ON $BP$-ALGEBRAS

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Abstract

In this paper, we introduce the notion of a $BP$-algebra, and discuss some relations with several algebras. Moreover, we discuss a quadratic $BP$-algebra and show that the quadratic $BP$-algebra is equivalent to several quadratic algebras.

Keywords: $B$-algebra, 0-commutative, $BF$-algebra, $BP$-algebra, $BH$-algebra, (normal) subalgebra.

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1. Introduction

Y. Imai and K. Iséki introduced two classes of abstract algebras: $BCK$-algebras and $BCI$-algebras ([3, 4]). It is known that the class of $BCK$-algebras is a proper subclass of the class of $BCI$-algebras. In [1, 2] Q. P. Hu and X. Li introduced a wide class of abstract algebras: $BCH$-algebras. They have shown that the class of $BCI$-algebras is a proper subclass of the class of $BCH$-algebras. J. Neggers and H. S. Kim ([11]) introduced the notion of $d$-algebras which is another generalization of $BCK$-algebras, and then they investigated several relations between $d$-algebras and $BCK$-algebras as well as some other interesting relations between $d$-algebras and oriented digraphs. Also they introduced the notion of $B$-algebras ([9, 12, 13]), i.e., (I) $x \ast x = e$; (II) $x \ast e = x$; (III) $(x \ast y) \ast z = x \ast (z \ast (e \ast y))$, for any $x, y, z \in X$. A. Walendziak ([14]) obtained another axiomatization of $B$-algebras. Y. B. Jun, E. H. Roh and H. S. Kim ([5]) introduced a new notion, called a $BH$-algebra which is a generalization of $BCH/BC1/BCK$-algebras. A. Walendziak ([15]) introduced a new notion, called an $BF$-algebra, i.e., (I); (II) and (IV) $e \ast (x \ast y) = y \ast x$ for any $x, y \in X$. In ([15]) it was shown that a $BF$-algebra is a generalizations of a $B$-algebra. H. S. Kim and N. R. Kye ([7]) introduced the notion of a quadratic $BF$-algebra, and obtained that quadratic $BF$-algebras, quadratic $Q$-algebras, $BG$-algebras and $B$-algebras are equivalent nations on a field $X$ with $|X| \geq 3$, and hence every quadratic $BF$-algebra is a $BCI$-algebra. In this paper, we introduce the notion of

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