NEW INTEGRAL INEQUALITIES
VIA $(\alpha, m)$-CONVEXITY AND QUASI-CONVEXITY

Wenjun Liu*

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Abstract
In this paper, we establish some new integral inequalities involving Beta function via $(\alpha, m)$-convexity and quasi-convexity, respectively. Our results in special cases recapture known results.

Keywords: Hermite’s inequality, Euler Beta function, Hölder’s inequality, $(\alpha, m)$-convexity, quasi-convexity


1. Introduction
Let $I$ be an interval in $\mathbb{R}$. Then $f : I \rightarrow \mathbb{R}$ is said to be convex (see [17, P.1]) if

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$$

holds for all $x, y \in I$ and $t \in [0, 1]$.

In [27], Toader defined $m$-convexity as follows:

1.1. Definition. The function $f : [0, b] \rightarrow \mathbb{R}$, $b > 0$ is said to be $m$-convex, where $m \in [0, 1]$, if

$$f(tx + m(1-t)y) \leq tf(x) + m(1-t)f(y)$$

holds for all $x, y \in [0, b]$ and $t \in [0, 1]$. We say that $f$ is $m$–concave if $-f$ is $m$–convex.

In [18], Mihešan defined $(\alpha, m)$– convexity as follows:

1.2. Definition. The function $f : [0, b] \rightarrow \mathbb{R}$, $b > 0$, is said to be $(\alpha, m)$– convex, where $(\alpha, m) \in [0, 1]^2$, if

$$f(tx + m(1-t)y) \leq t^\alpha f(x) + m(1-t)^\alpha f(y)$$

holds for all $x, y \in [0, b]$ and $t \in [0, 1]$.

*College of Mathematics and Statistics, Nanjing University of Information Science and Technology, Nanjing 210044, China E-mail: wjliu@nuist.edu.cn