various functional equations and differential equations can be derived. New proofs both for a recursive definition of the Bernstein type basis functions and for derivatives of the \( n \)th degree Bernstein type polynomials can be given using these equations. This paper presents a novel method for deriving various new identities and properties for the Bernstein type basis functions by using not only these generating functions but also these equations. By applying the Fourier transform and the Laplace transform to the generating functions, we derive interesting series representations for the Bernstein type basis functions. Furthermore, we discuss analytic representations for the generalized Bernstein polynomials through the binomial or Newton distribution and Poisson distribution with mean and variance. By using the mean and the variance, we generalize Szasz-Mirakjan type basis functions.

Keywords: Bernstein polynomials; Generating function; Szasz-Mirakjan basis functions; Bezier curves; Binomial distribution; Poisson distribution; Fourier transform; Laplace transform; Functional equation.

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